B.
Pattern Formation

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Differentiation & Pattern Formation

- A central problem in development: How do cells differentiate to fulfill different purposes?
- How do complex systems generate spatial & temporal structure?
- CAs are natural models of intercellular communication

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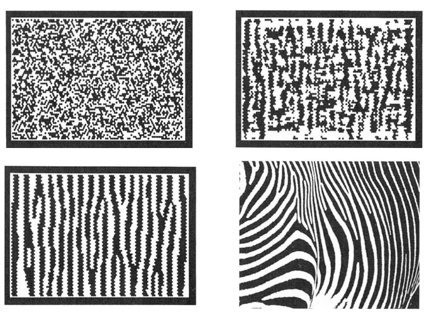
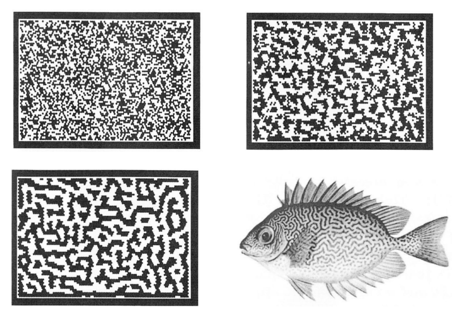
Zebra



9/8/09 figs. from Camazine & al.: *Self-Org. Biol. Sys.* 3

Vermiculated Rabbit Fish



9/8/09 figs. from Camazine & al.: *Self-Org. Biol. Sys.* 4

Activation & Inhibition in Pattern Formation

- Color patterns typically have a characteristic length scale
- Independent of cell size and animal size
- Achieved by:
  - short-range activation ⇒ local uniformity
  - long-range inhibition ⇒ separation

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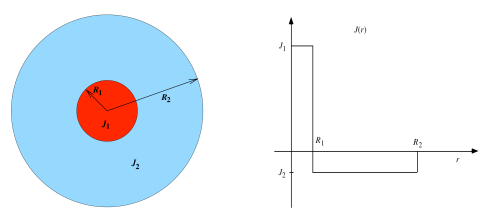
Interaction Parameters



- R_1 and R_2 are the interaction ranges
- J_1 and J_2 are the interaction strengths

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CA Activation/Inhibition Model

- Let states $s_i \in \{-1, +1\}$
- and h be a bias parameter
- and r_{ij} be the distance between cells i and j
- Then the state update rule is:

$$s_i(t+1) = \text{sign}\left[h + J_1 \sum_{r_{ij} < R_1} s_j(t) + J_2 \sum_{R_1 \le r_{ij} < R_2} s_j(t)\right]$$

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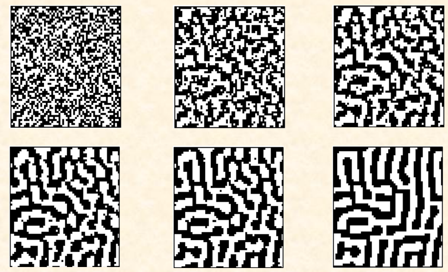
Example

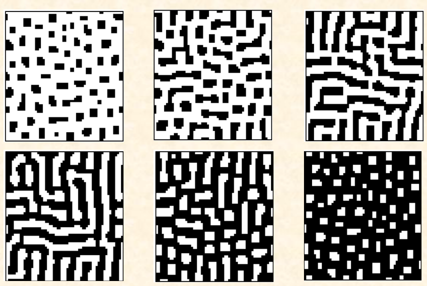
(R_1=1, R_2=6, J_1=1, J_2=–0.1, h=0)



9/8/09 figs. from Bar-Yam 8

Effect of Bias

(h = –6, –3, –1; 1, 3, 6)

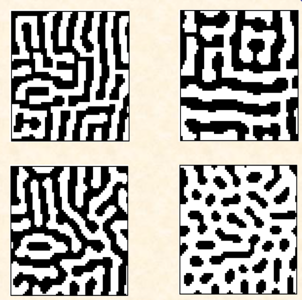


9/8/09 figs. from Bar-Yam 9

Effect of Interaction Ranges

$R_2 = 6$, $R_1 = 1$, $h = 0$

$R_2 = 8$, $R_1 = 1$, $h = 0$



$R_2 = 6$, $R_1 = 1.5$, $h = 0$

$R_2 = 6$, $R_1 = 1.5$, $h = -3$

9/8/09 figs. from Bar-Yam 10

Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation: Fur

RunAICA.nlogo

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Differential Interaction Ranges

- How can a system using strictly local interactions discriminate between states at long and short range?
- E.g. cells in developing organism
- Can use two different *morphogens* diffusing at two different rates
  - activator diffuses slowly (short range)
  - inhibitor diffuses rapidly (long range)

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Digression on Diffusion

- Simple 2-D diffusion equation:

$$\dot{A}(x,y) = c\nabla^2 A(x,y)$$

- Recall the 2-D Laplacian:

$$\nabla^2 A(x,y) = \frac{\partial^2 A(x,y)}{\partial x^2} + \frac{\partial^2 A(x,y)}{\partial y^2}$$

- The Laplacian (like 2nd derivative) is:
  - positive in a local minimum
  - negative in a local maximum

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Reaction-Diffusion System

diffusion

$$\frac{\partial A}{\partial t} = d_A \nabla^2 A + f_A(A,I)$$
$$\frac{\partial I}{\partial t} = d_I \nabla^2 I + f_I(A,I)$$

reaction

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ I \end{pmatrix} = \begin{pmatrix} d_A & 0 \\ 0 & d_I \end{pmatrix}\begin{pmatrix} \nabla^2 A \\ \nabla^2 I \end{pmatrix} + \begin{pmatrix} f_A(A,I) \\ f_I(A,I) \end{pmatrix}$$

$$\dot{\mathbf{c}} = \mathbf{D}\nabla^2 \mathbf{c} + \mathbf{f}(\mathbf{c}), \text{ where } \mathbf{c} = \begin{pmatrix} A \\ I \end{pmatrix}$$

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Example: Activation-Inhibition System

- Let σ be some kind of threshold function
- Activator A and inhibitor I may diffuse at different rates in x and y directions
- Cell is "on" if activator + bias exceeds inhibitor

$$\frac{\partial A}{\partial t} = d_{Ax}\frac{\partial^2 A}{\partial x^2} + d_{Ay}\frac{\partial^2 A}{\partial y^2} + k_A \sigma(A + B - I)A$$
$$\frac{\partial I}{\partial t} = d_{Ix}\frac{\partial^2 I}{\partial x^2} + d_{Iy}\frac{\partial^2 I}{\partial y^2} + k_I \sigma(A + B - I)I$$

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NetLogo Simulation of Reaction-Diffusion System

1. Diffuse activator in X and Y directions
2. Diffuse inhibitor in X and Y directions
3. Each patch performs:

stimulation = bias + activator – inhibitor + noise
if stimulation > 0 then
    set activator and inhibitor to 100
else
    set activator and inhibitor to 0

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Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation

Run Pattern.nlogo

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Turing Patterns

- Alan Turing studied the mathematics of reaction-diffusion systems
- Turing, A. (1952). The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society* **B 237**: 37–72.
- The resulting patterns are known as *Turing patterns*

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Abstract Activation/Inhibition Spaces

- Consider two axes of cultural preference
  - E.g. hair length & interpersonal distance
  - Fictitious example!
- Suppose there are no objective reasons for preferences
- Suppose people approve/encourage those with similar preferences
- Suppose people disapprove/discourage those with different preferences
- What is the result?

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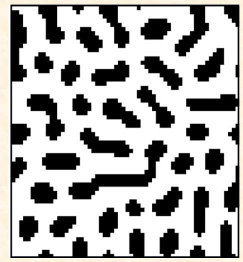
Emergent Regions of Acceptable Variation



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A Key Element of Self-Organization

- Activation vs. Inhibition
- Cooperation vs. Competition
- Amplification vs. Stabilization
- Growth vs. Limit
- Positive Feedback vs. Negative Feedback
  - Positive feedback creates
  - Negative feedback shapes

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Reaction-Diffusion Computing

- Has been used for image processing
  - diffusion ⇒ noise filtering
  - reaction ⇒ contrast enhancement
- Depending on parameters, RD computing can:
  - restore broken contours
  - detect edges
  - improve contrast

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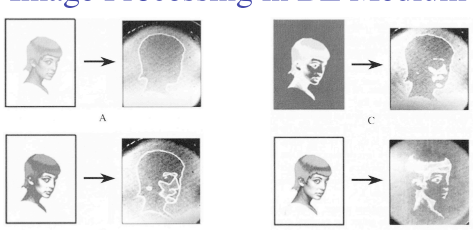
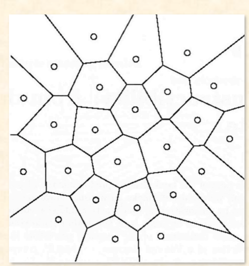
Image Processing in BZ Medium



- (A) boundary detection, (B) contour enhancement, (C) shape enhancement, (D) feature enhancement

9/8/09 Image < Adamatzky, *Comp. in Nonlinear Media & Autom. Coll.* 23

Voronoi Diagrams



- Given a set of generating points:
- Construct polygon around each gen. point of set, so all points in poly. are closer to its generating point than to any other generating points.

9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 24

Some Uses of Voronoi Diagrams

- Collision-free path planning
- Determination of service areas for power substations
- Nearest-neighbor pattern classification
- Determination of largest empty figure

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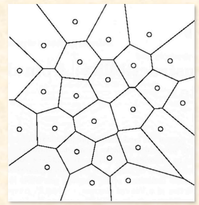
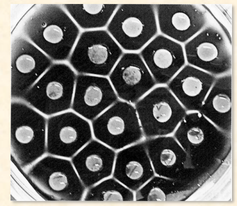
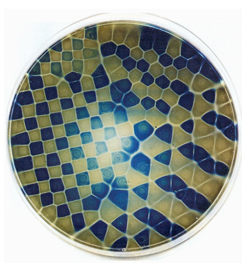
Computation of Voronoi Diagram by Reaction-Diffusion Processor





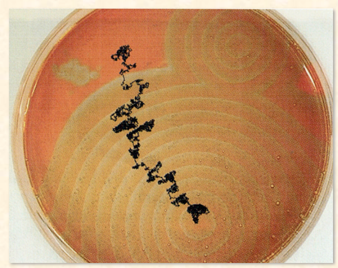
9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 26

Mixed Cell Voronoi Diagram



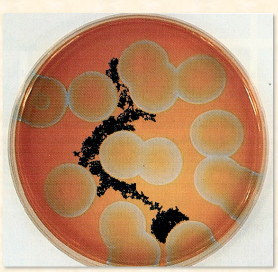
9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 27

Path Planning via BZ medium: No Obstacles



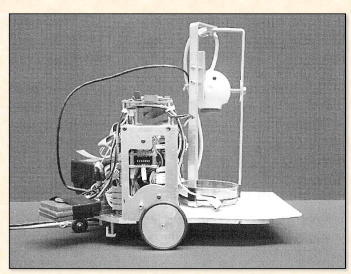
9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 28

Path Planning via BZ medium: Circular Obstacles



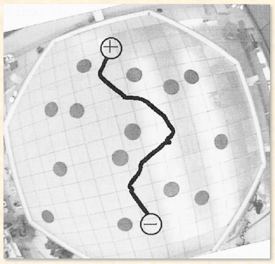
9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 29

Mobile Robot with Onboard Chemical Reactor



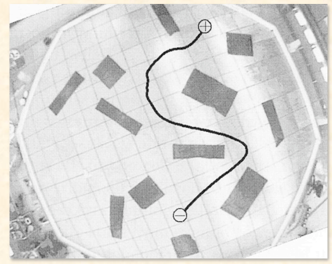
9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 30

Actual Path: Pd Processor



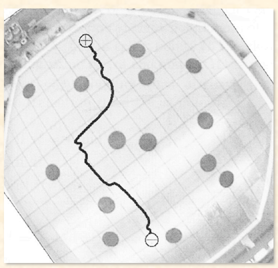
9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 31

Actual Path: Pd Processor



9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 32

Actual Path: BZ Processor



9/8/09 Image < Adamatzky & al., *Reaction-Diffusion Computers* 33

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Segmentation

(in embryological development)

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Vertebrae

- Humans: 33, chickens: 55, mice: 65, corn snake: 315
- Characteristic of species
- How does an embryo "count" them?
- "Clock and wavefront model" of Cooke & Zeeman (1976).

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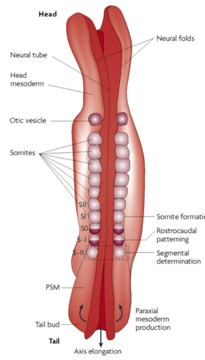

Nature Reviews | Genetics

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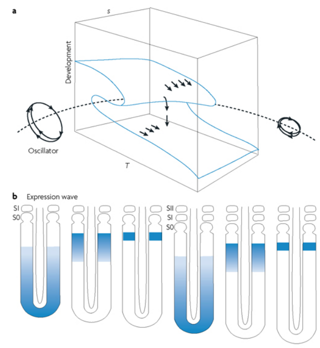

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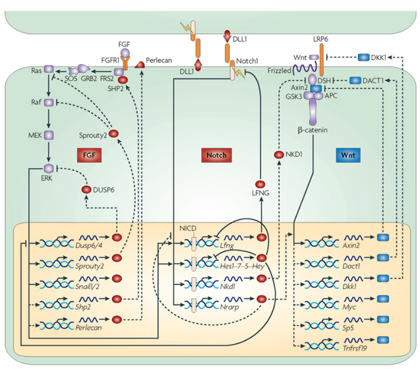

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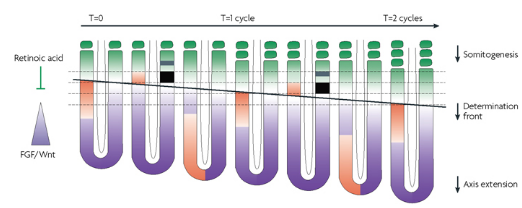

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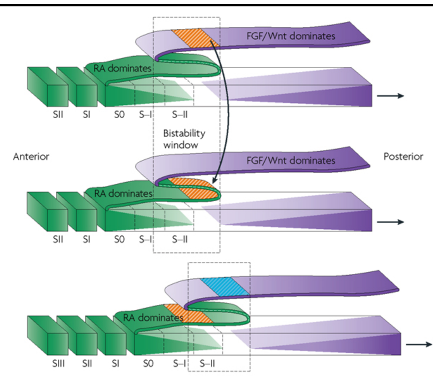

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NetLogo Simulation of Segmentation

Run Segmentation.nlogo

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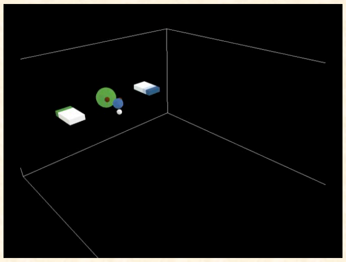
Simulated Segmentation by Clock-and-Wavefront Process



9/8/09 Run Segmentation-cells-3D.nlogo 43

2D Simulation of Clock-and-Wavefront Process

9/8/09 Run Segmentation-cells.nlogo 44

Effect of Growth Rate

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