

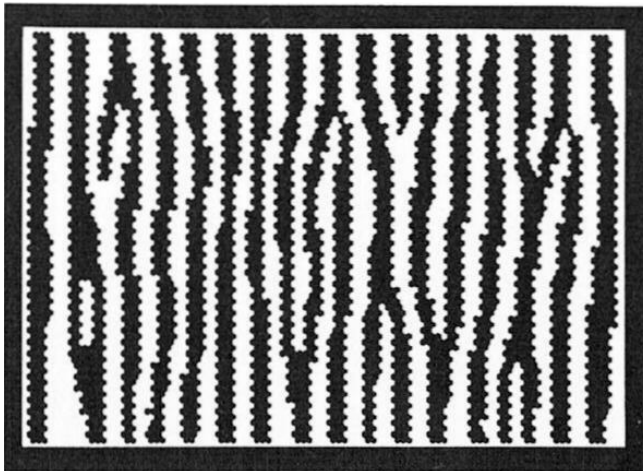
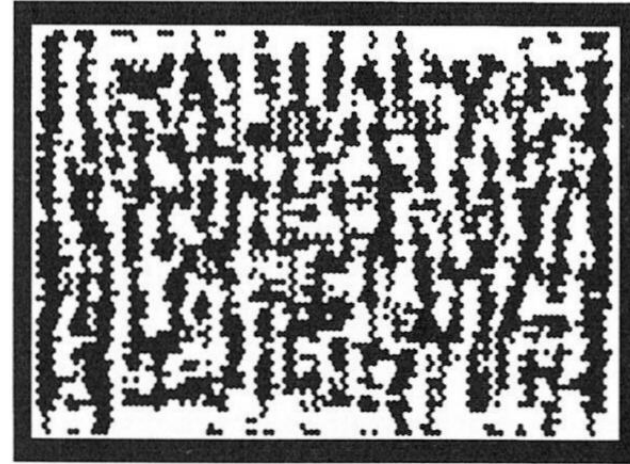
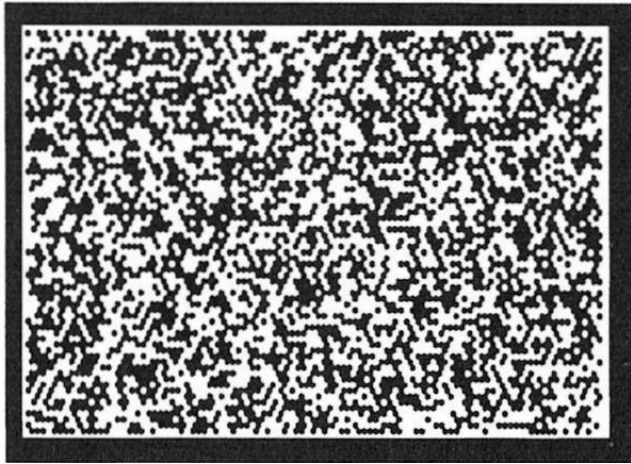
B. Pattern Formation

Differentiation & Pattern Formation

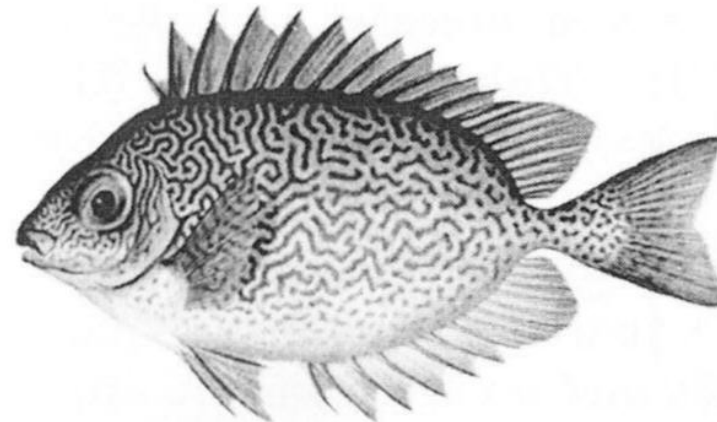
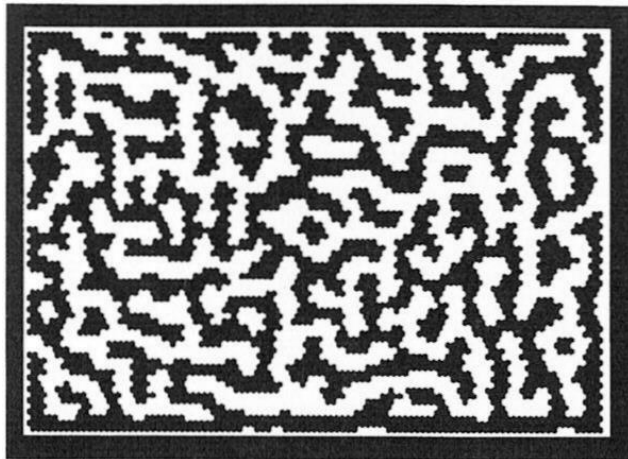
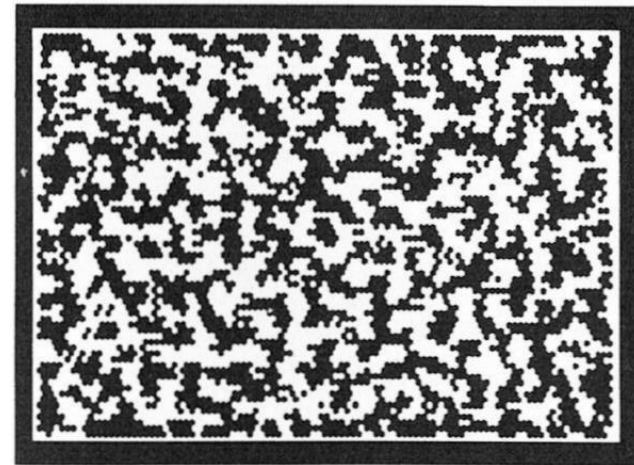
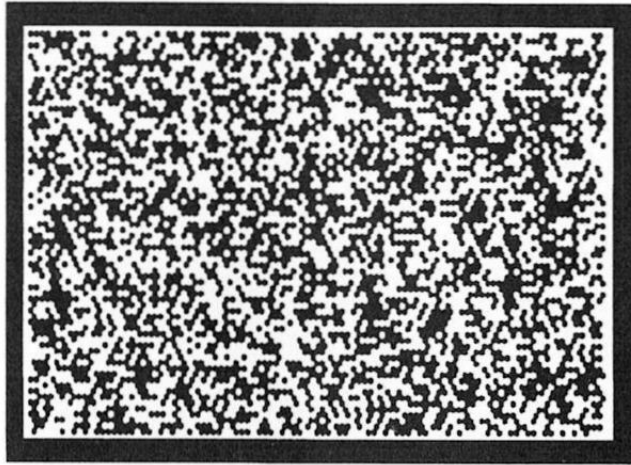


- A central problem in development: How do cells differentiate to fulfill different purposes?
- How do complex systems generate spatial & temporal structure?
- CAs are natural models of intercellular communication

Zebra



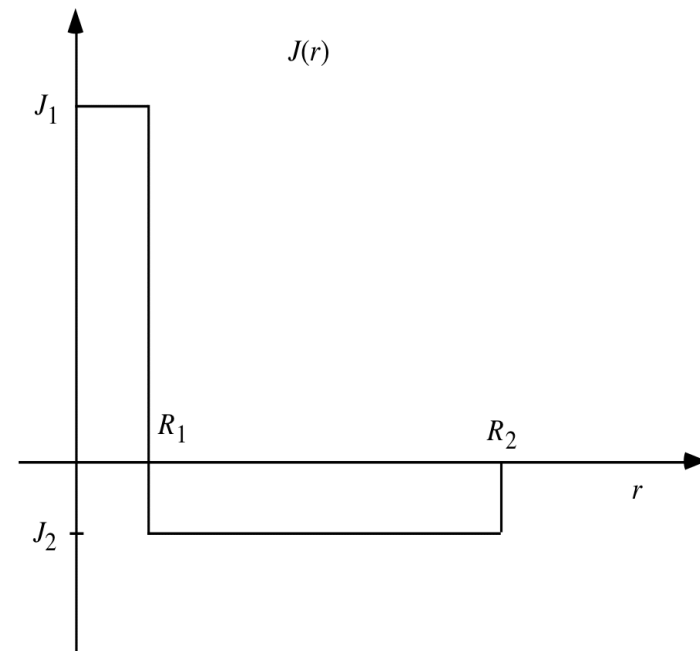
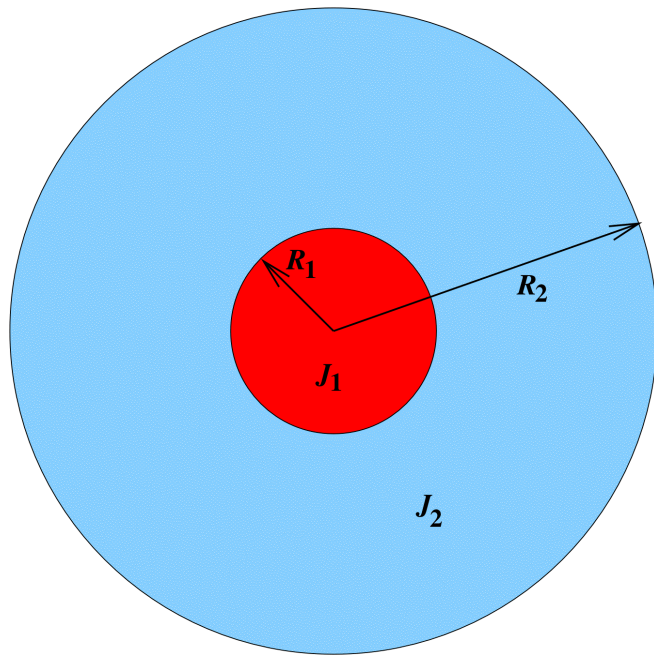
Vermiculated Rabbit Fish



Activation & Inhibition in Pattern Formation

- Color patterns typically have a characteristic length scale
- Independent of cell size and animal size
- Achieved by:
 - short-range activation \Rightarrow local uniformity
 - long-range inhibition \Rightarrow separation

Interaction Parameters



- R_1 and R_2 are the interaction ranges
- J_1 and J_2 are the interaction strengths

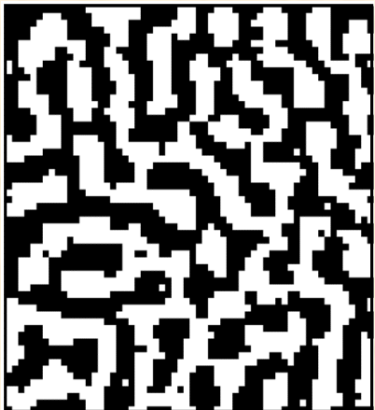
CA Activation/Inhibition Model

- Let states $s_i \in \{-1, +1\}$
- and h be a bias parameter
- and r_{ij} be the distance between cells i and j
- Then the state update rule is:

$$s_i(t+1) = \text{sign} \left[h + J_1 \sum_{r_{ij} < R_1} s_j(t) + J_2 \sum_{R_1 \leq r_{ij} < R_2} s_j(t) \right]$$

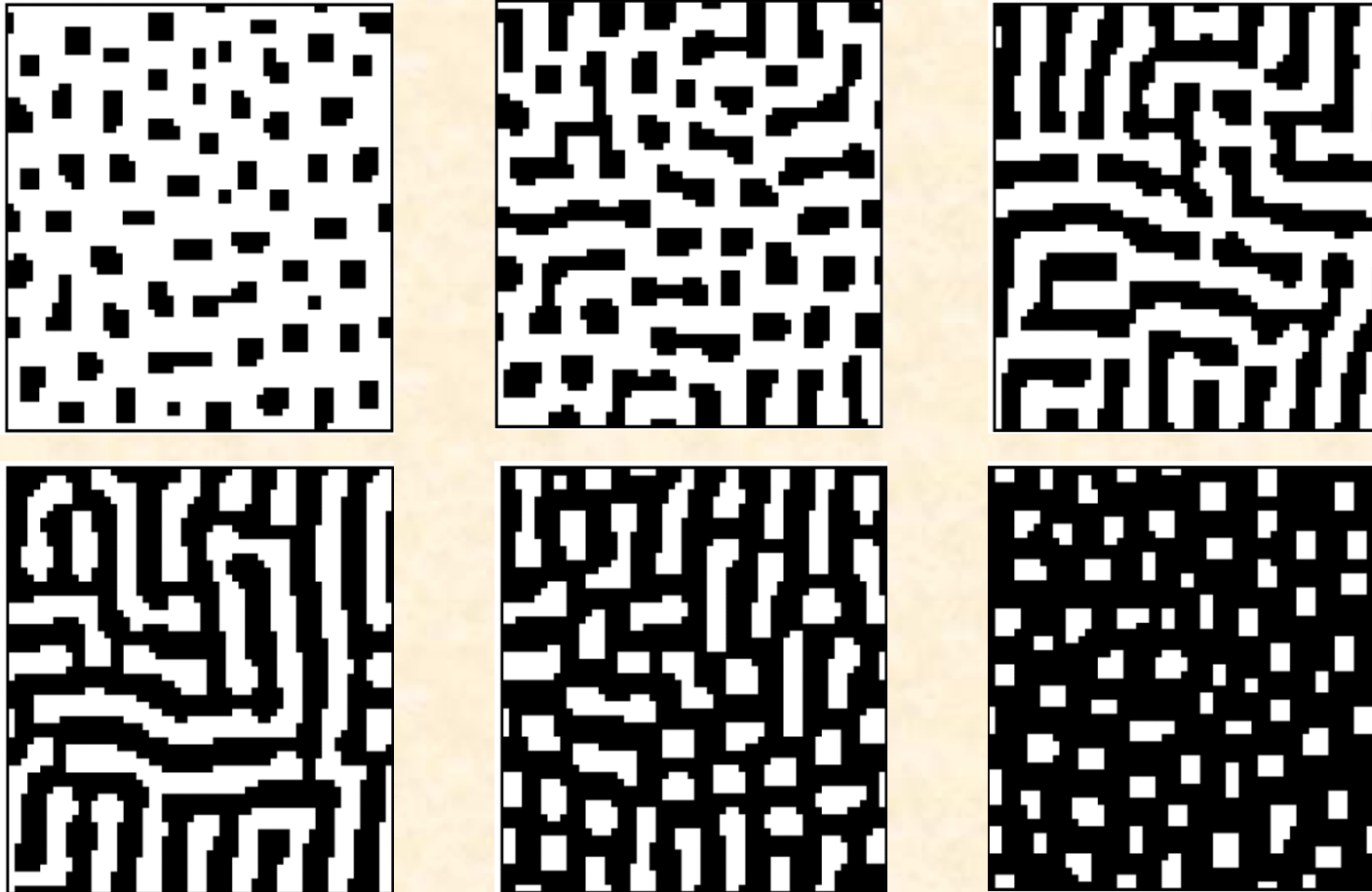
Example

$$(R_1=1, R_2=6, J_1=1, J_2=-0.1, h=0)$$



Effect of Bias

$(h = -6, -3, -1; 1, 3, 6)$



Effect of Interaction Ranges

$$\begin{aligned} R_2 &= 6 \\ R_1 &= 1 \\ h &= 0 \end{aligned}$$



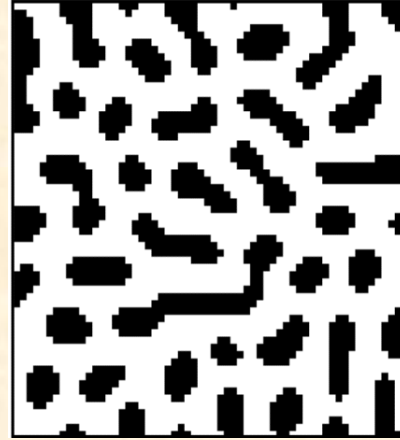
$$\begin{aligned} R_2 &= 8 \\ R_1 &= 1 \\ h &= 0 \end{aligned}$$



$$\begin{aligned} R_2 &= 6 \\ R_1 &= 1.5 \\ h &= 0 \end{aligned}$$



$$\begin{aligned} R_2 &= 6 \\ R_1 &= 1.5 \\ h &= -3 \end{aligned}$$



Demonstration of NetLogo
Program for Activation/Inhibition
Pattern Formation:
Fur

[RunAICA.nlogo](#)

Differential Interaction Ranges

- How can a system using strictly local interactions discriminate between states at long and short range?
- E.g. cells in developing organism
- Can use two different *morphogens* diffusing at two different rates
 - activator diffuses slowly (short range)
 - inhibitor diffuses rapidly (long range)

Digression on Diffusion

- Simple 2-D diffusion equation:

$$\dot{A}(x, y) = c \nabla^2 A(x, y)$$

- Recall the 2-D Laplacian:

$$\nabla^2 A(x, y) = \frac{\partial^2 A(x, y)}{\partial x^2} + \frac{\partial^2 A(x, y)}{\partial y^2}$$

- The Laplacian (like 2nd derivative) is:
 - positive in a local minimum
 - negative in a local maximum

Reaction-Diffusion System

diffusion

$$\begin{aligned}\frac{\partial A}{\partial t} &= d_A \nabla^2 A + f_A(A, I) \\ \frac{\partial I}{\partial t} &= d_I \nabla^2 I + f_I(A, I)\end{aligned}$$

reaction

$$\frac{\partial}{\partial t} \begin{pmatrix} A \\ I \end{pmatrix} = \begin{pmatrix} d_A & 0 \\ 0 & d_I \end{pmatrix} \begin{pmatrix} \nabla^2 A \\ \nabla^2 I \end{pmatrix} + \begin{pmatrix} f_A(A, I) \\ f_I(A, I) \end{pmatrix}$$

$$\dot{\mathbf{c}} = \mathbf{D} \nabla^2 \mathbf{c} + \mathbf{f}(\mathbf{c}), \quad \text{where } \mathbf{c} = \begin{pmatrix} A \\ I \end{pmatrix}$$

Continuous-time Activator-Inhibitor System

- Activator A and inhibitor I may diffuse at different rates in x and y directions
- Cell becomes more active if activator + bias exceeds inhibitor
- Otherwise, less active

$$\frac{\partial A}{\partial t} = d_{Ax} \frac{\partial^2 A}{\partial x^2} + d_{Ay} \frac{\partial^2 A}{\partial y^2} + k_A(A + B - I)$$

$$\frac{\partial I}{\partial t} = d_{Ix} \frac{\partial^2 I}{\partial x^2} + d_{Iy} \frac{\partial^2 I}{\partial y^2} + k_I(A + B - I)$$

NetLogo Simulation of Reaction-Diffusion System

1. Diffuse activator in X and Y directions
2. Diffuse inhibitor in X and Y directions
3. Each patch performs:
stimulation = bias + activator – inhibitor + noise
if stimulation > 0 then
 set activator and inhibitor to 100
else
 set activator and inhibitor to 0

Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation

[Run Pattern.nlogo](#)

Demonstration of NetLogo
Program for Activator/Inhibitor
Pattern Formation
with Continuous State Change

[Run Activator-Inhibitor.nlogo](#)

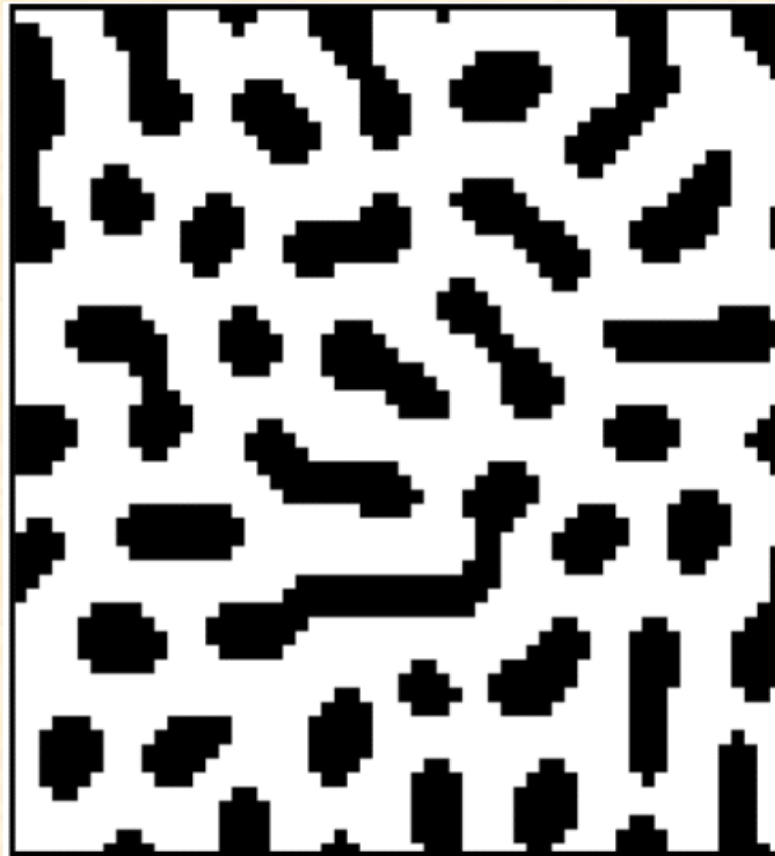
Turing Patterns

- Alan Turing studied the mathematics of reaction-diffusion systems
- Turing, A. (1952). The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society* **B 237**: 37–72.
- The resulting patterns are known as *Turing patterns*

Abstract Activation/Inhibition Spaces

- Consider two axes of cultural preference
 - E.g. hair length & interpersonal distance
 - Fictitious example!
- Suppose there are no objective reasons for preferences
- Suppose people approve/encourage those with similar preferences
- Suppose people disapprove/discourage those with different preferences
- What is the result?

Emergent Regions of Acceptable Variation



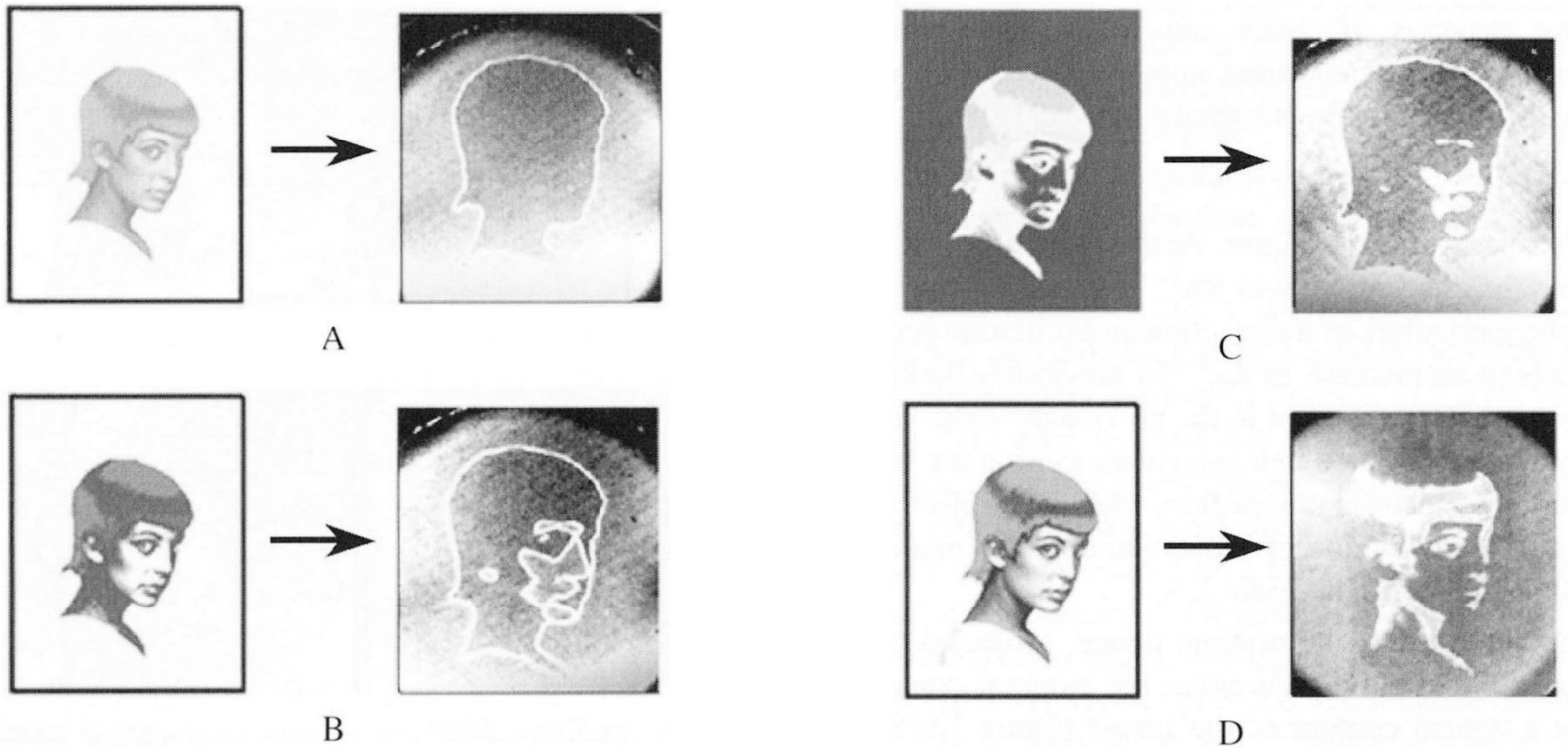
A Key Element of Self-Organization

- Activation vs. Inhibition
- Cooperation vs. Competition
- Amplification vs. Stabilization
- Growth vs. Limit
- Positive Feedback vs. Negative Feedback
 - Positive feedback creates
 - Negative feedback shapes

Reaction-Diffusion Computing

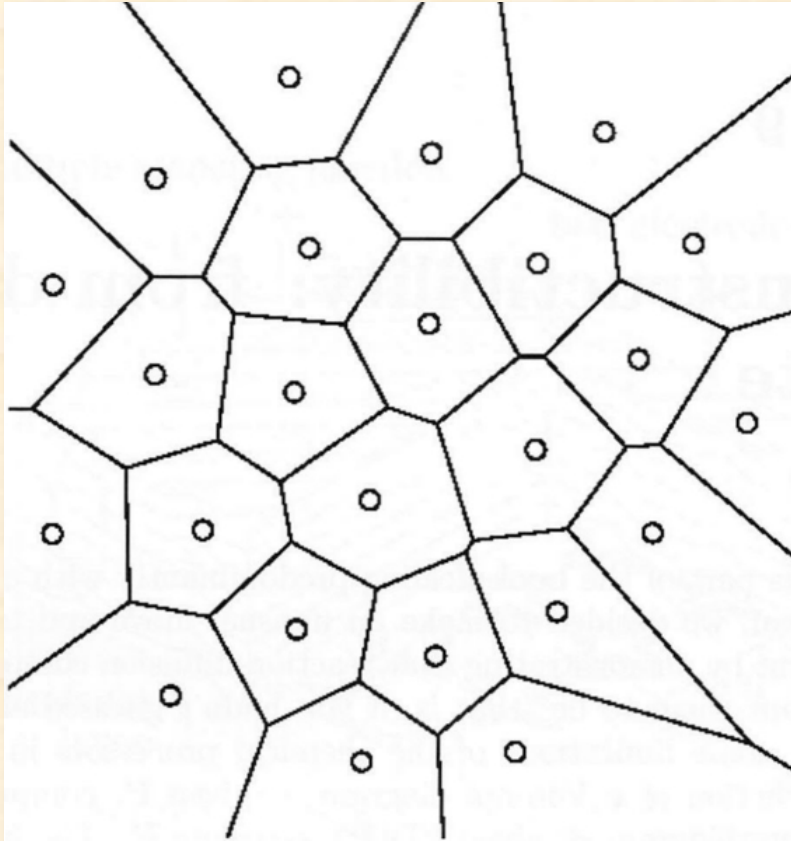
- Has been used for image processing
 - diffusion \Rightarrow noise filtering
 - reaction \Rightarrow contrast enhancement
- Depending on parameters, RD computing can:
 - restore broken contours
 - detect edges
 - improve contrast

Image Processing in BZ Medium



- (A) boundary detection, (B) contour enhancement, (C) shape enhancement, (D) feature enhancement

Voronoi Diagrams

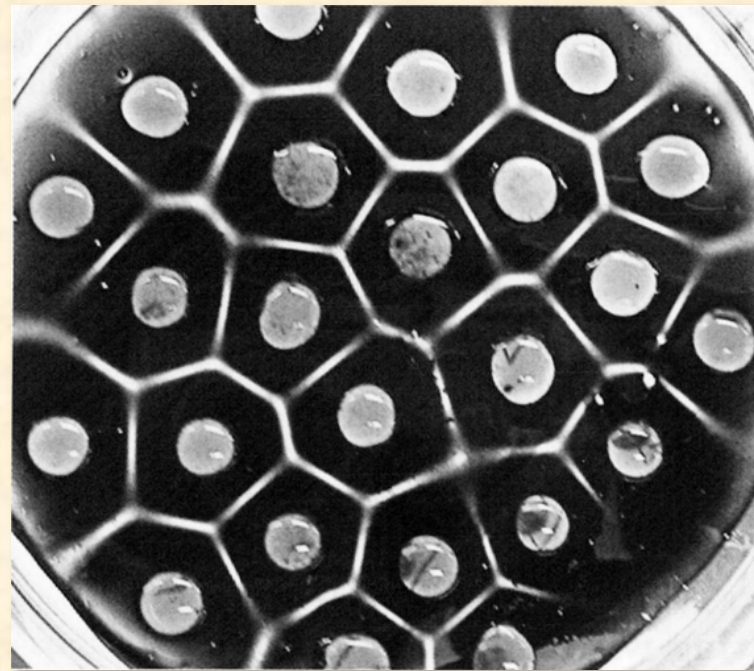
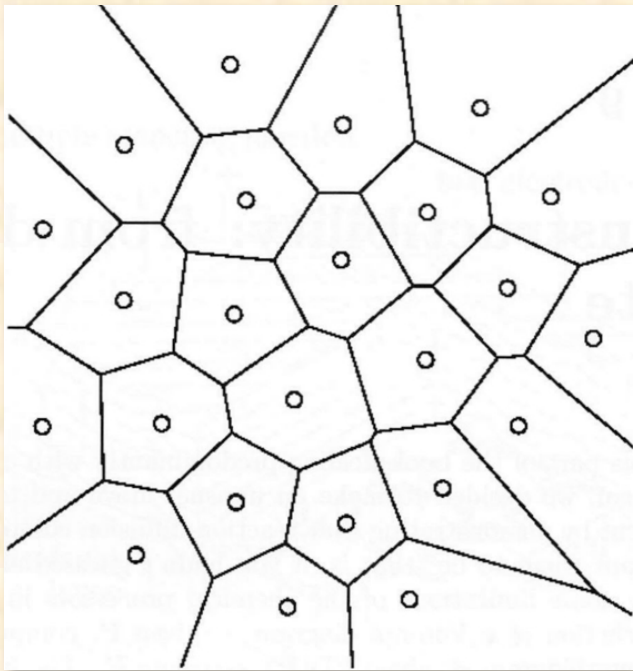


- Given a set of generating points:
- Construct a polygon around each generating point of set, so all points in a polygon are closer to its generating point than to any other generating points.

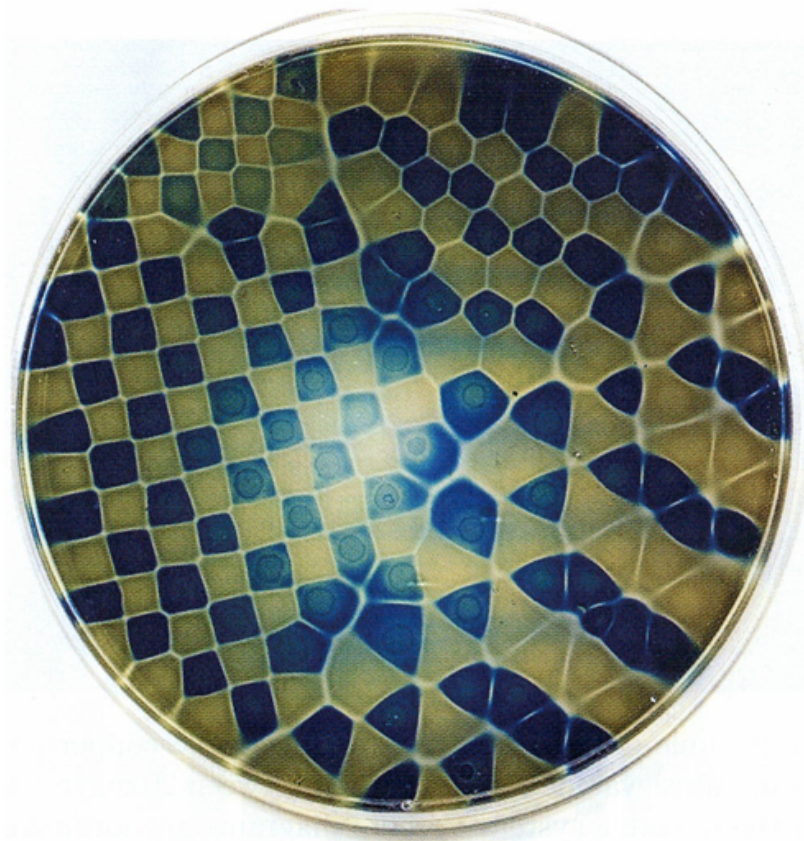
Some Uses of Voronoi Diagrams

- Collision-free path planning
- Determination of service areas for power substations
- Nearest-neighbor pattern classification
- Determination of largest empty figure

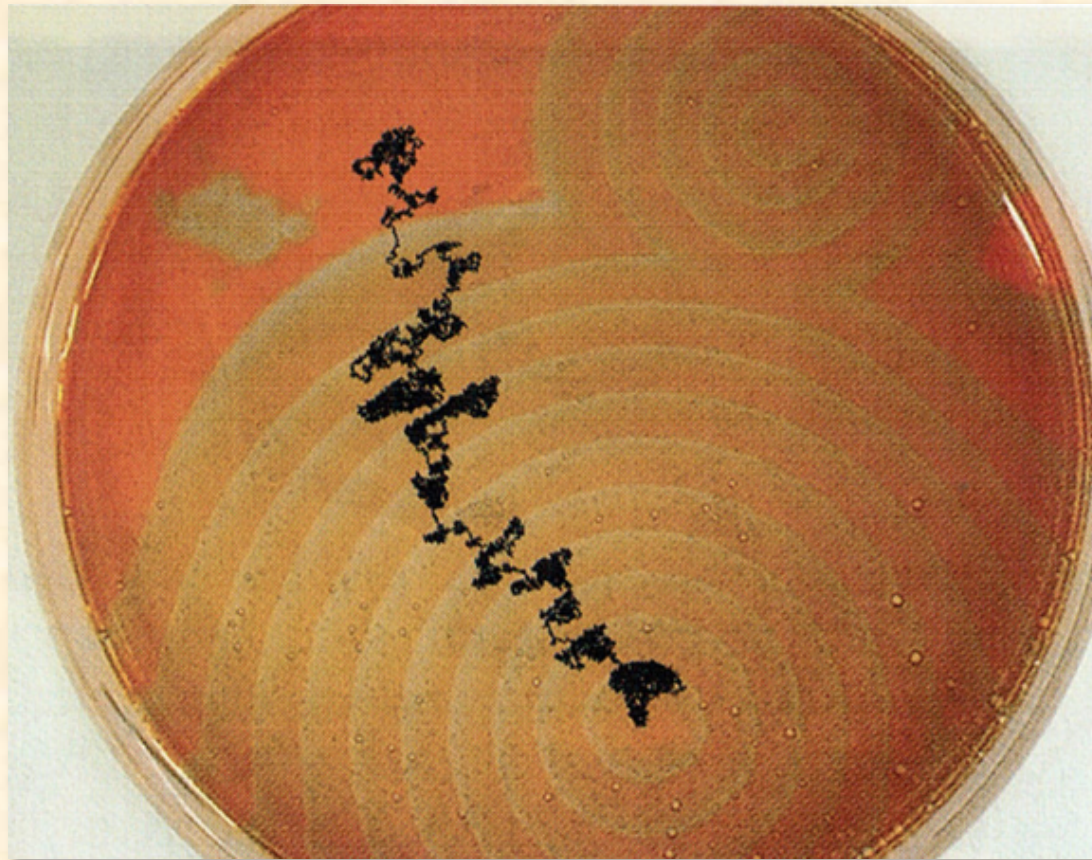
Computation of Voronoi Diagram by Reaction-Diffusion Processor



Mixed Cell Voronoi Diagram



Path Planning via BZ medium: No Obstacles

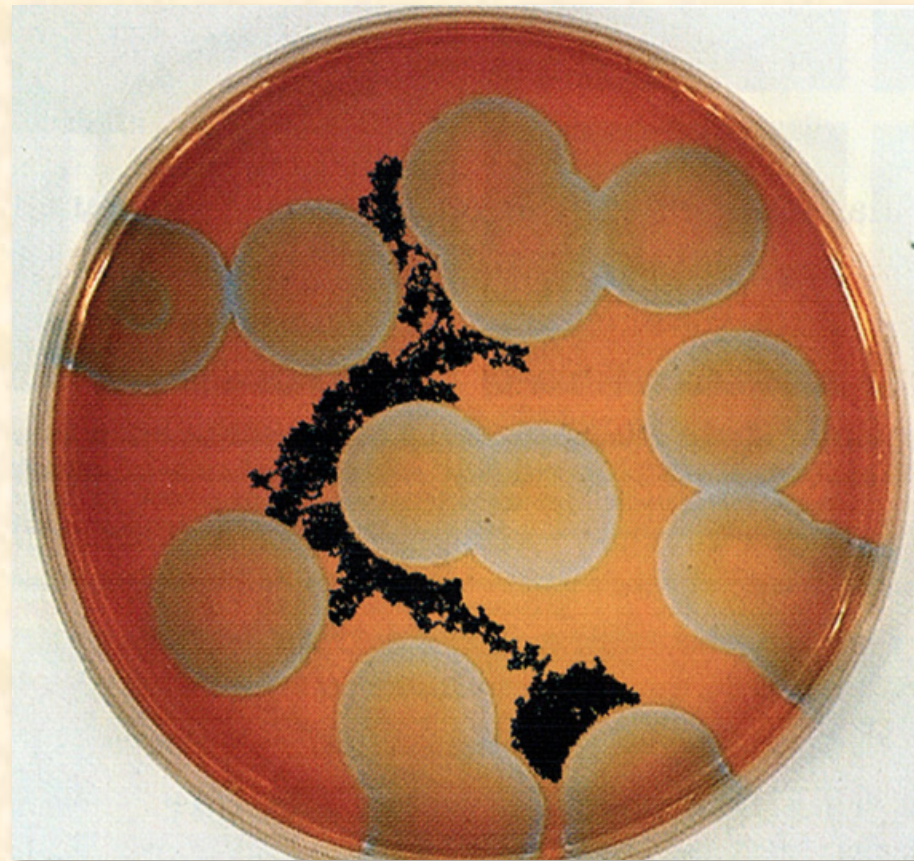


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Image < Adamatzky & al., *Reaction-Diffusion Computers*

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Path Planning via BZ medium: Circular Obstacles

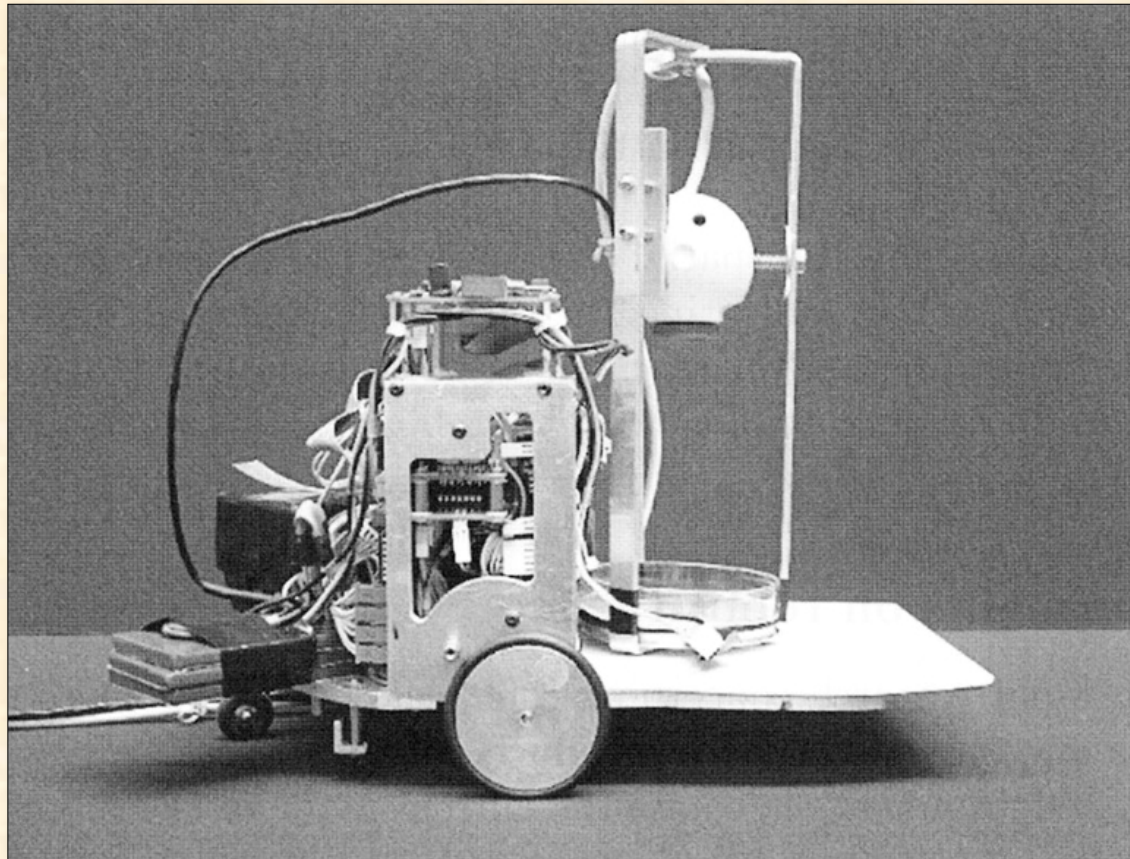


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Image < Adamatzky & al., *Reaction-Diffusion Computers*

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Mobile Robot with Onboard Chemical Reactor

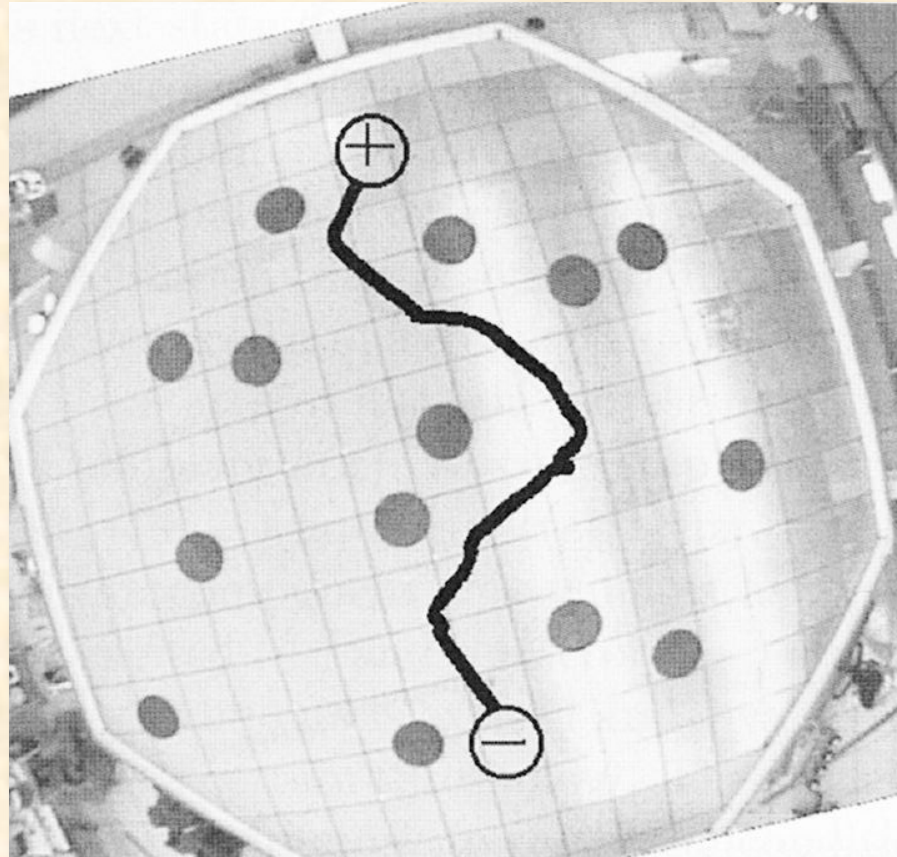


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Image < Adamatzky & al., *Reaction-Diffusion Computers*

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Actual Path: Pd Processor

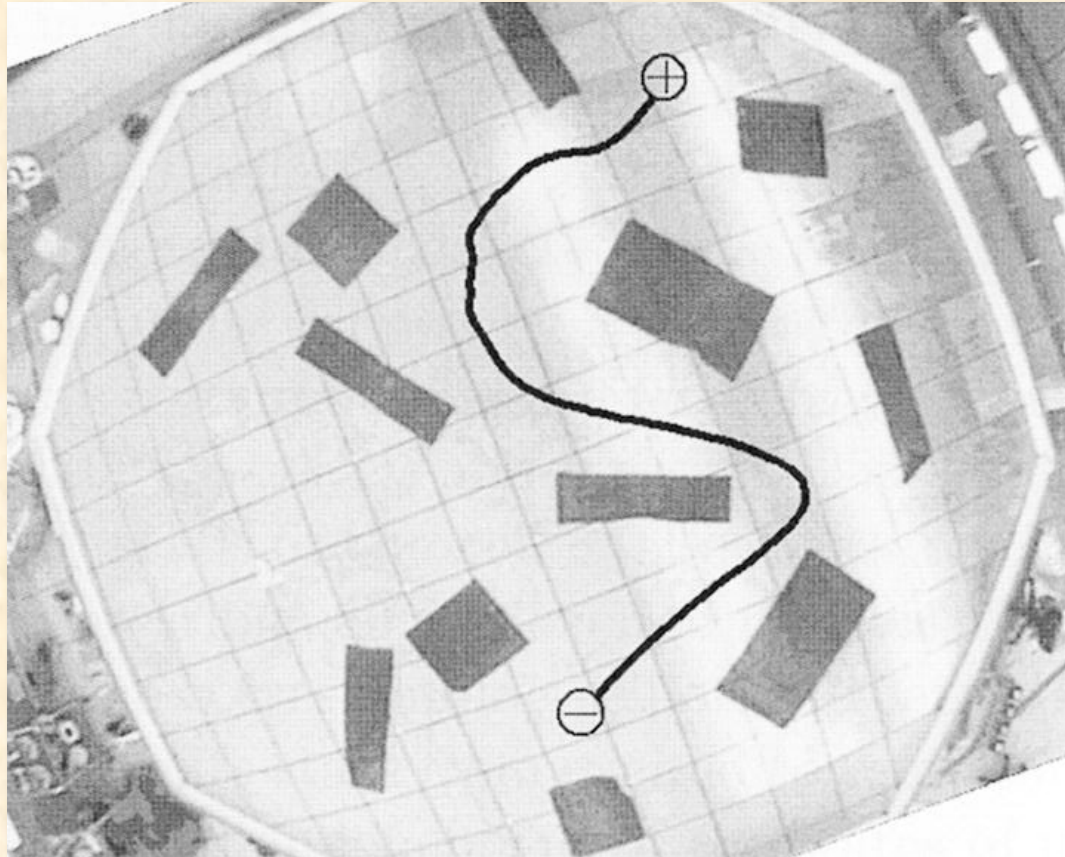


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Image < Adamatzky & al., *Reaction-Diffusion Computers*

32

Actual Path: Pd Processor

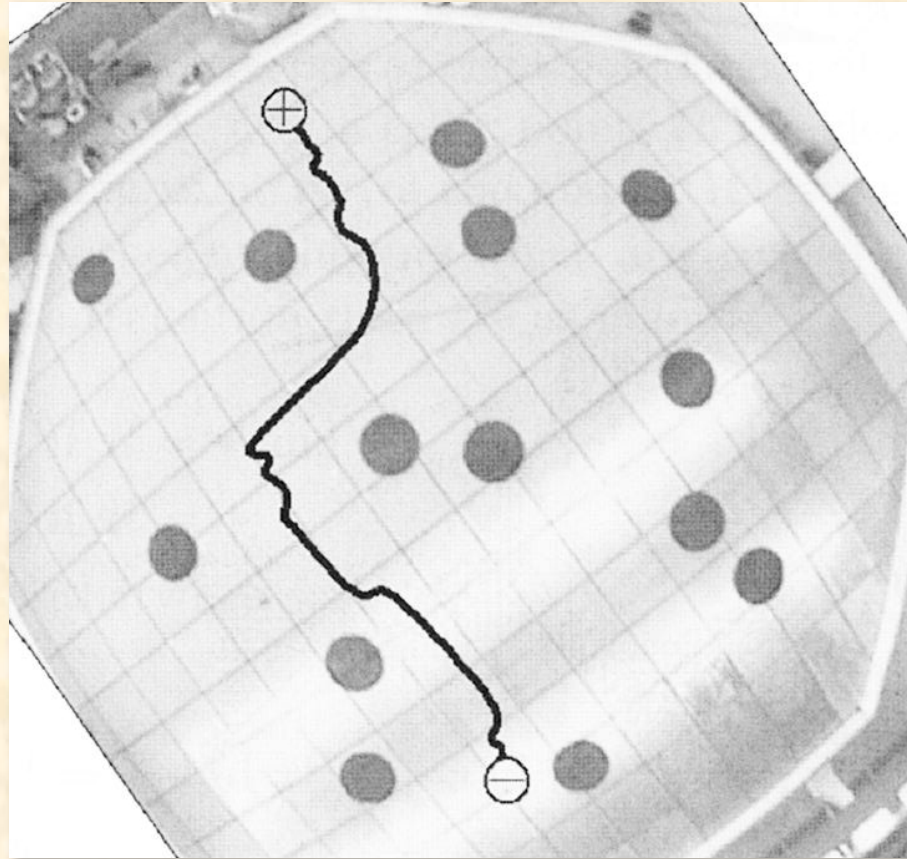


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Image < Adamatzky & al., *Reaction-Diffusion Computers*

33

Actual Path: BZ Processor



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Image < Adamatzky & al., *Reaction-Diffusion Computers*

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Bibliography for Reaction-Diffusion Computing

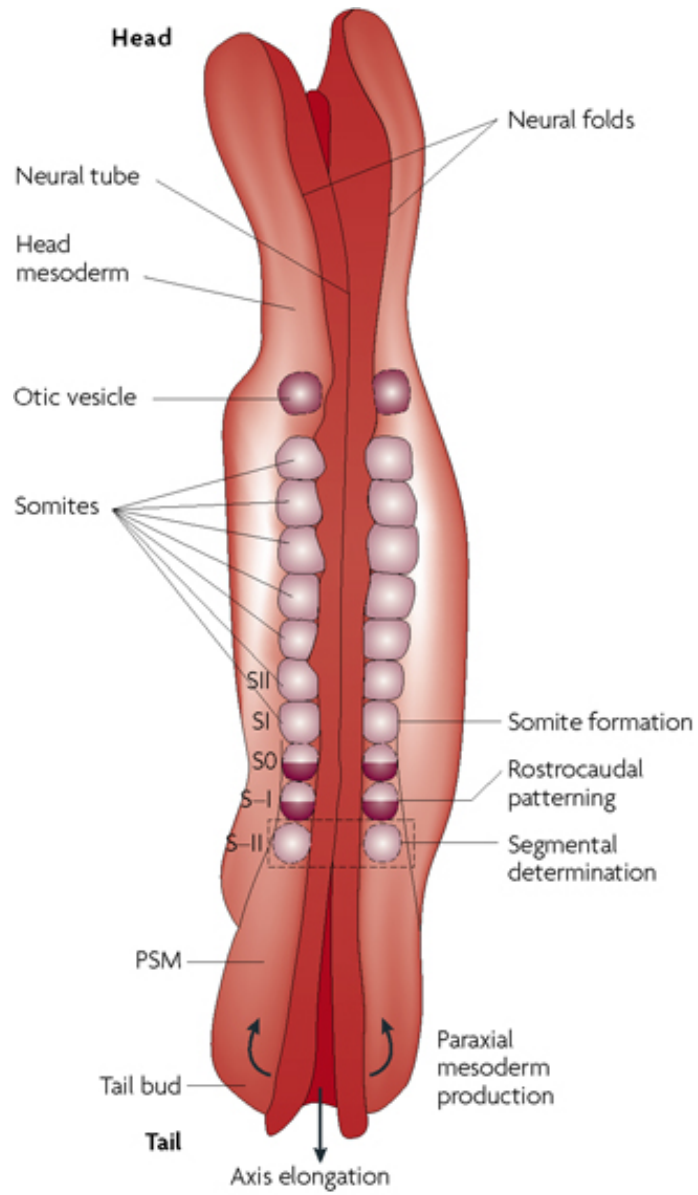
1. Adamatzky, Adam. *Computing in Nonlinear Media and Automata Collectives*. Bristol: Inst. of Physics Publ., 2001.
2. Adamatzky, Adam, De Lacy Costello, Ben, & Asai, Tetsuya. *Reaction Diffusion Computers*. Amsterdam: Elsevier, 2005.

Segmentation

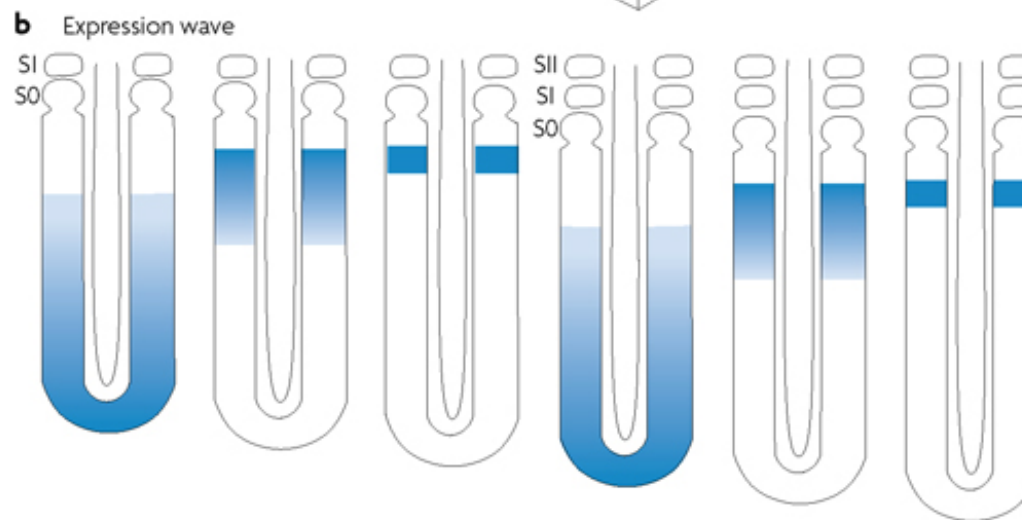
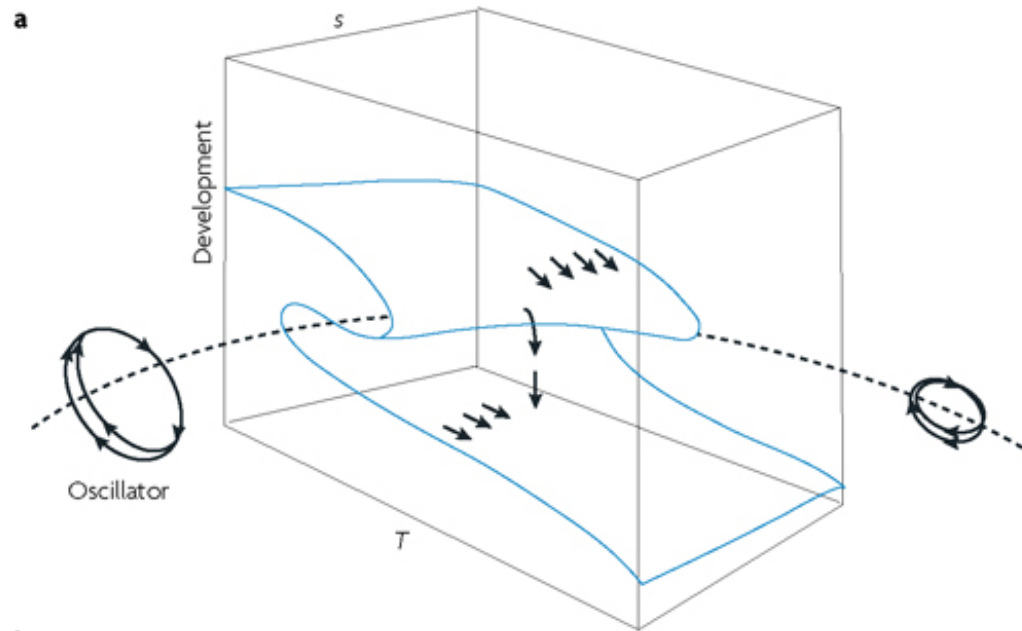
(in embryological development)

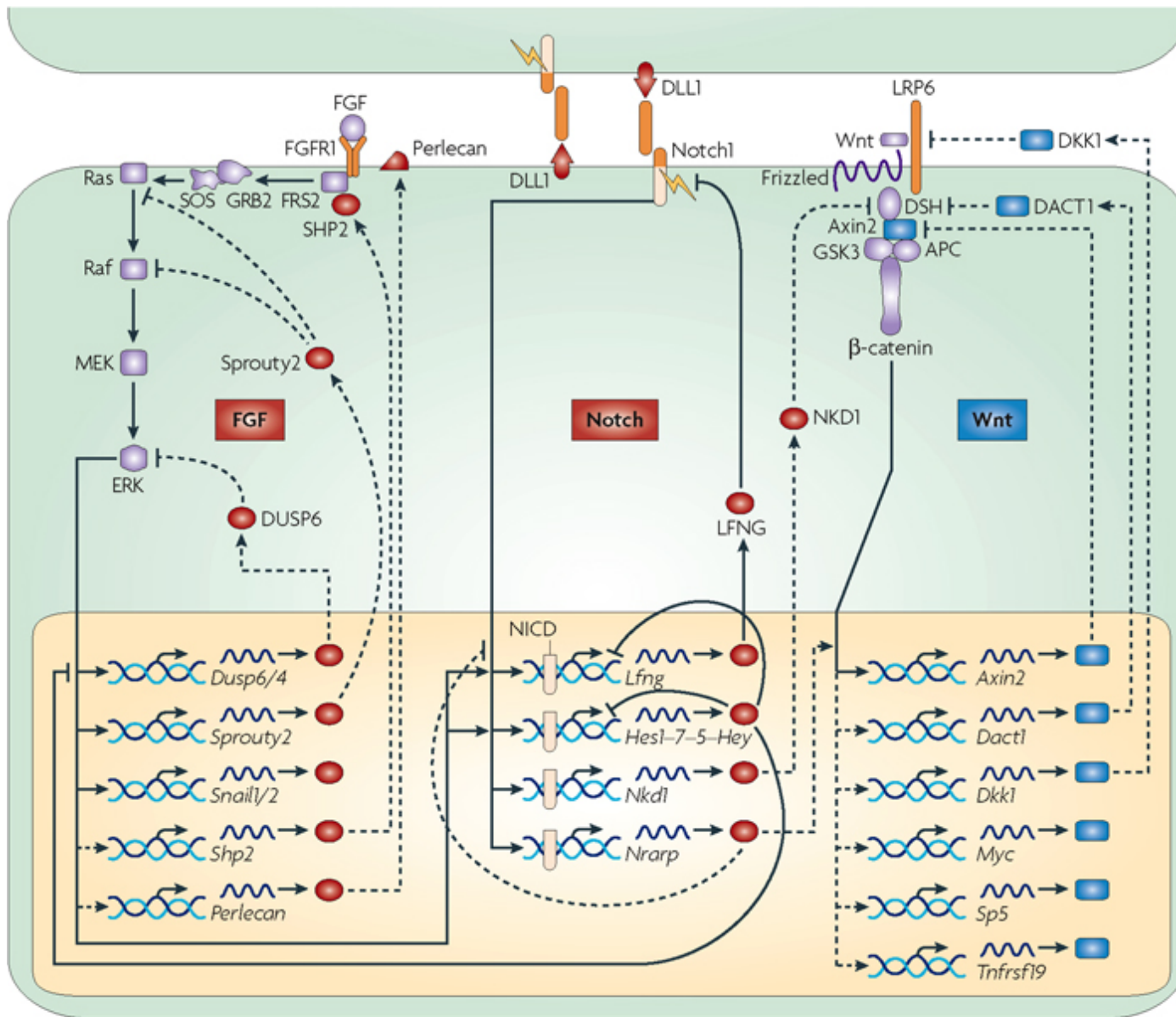
Vertebrae

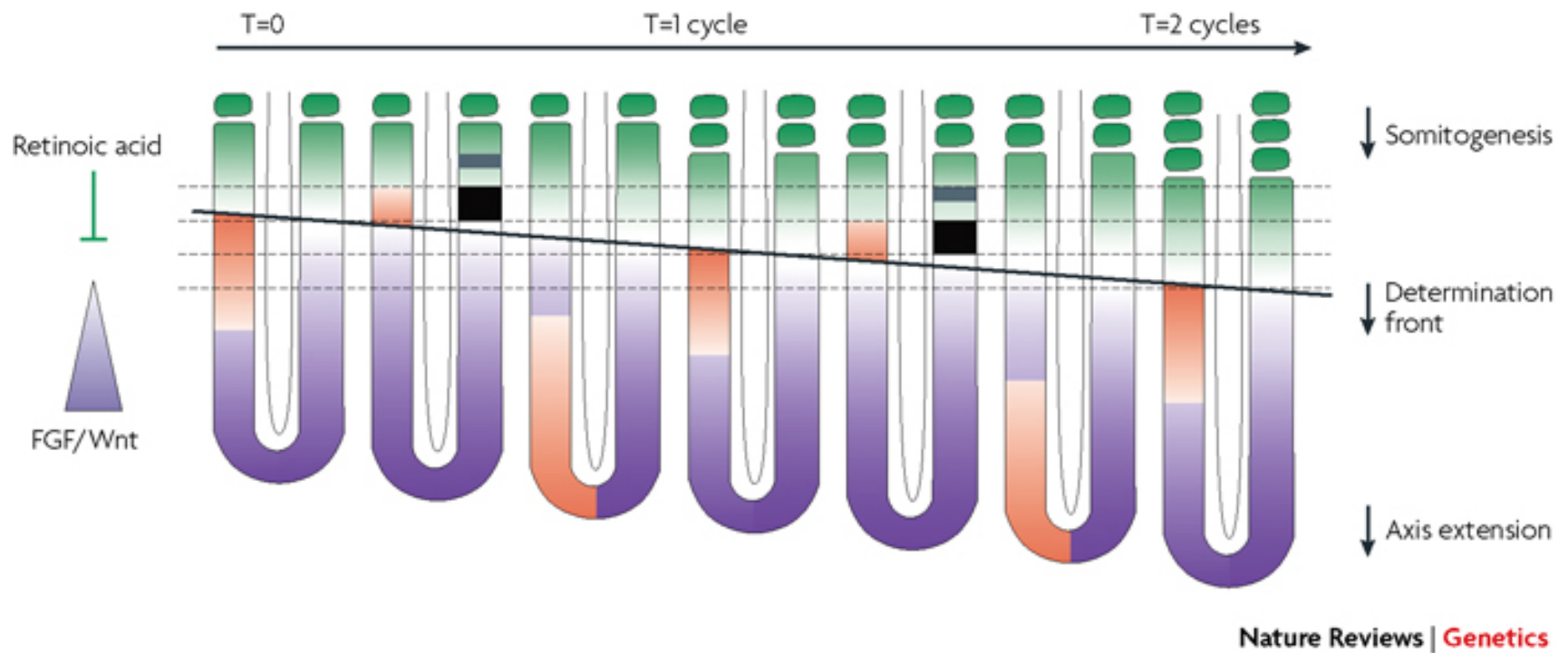
- Humans: 33, chickens: 55, mice: 65, corn snake: 315
- Characteristic of species
- How does an embryo “count” them?
- “Clock and wavefront model” of Cooke & Zeeman (1976).

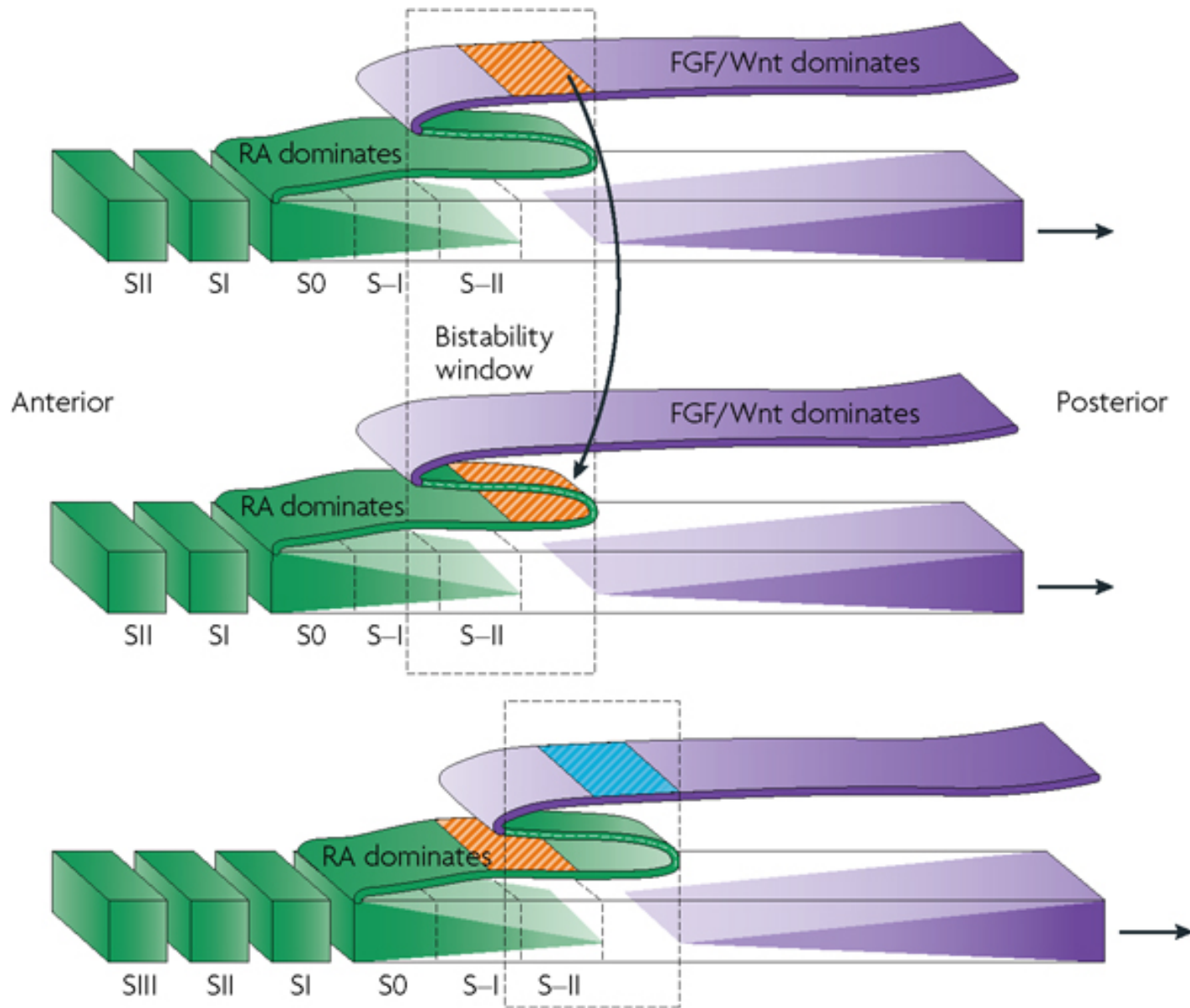


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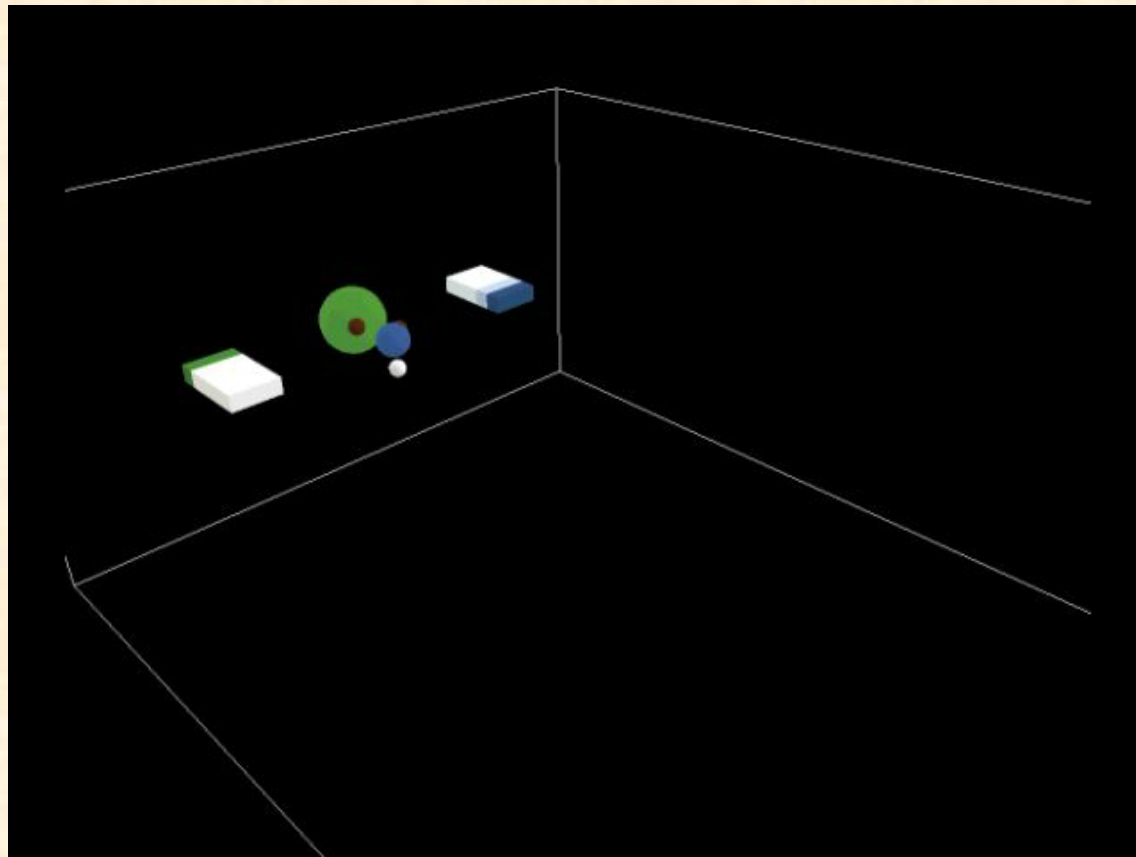








Simulated Segmentation by Clock-and-Wavefront Process



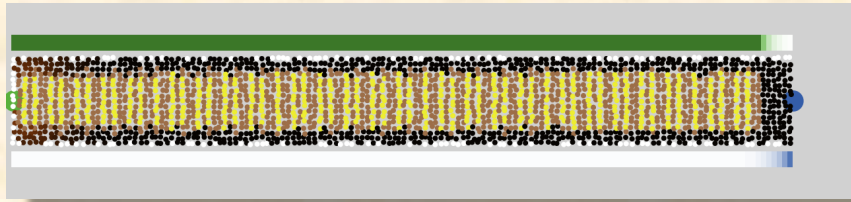
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Run Segmentation-cells-3D.nlogo

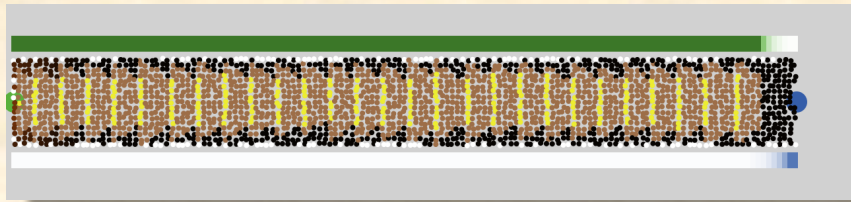
43

2D Simulation of Clock-and-Wavefront Process

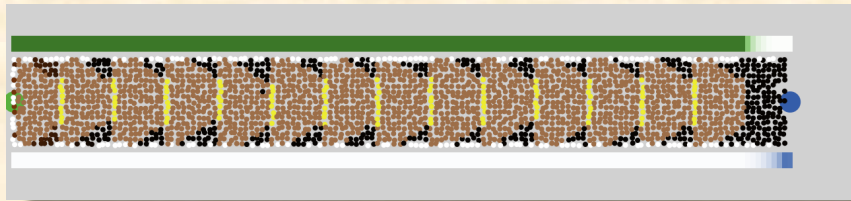




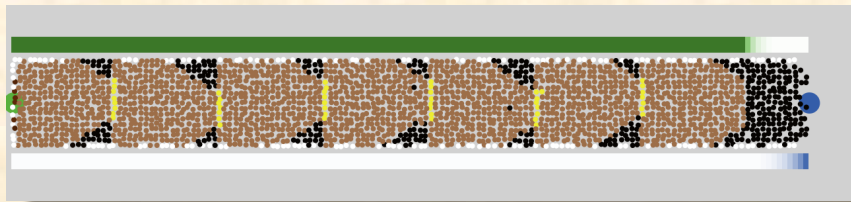
500



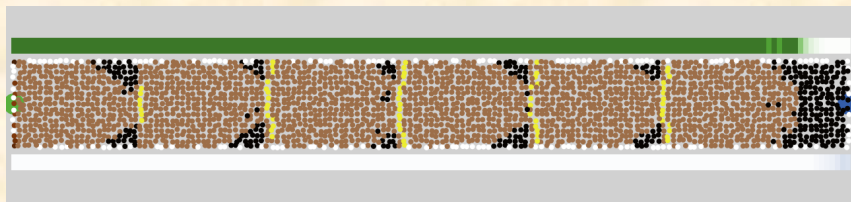
1000



2000



4000



5000

Effect of Growth Rate

NetLogo Simulation of Segmentation

[Run Segmentation.nlogo](#)

Segmentation References

1. Cooke, J., & Zeeman, E.C. (1976). A clock and wavefront model for control of the number of repeated structures during animal morphogenesis. *J. Theor. Biol.* **58**: 455–76.
2. Dequéant, M.-L., & Pourquié, O. (2008). Segmental patterning of the vertebrate embryonic axis. *Nature Reviews Genetics* **9**: 370–82.
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3. Tyson, J. J., & Keener, J. P. “Singular Perturbation Theory of Traveling Waves in Excitable Media (A Review),” *Physica D* **32** (1988): 327-61.
4. Camazine, S., Deneubourg, J.-L., Franks, N. R., Sneyd, J., Theraulaz, G., & Bonabeau, E. *Self-Organization in Biological Systems*. Princeton, 2001.
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6. Solé, R., & Goodwin, B. *Signs of Life: How Complexity Pervades Biology*. Basic Books, 2000.