

# V. Evolutionary Computing

## A. Genetic Algorithms

# Genetic Algorithms

- Developed by John Holland in '60s
- Did not become popular until late '80s
- A simplified model of genetics and evolution by natural selection
- Most widely applied to optimization problems (maximize “fitness”)

# Assumptions

- Existence of fitness function to quantify merit of potential solutions
  - this “fitness” is what the GA will maximize
- A mapping from bit-strings to potential solutions
  - best if each possible string generates a legal potential solution
  - choice of mapping is important
  - can use strings over other finite alphabets

# Outline of Simplified GA

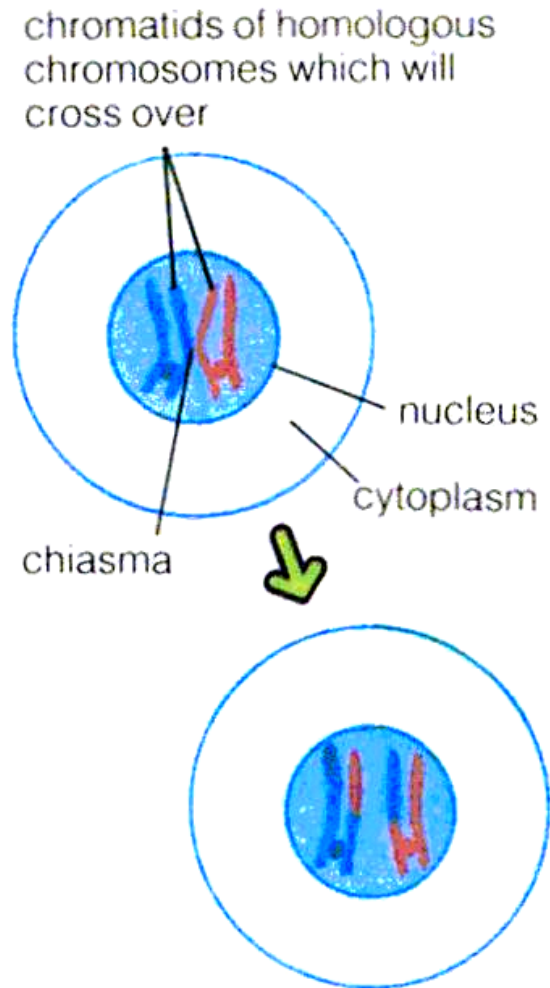
1. Random initial population  $P(0)$
2. Repeat for  $t = 0, \dots, t_{\max}$  or until converges:
  - a) create empty population  $P(t + 1)$
  - b) repeat until  $P(t + 1)$  is full:
    - 1) select two individuals from  $P(t)$  based on fitness
    - 2) optionally mate & replace with offspring
    - 3) optionally mutate offspring
    - 4) add two individuals to  $P(t + 1)$

# Fitness-Biased Selection

- Want the more “fit” to be more likely to reproduce
  - always selecting the best
    - ⇒ premature convergence
  - probabilistic selection ⇒ better exploration
- Roulette-wheel selection: probability  $\propto$  relative fitness:

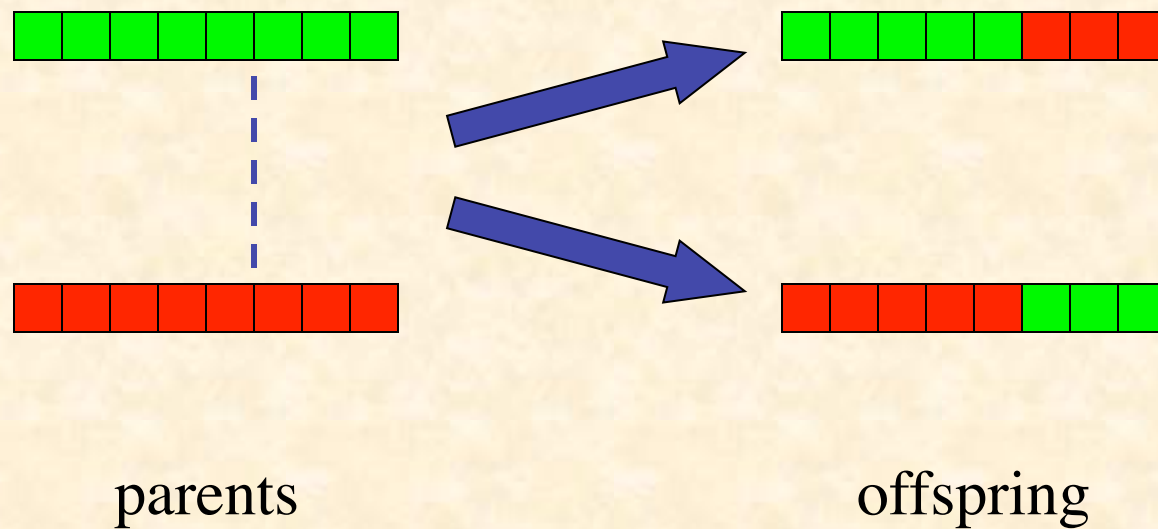
$$\Pr\{i \text{ mates}\} = \frac{f_i}{\sum_{j=1}^n f_j}$$

# Crossover: Biological Inspiration

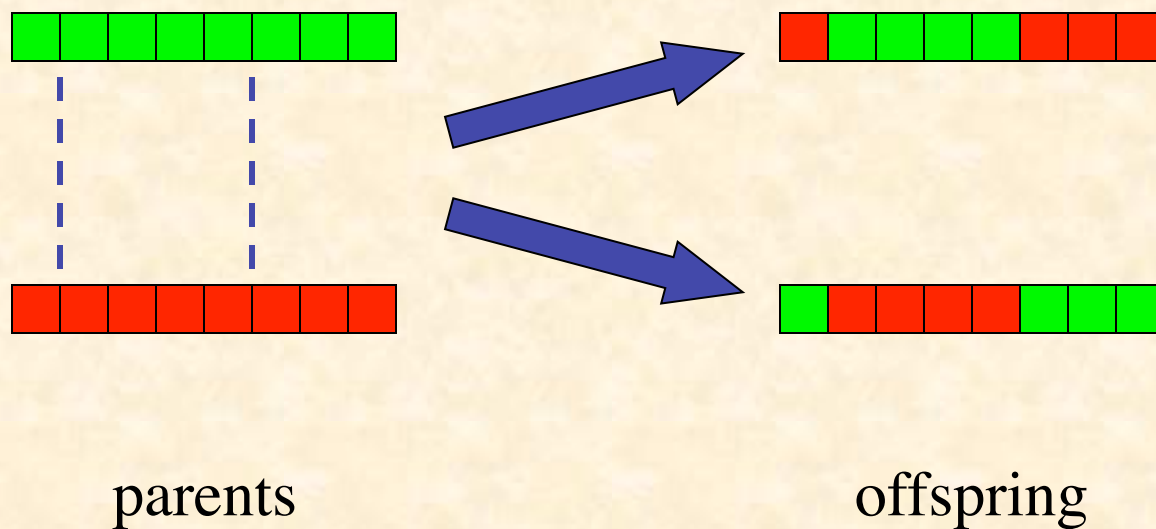


- Occurs during meiosis, when haploid gametes are formed
- Randomly mixes genes from two parents
- Creates genetic variation in gametes

# GAs: One-point Crossover

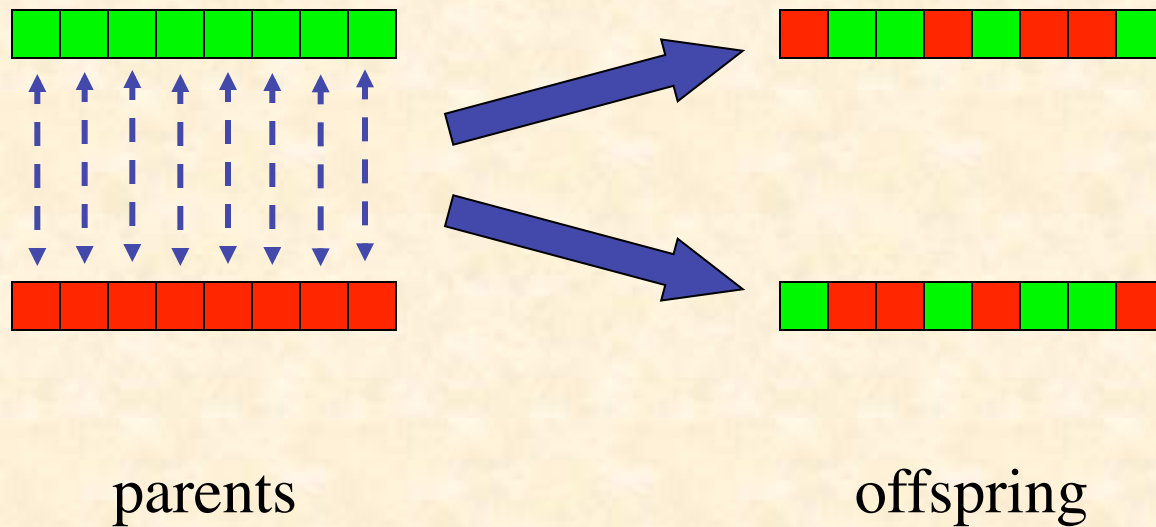


# GAs: Two-point Crossover



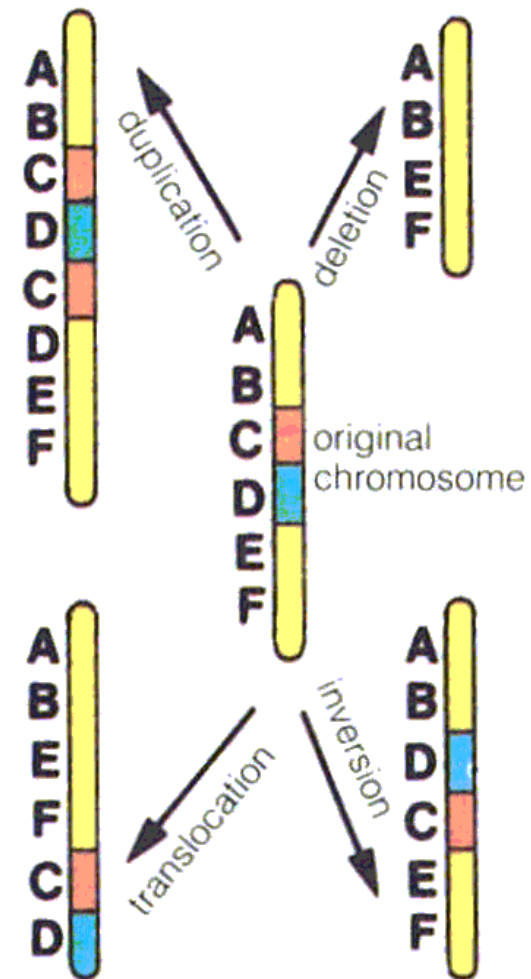


# GAs: $N$ -point Crossover

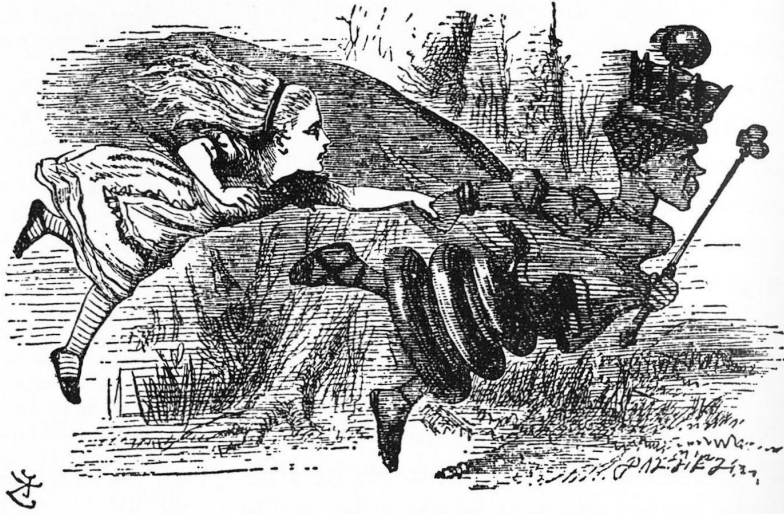


# Mutation: Biological Inspiration

- **Chromosome mutation**  $\Rightarrow$
- **Gene mutation**: alteration of the DNA in a gene
  - inspiration for mutation in GAs
- In typical GA each bit has a low probability of changing
- Some GAs models rearrange bits



# The Red Queen Hypothesis



“Now, *here*, you see, it takes all the running *you* can do, to keep in the same place.”

— *Through the Looking-Glass and What Alice Found There*

- *Observation*: a species probability of extinction is independent of time it has existed
- *Hypothesis*: species continually adapt to each other
- Extinction occurs with insufficient variability for further adaptation

# Demonstration of GA: Finding Maximum of Fitness Landscape

Run Genetic Algorithms — An Intuitive  
Introduction  
by Pascal Glauser

[<www.glauserweb.ch/gentore.htm>](http://www.glauserweb.ch/gentore.htm)

# Demonstration of GA: Evolving to Generate a Pre-specified Shape (Phenotype)

Run Genetic Algorithm Viewer

[www.rennard.org/alife/english/gavgb.html](http://www.rennard.org/alife/english/gavgb.html)

# Demonstration of GA: Eaters Seeking Food

<http://math.hws.edu/xJava/GA/>

# Morphology Project

by Michael “Flux” Chang

- Senior Independent Study project at UCLA
  - [users.design.ucla.edu/~mflux/morphology](http://users.design.ucla.edu/~mflux/morphology)
- Researched and programmed in 10 weeks
- Programmed in **Processing** language
  - [www.processing.org](http://www.processing.org)

# Genotype $\Rightarrow$ Phenotype

- Cells are “grown,” not specified individually
- Each gene specifies information such as:
  - angle
  - distance
  - type of cell
  - how many times to replicate
  - following gene
- Cells connected by “springs”
- Run **phenome**:  
[users.design.ucla.edu/~mflux/morphology/gallery/sketches/phenome](http://users.design.ucla.edu/~mflux/morphology/gallery/sketches/phenome)



# Complete Creature

- Neural nets for control (**blue**)
  - integrate-and-fire neurons
- Muscles (**red**)
  - decrease “spring length” when fire
- Sensors (**green**)
  - fire when exposed to “light”
- Structural elements (grey)
  - anchor other cells together
- Creature embedded in a fluid
- Run [users.design.ucla.edu/~mflux/morphology/gallery/sketches/creature](http://users.design.ucla.edu/~mflux/morphology/gallery/sketches/creature)

# Effects of Mutation

- Neural nets for control (**blue**)
- Muscles (**red**)
- Sensors (**green**)
- Structural elements (grey)
- Creature embedded in a fluid
- Run

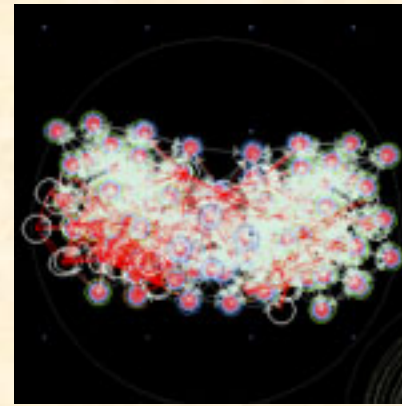
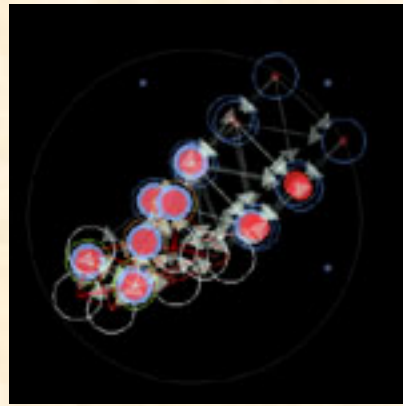
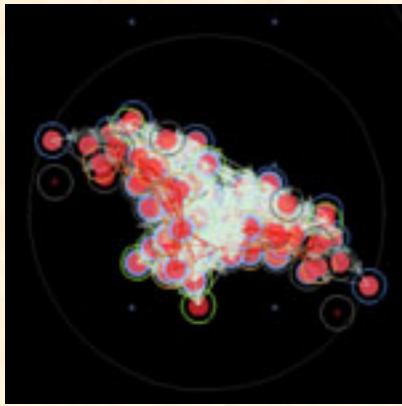
[users.design.ucla.edu/~mflux/morphology/gallery/sketches/creaturepack](http://users.design.ucla.edu/~mflux/morphology/gallery/sketches/creaturepack)

# Evolution

- Population: 150–200
- Nonviable & nonresponsive creatures eliminated
- Fitness based on speed or light-following
- 30% of new pop. are mutated copies of best
- 70% are random
- No crossover



# Gallery of Evolved Creatures



- Selected for speed of movement
- Run

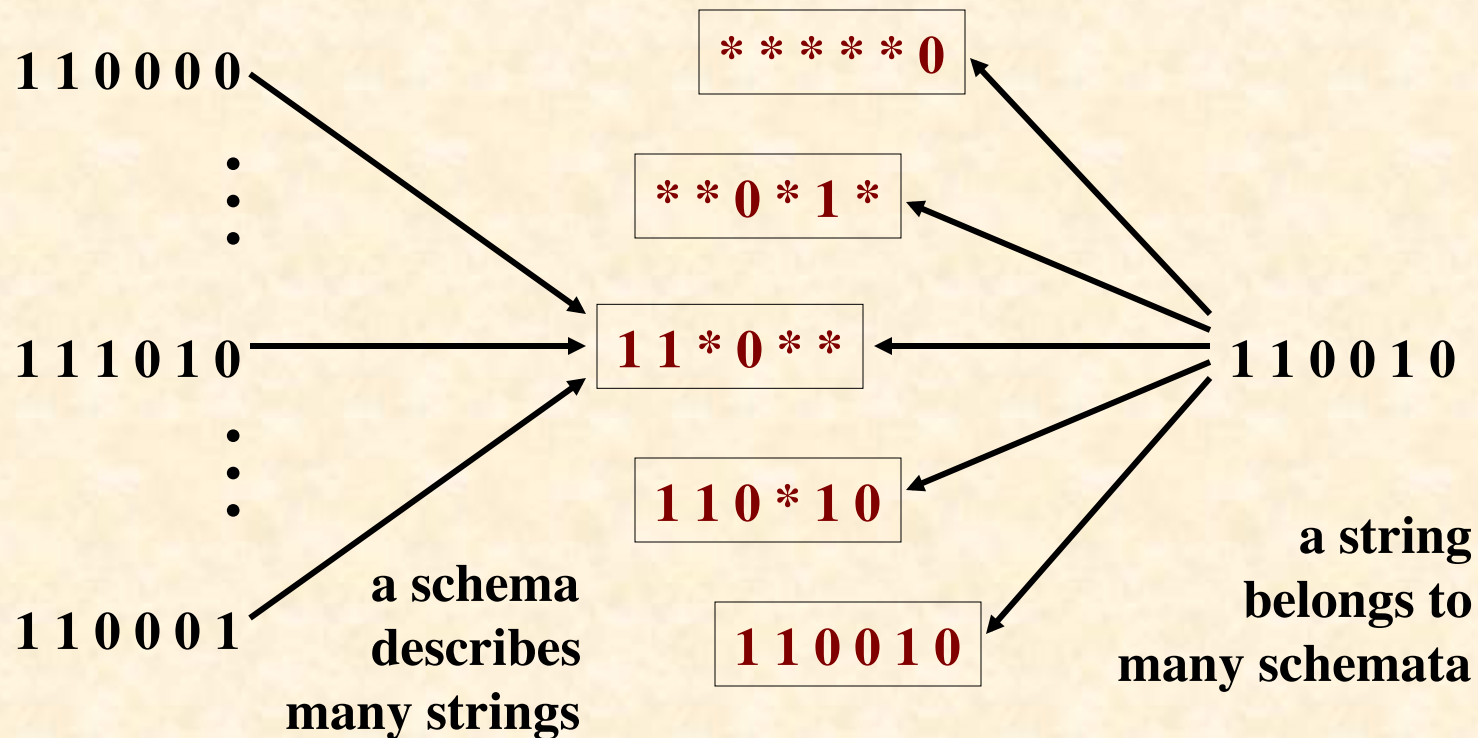
[users.design.ucla.edu/~mflux/morphology/gallery/sketches/creaturegallery](http://users.design.ucla.edu/~mflux/morphology/gallery/sketches/creaturegallery)

# Why Does the GA Work?

## The Schema Theorem

# Schemata

A **schema** is a description of certain patterns of bits in a genetic string



# The Fitness of Schemata

- The schemata are the **building blocks** of solutions
- We would like to know the average fitness of all possible strings belonging to a schema
- We cannot, but the strings in a population that belong to a schema give an estimate of the fitness of that schema
- Each string in a population is giving information about all the schemata to which it belongs (**implicit parallelism**)

# Effect of Selection

Let  $n$  = size of population

Let  $m(S, t)$  = number of instances of schema  $S$  at time  $t$

String  $i$  gets picked with probability  $\frac{f_i}{\sum_j f_j}$

Let  $f(S)$  = avg fitness of instances of  $S$  at time  $t$

So expected  $m(S, t + 1) = m(S, t) \cdot n \cdot \frac{f(S)}{\sum_j f_j}$

Since  $f_{\text{av}} = \frac{\sum_j f_j}{n}$ ,  $m(S, t + 1) = m(S, t) \frac{f(S)}{f_{\text{av}}}$



# Exponential Growth

- We have discovered:

$$m(S, t+1) = m(S, t) \cdot f(S) / f_{\text{av}}$$

- Suppose  $f(S) = f_{\text{av}} (1 + c)$
- Then  $m(S, t) = m(S, 0) (1 + c)^t$
- That is, **exponential growth** in above-average schemata

# Effect of Crossover

\*\*1 ... 0\*\*  
|←δ→|

- Let  $\lambda$  = length of genetic strings
- Let  $\delta(S)$  = defining length of schema  $S$
- Probability {crossover destroys  $S$ }:  
 $p_d \leq \delta(S) / (\lambda - 1)$
- Let  $p_c$  = probability of crossover
- Probability schema survives:

$$p_s \geq 1 - p_c \frac{\delta(S)}{\lambda - 1}$$

# Selection & Crossover Together

$$m(S, t + 1) \geq m(S, t) \frac{f(S)}{f_{\text{av}}} \left[ 1 - p_c \frac{\delta(S)}{\lambda - 1} \right]$$

# Effect of Mutation

- Let  $p_m$  = probability of mutation
- So  $1 - p_m$  = probability an allele survives
- Let  $o(S)$  = number of fixed positions in  $S$
- The probability they all survive is  
 $(1 - p_m)^{o(S)}$
- If  $p_m \ll 1$ ,  $(1 - p_m)^{o(S)} \approx 1 - o(S) p_m$

# Schema Theorem: “Fundamental Theorem of GAs”

$$m(S, t + 1) \geq m(S, t) \frac{f(S)}{f_{av}} \left[ 1 - p_c \frac{\delta(S)}{\lambda - 1} - o(S) p_m \right]$$

# The Bandit Problem

- Two-armed bandit:
  - random payoffs with (unknown) means  $m_1, m_2$  and variances  $\sigma_1, \sigma_2$
  - optimal strategy: allocate exponentially greater number of trials to apparently better lever
- $k$ -armed bandit: similar analysis applies
- Analogous to allocation of population to schemata
- Suggests GA may allocate trials optimally

# Goldberg's Analysis of Competent & Efficient GAs

# Paradox of GAs

- Individually uninteresting operators:
  - selection, recombination, mutation
- Selection + mutation  $\Rightarrow$  continual improvement
- Selection + recombination  $\Rightarrow$  innovation
  - fundamental to invention:  
*generation vs. evaluation*
- Fundamental intuition of GAs: the three work well together



# Race Between Selection & Innovation: Takeover Time

- Takeover time  $t^*$  = average time for most fit to take over population
- Transaction selection: population replaced by  $s$  copies of top  $1/s$
- $s$  quantifies selective pressure
- Estimate  $t^* \approx \ln n / \ln s$

# Innovation Time

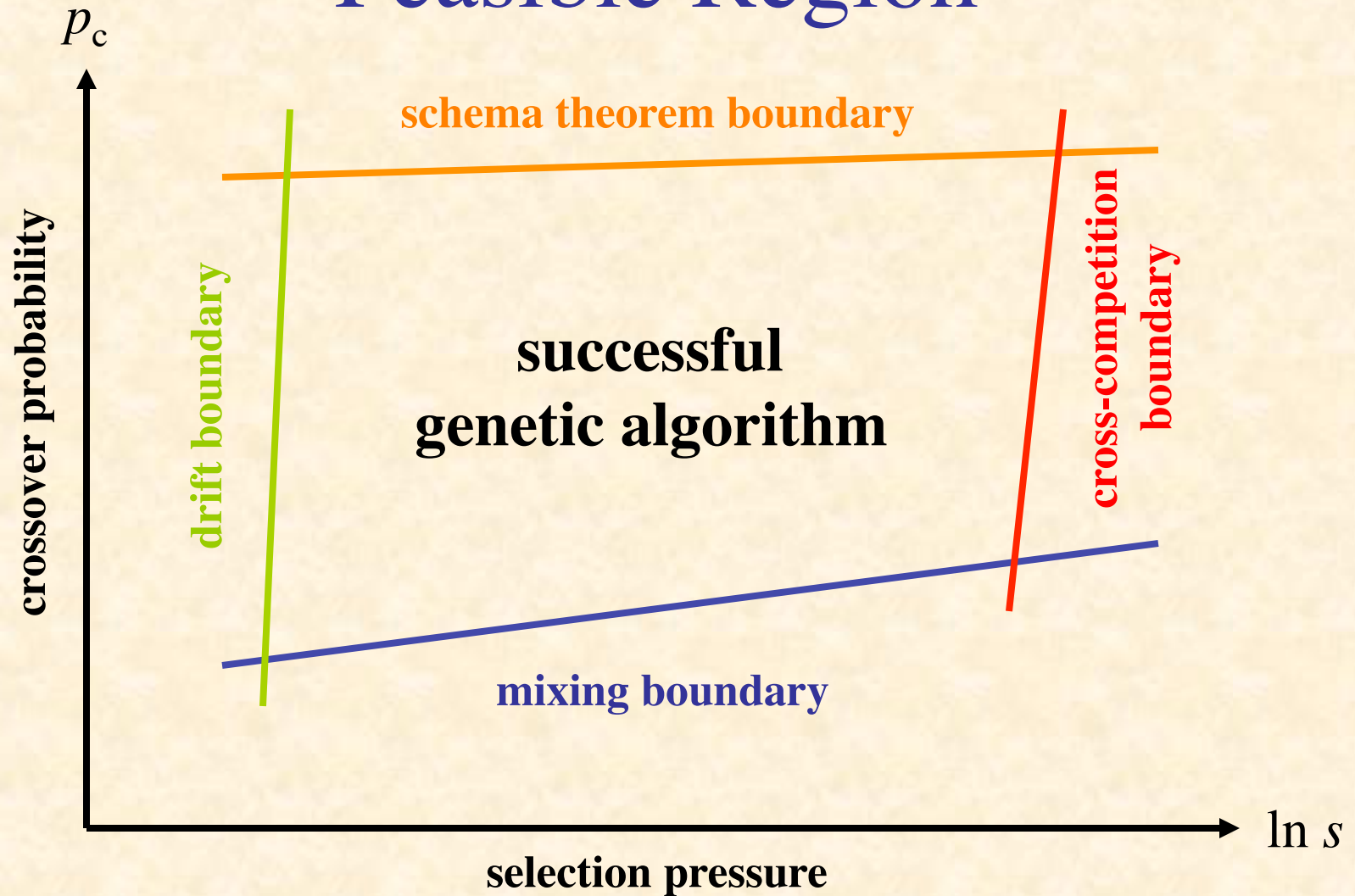
- Innovation time  $t_i$  = average time to get a better individual through crossover & mutation
- Let  $p_i$  = probability a single crossover produces a better individual
- Number of individuals undergoing crossover =  $p_c n$
- Number of probable improvements =  $p_i p_c n$
- Estimate:  $t_i \approx 1 / (p_c p_i n)$

# Steady State Innovation

- Bad:  $t^* < t_i$ 
  - because once you have takeover, crossover does no good
- Good:  $t_i < t^*$ 
  - because each time a better individual is produced, the  $t^*$  clock resets
  - *steady state innovation*
- Innovation number:

$$\text{Iv} = \frac{t^*}{t_i} = p_c p_i \frac{n \ln n}{\ln s} > 1$$

# Feasible Region



# Other Algorithms Inspired by Genetics and Evolution

- Evolutionary Programming
  - natural representation, no crossover, time-varying continuous mutation
- Evolutionary Strategies
  - similar, but with a kind of recombination
- Genetic Programming
  - like GA, but program trees instead of strings
- Classifier Systems
  - GA + rules + bids/payments
- and many variants & combinations...

# Next: Agents and Self-Organization

(Re)read ch. 16

# Additional Bibliography

1. Goldberg, D.E. *The Design of Innovation: Lessons from and for Competent Genetic Algorithms*. Kluwer, 2002.
2. Milner, R. *The Encyclopedia of Evolution*. Facts on File, 1990.

