

# Fairy Circles

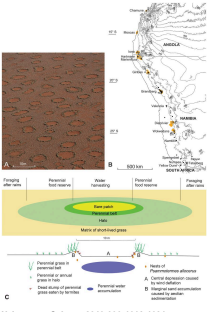
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# Fairy Circles




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**Fig. 1 (A) Spatial pattern of FCs. (B) Geographical distribution of FCs (black dots) and hotspots of FC occurrences at wider landscape scale (yellow clusters).**



N Juergens Science 2013;339:1618-1621

Published by AAAS



# Part C


## Nest Building

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# Nest Building by Termites (Natural and Artificial)

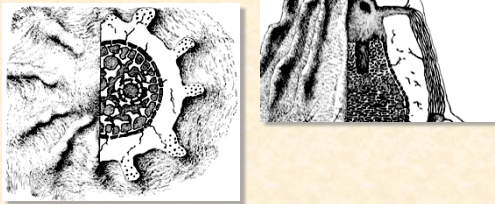
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# Mound Building by *Macrotermes* Termites



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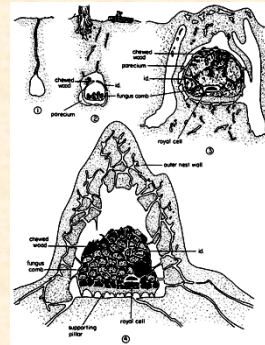
### Structure of Mound



2013/4/12 figs. from Lüscher (1961)

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### Construction of Mound

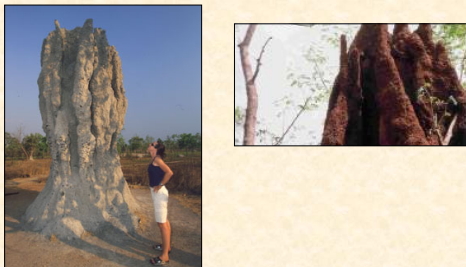


- (1) First chamber made by royal couple
- (2, 3) Intermediate stages of development
- (4) Fully developed nest

2013/4/12 Fig. from Wilson (1971)

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### Termite Nests



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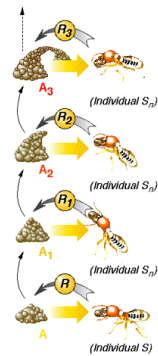
### Alternatives to Self-Organization

- Leader
  - directs building activity of group
- Blueprint (image of completion)
  - compact representation of spatial/temporal relationships of parts
- Recipe (program)
  - sequential instructions specify spatial/temporal actions of individual
- Template
  - full-sized guide or mold that specifies final pattern

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### Basic Mechanism of Construction (Stigmergy)



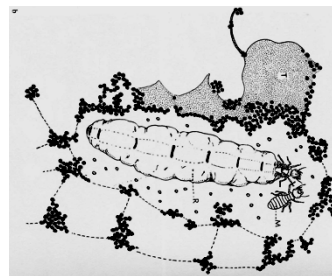
- Worker picks up soil granule
- Mixes saliva to make cement
- Cement contains pheromone
- Other workers attracted by pheromone to bring more granules
- There are also trail and queen pheromones

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Fig. from Solé & Goodwin

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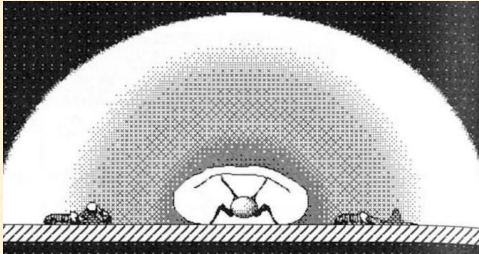
### Construction of Royal Chamber



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## Construction of Arch (1)

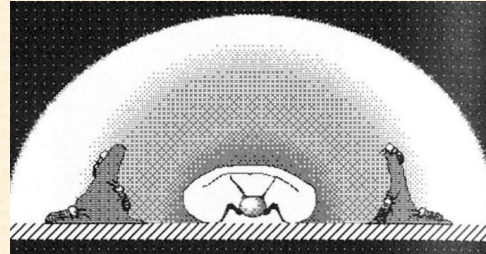


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Fig. from Bonabeau, Dorigo &amp; Theraulaz

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## Construction of Arch (2)

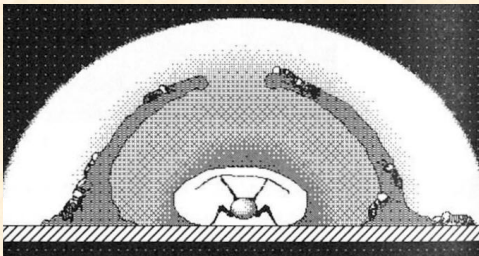


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Fig. from Bonabeau, Dorigo &amp; Theraulaz

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## Construction of Arch (3)



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Fig. from Bonabeau, Dorigo &amp; Theraulaz

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## Basic Principles

- Continuous (quantitative) stigmergy
- Positive feedback:
  - via pheromone deposition
- Negative feedback:
  - depletion of soil granules & competition between pillars
  - pheromone decay

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## Deneubourg Model

- $H(r, t)$  = concentration of cement pheromone in air at location  $r$  & time  $t$
- $P(r, t)$  = amount of deposited cement with still active pheromone at  $r, t$
- $C(r, t)$  = density of laden termites at  $r, t$
- $\Phi$  = constant flow of laden termites into system

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Equation for  $P$   
(Deposited Cement with Pheromone)

- $\partial_t P$  (rate of change of active cement) =
- $k_1 C$  (rate of cement deposition by termites)
- $k_2 P$  (rate of pheromone loss to air)

$$\partial_t P = k_1 C - k_2 P$$

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### Equation for $H$ (Concentration of Pheromone)

$\partial_t H$  (rate of change of concentration) =  
 $k_2 P$  (pheromone from deposited material)  
 $- k_4 H$  (pheromone decay)  
 $+ D_H \nabla^2 H$  (pheromone diffusion)

$$\partial_t H = k_2 P - k_4 H + D_H \nabla^2 H$$

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### Equation for $C$ (Density of Laden Termites)

$\partial_t C$  (rate of change of concentration) =  
 $\Phi$  (flux of laden termites)  
 $- k_1 C$  (unloading of termites)  
 $+ D_C \nabla^2 C$  (random walk)  
 $- \gamma \nabla \cdot (C \nabla H)$  (chemotaxis: response to pheromone gradient)

$$\partial_t C = \Phi - k_1 C + D_C \nabla^2 C - \gamma \nabla \cdot (C \nabla H)$$

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### Explanation of Divergence

- velocity field =  $\mathbf{V}(x,y) = \mathbf{i}V_x(x,y) + \mathbf{j}V_y(x,y)$
- $C(x,y)$  = density
- outflow rate =  $\Delta_x(CV_x) \Delta y + \Delta_y(CV_y) \Delta x$
- outflow rate / unit area =  $\frac{\Delta_x(CV_x)}{\Delta x} + \frac{\Delta_y(CV_y)}{\Delta y}$

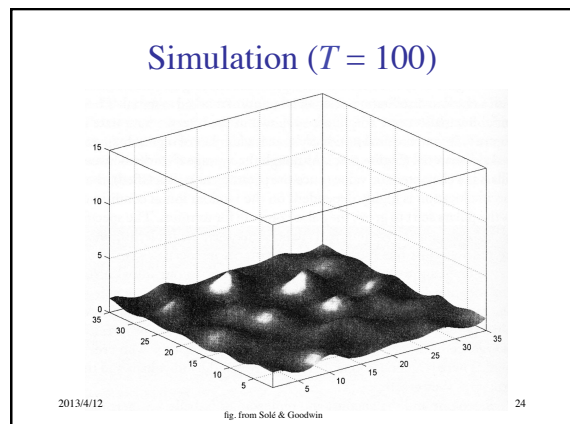
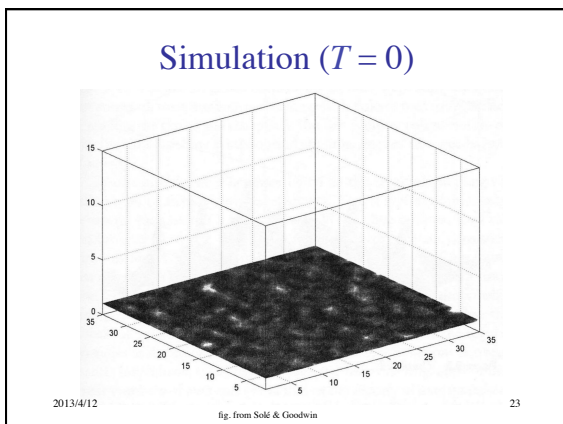
$$\rightarrow \frac{\partial(CV_x)}{\partial x} + \frac{\partial(CV_y)}{\partial y} = \nabla \cdot C\mathbf{V}$$

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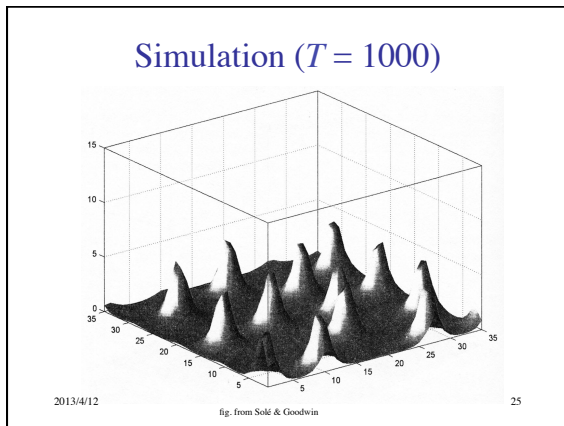
### Explanation of Chemotaxis Term

- The termite flow *into* a region is the *negative* divergence of the flux through it  
 $-\nabla \cdot \mathbf{J} = -(\partial J_x / \partial x + \partial J_y / \partial y)$
- The flux velocity is proportional to the pheromone gradient  
 $\mathbf{J} \propto \nabla H$
- The flux density is proportional to the number of moving termites  
 $\mathbf{J} \propto C$
- Hence,  $-\gamma \nabla \cdot \mathbf{J} = -\gamma \nabla \cdot (C \nabla H)$

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### Conditions for Self-Organized Pillars

- Will not produce regularly spaced pillars if:
  - density of termites is too low
  - rate of deposition is too low
- A homogeneous stable state results

$$C_0 = \frac{\Phi}{k_1}, \quad H_0 = \frac{\Phi}{k_4}, \quad P_0 = \frac{\Phi}{k_2}$$

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### NetLogo Simulation of Deneubourg Model

[Run Pillars3D.nlogo](#)

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### Interaction of Three Pheromones

- Queen pheromone governs size and shape of queen chamber (template)
- Cement pheromone governs construction and spacing of pillars & arches (stigmergy)
- Trail pheromone:
  - attracts workers to construction sites (stigmergy)
  - encourages soil pickup (stigmergy)
  - governs sizes of galleries (template)

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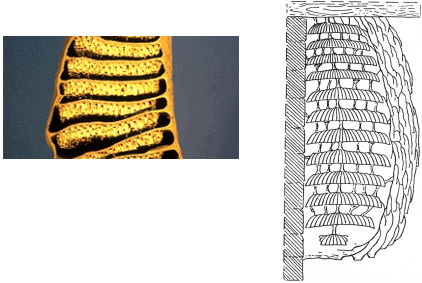
### Wasp Nest Building and Discrete Stigmergy

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Fig. from Solé & Goodwin

### Structure of Some Wasp Nests


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Fig. from *Self-Org. Biol. Sys.*

### Adaptive Function of Nests



2013/4/12      Figs. from Self-Org. Biol. Sys.      31

### How Do They Do It?



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### Lattice Swarms

(developed by Theraulaz & Bonabeau)

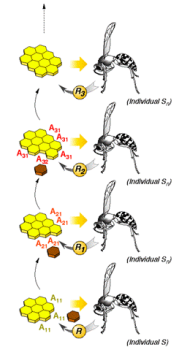
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### Discrete vs. Continuous Stigmergy

- Recall: *stigmergy* is the coordination of activities through the environment
- *Continuous* or *quantitative* stigmergy
  - quantitatively different stimuli trigger quantitatively different behaviors
- *Discrete* or *qualitative* stigmergy
  - stimuli are classified into distinct classes, which trigger distinct behaviors

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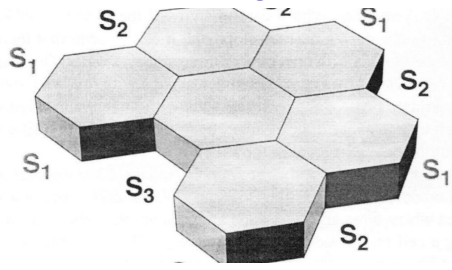
### Discrete Stigmergy in Comb Construction



- Initially all sites are equivalent
- After addition of cell, qualitatively different sites created

2013/4/12      Fig. from Self-Org. Biol. Sys.      35

### Numbers and Kinds of Building Sites



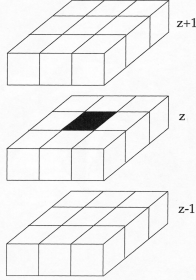
2013/4/12      Fig. from Self-Org. Biol. Sys.      36

### Lattice Swarm Model

- Random movement by wasps in a 3D lattice  
– cubic or hexagonal
- Wasps obey a 3D CA-like rule set
- Depending on configuration, wasp deposits one of several types of “bricks”
- Once deposited, it cannot be removed
- May be deterministic or probabilistic
- Start with a single brick

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### Cubic Neighborhood

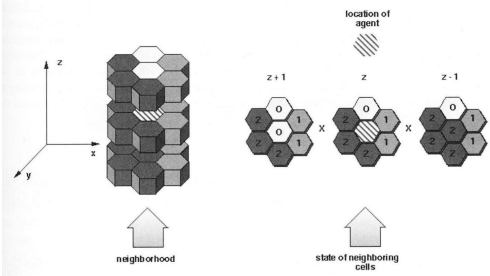


- Deposited brick depends on states of 26 surrounding cells
- Configuration of surrounding cells may be represented by matrices:

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 1 & \cdot & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

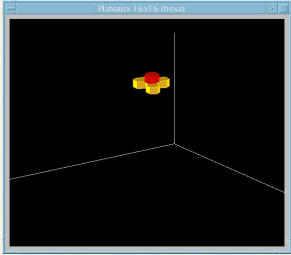
2013/4/12 Fig. from Solé & Goodwin 38

### Hexagonal Neighborhood



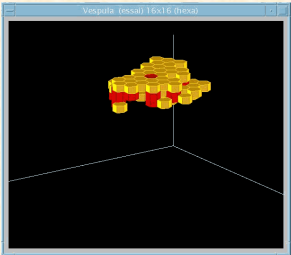
2013/4/12 39  
Fig. from Bonabeau, Dorigo & Theraulaz

### Example Construction



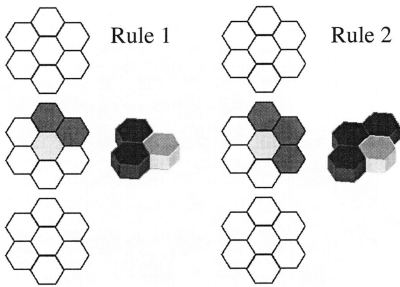
2013/4/12 40  
Fig. from IASC Dept., ENST de Bretagne.

### Another Example

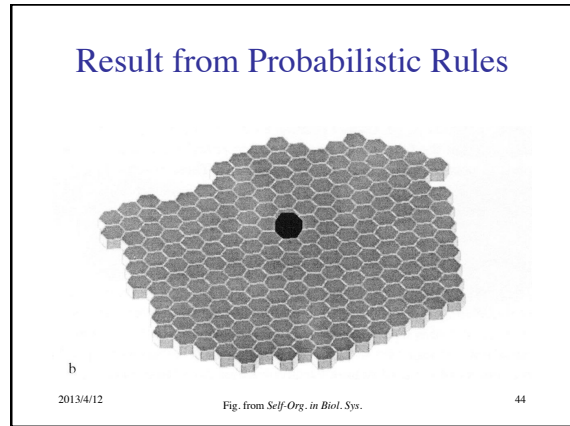
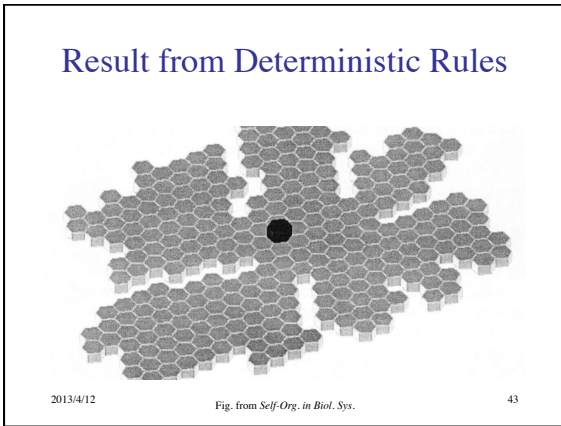


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fig. from IASC Dept., ENST de Bretagne.

### A Simple Pair of Rules



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Fig. from Self-Org. in Biol. Sys.



### Example Rules for a More Complex Architecture

The following stimulus configurations cause the agent to deposit a type-1 brick:

$$(1.1) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(1.2) \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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### Second Group of Rules

For these configurations, deposit a type-2 brick

**B**

(2.1) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	(2.10) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(2.2) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	(2.11) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(2.3) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	(2.12) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(2.4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	(2.13) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(2.5) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	(2.14) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(2.6) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	(2.15) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(2.7) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	(2.16) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(2.8) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	(2.17) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(2.9) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	(2.18) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

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### Result

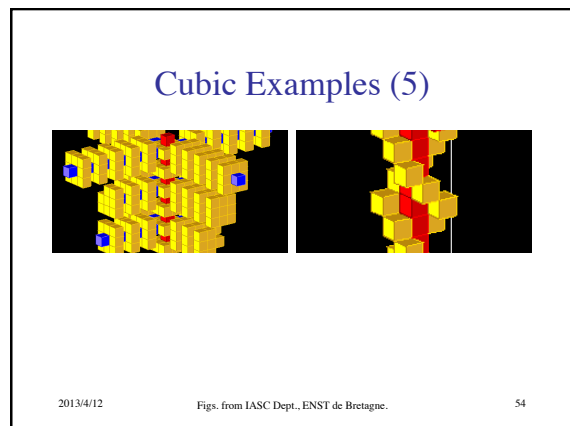
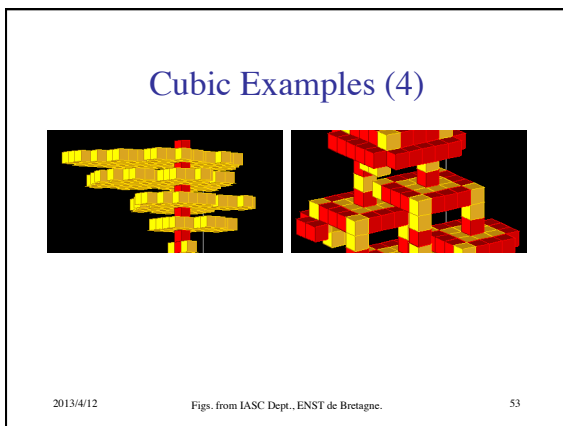
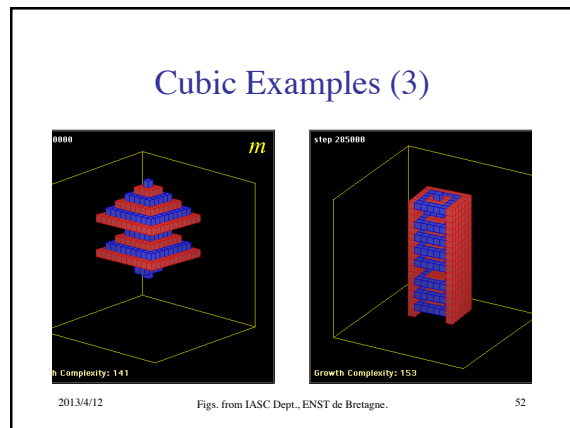
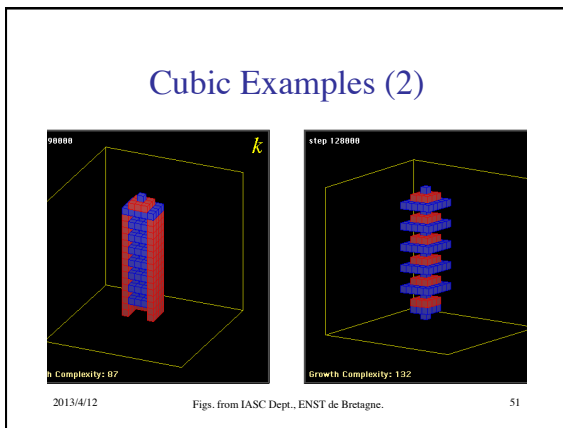
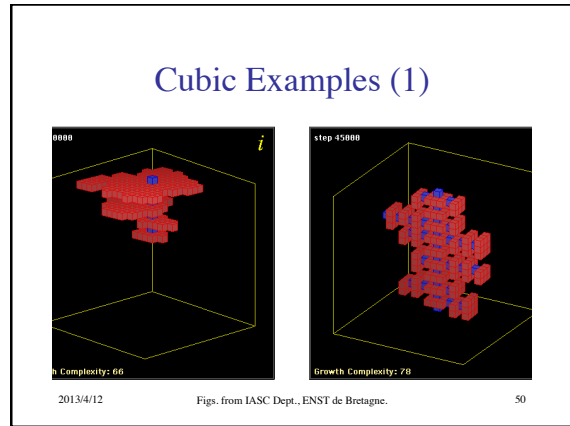
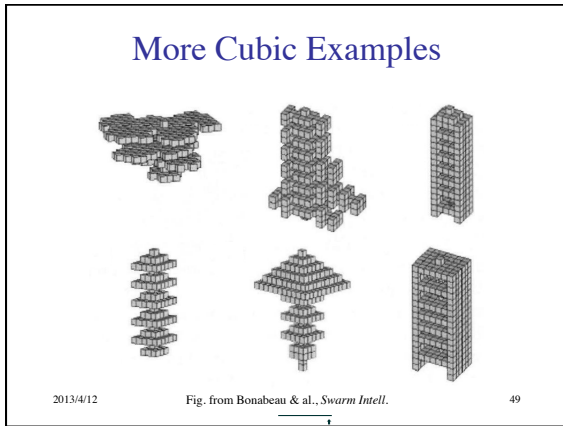
- 20x20x20 lattice
- 10 wasps
- After 20 000 simulation steps
- Axis and plateaus
- Resembles nest of *Parachartergus*

2013/4/12 Fig. from Bonabeau & al., *Swarm Intell.* 47

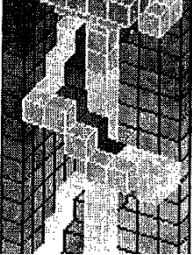
### Architectures Generated from Other Rule Sets

2013/4/12 Fig. from Bonabeau & al., *Swarm Intell.* 48





### An Interesting Example



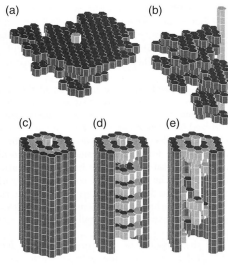
Includes

- central axis
- external envelope
- long-range helical ramp

Similar to *Apicotermes* termite nest

2013/4/12 Fig. from Theraulaz & Bonabeau (1995) 55

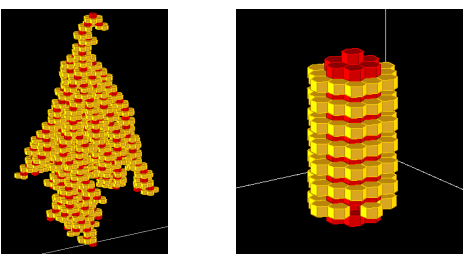
### Similar Results with Hexagonal Lattice



- 20x20x20 lattice
- 10 wasps
- All resemble nests of wasp species
- (d) is (c) with envelope cut away
- (e) has envelope cut away


2013/4/12 Fig. from Bonabeau & al., *Swarm Intell.* 56

### More Hexagonal Examples



2013/4/12 Figs. from IASC Dept., ENST de Bretagne. 57

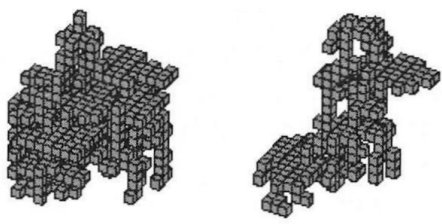
### Effects of Randomness (Coordinated Algorithm)



- Specifically different (i.e., different in details)
- Generically the same (qualitatively identical)
- Sometimes results are fully constrained

2013/4/12 Fig. from Bonabeau & al., *Swarm Intell.* 58

### Effects of Randomness (Non-coordinated Algorithm)



2013/4/12 Fig. from Bonabeau & al., *Swarm Intell.* 59

### Non-coordinated Algorithms

- Stimulating configurations are not ordered in time and space
- Many of them overlap
- Architecture grows without any coherence
- May be convergent, but are still unstructured

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### Coordinated Algorithm

- Non-conflicting rules
  - can't prescribe two different actions for the same configuration
- Stimulating configurations for different building stages cannot overlap
- At each stage, “handshakes” and “interlocks” are required to prevent conflicts in parallel assembly

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### More Formally...

- Let  $C = \{c_1, c_2, \dots, c_n\}$  be the set of local stimulating configurations
- Let  $(S_1, S_2, \dots, S_m)$  be a sequence of assembly stages
- These stages partition  $C$  into mutually disjoint subsets  $C(S_p)$
- Completion of  $S_p$  signaled by appearance of a configuration in  $C(S_{p+1})$

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Example

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Fig. from Camazine & al., *Self-Org. Biol. Sys.*

Example

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fig. from IASC Dept., ENST de Bretagne.

### Modular Structure

- Recurrent states induce cycles in group behavior
- These cycles induce modular structure
- Each module is built during a cycle
- Modules are qualitatively similar

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Fig. from Camazine & al., *Self-Org. Biol. Sys.*

### Possible Termination Mechanisms

- Qualitative
  - the assembly process leads to a configuration that is not stimulating
- Quantitative
  - a separate rule inhibiting building when nest a certain size relative to population
  - “empty cells rule”: make new cells only when no empties available
  - growing nest may inhibit positive feedback mechanisms

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### Observations

- Random algorithms tend to lead to uninteresting structures
  - random or space-filling shapes
- Similar structured architectures tend to be generated by similar coordinated algorithms
- Algorithms that generate structured architectures seem to be confined to a small region of rule-space

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### Analysis

- Define matrix  $M$ :
  - 12 columns for 12 sample structured architectures
  - 211 rows for stimulating configurations
  - $M_{ij} = 1$  if architecture  $j$  requires configuration  $i$

Fig. from Bonabeau & al., *Swarm Intell.*

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### Factorial Correspondence Analysis

Fig. from Bonabeau & al., *Swarm Intell.*

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### Conclusions

- Simple rules that exploit discrete (qualitative) stigmergy can be used by autonomous agents to assemble complex, 3D structures
- The rules must be non-conflicting and coordinated according to stage of assembly
- The rules corresponding to interesting structures occupy a comparatively small region in rule-space

Part 7

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## The Termes Project

Wyss Institute for Biologically Inspired Engineering  
Harvard

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## Introduction

Self-Organizing Systems Research  
School of Engineering and Applied Sciences  
Harvard University

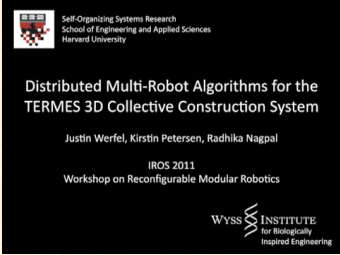
### TERMES: Simple Climbing Robots Building 3D Structures

WYSS INSTITUTE  
for Biologically Inspired Engineering

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## Algorithmic Assembly



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## The Robot



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## Additional Bibliography

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2. Bonabeau, E., Dorigo, M., & Theraulaz, G. *Swarm Intelligence: From Natural to Artificial Systems*. Oxford, 1999, chs. 2, 6.
3. Solé, R., & Goodwin, B. *Signs of Life: How Complexity Pervades Biology*. Basic Books, 2000, ch. 6.
4. Resnick, M. *Turtles, Termites, and Traffic Jams: Explorations in Massively Parallel Microworlds*. MIT Press, 1994, pp. 59-68, 75-81.
5. Kennedy, J., & Eberhart, R. "Particle Swarm Optimization," *Proc. IEEE Int'l. Conf. Neural Networks* (Perth, Australia), 1995. <http://www.engr.iupui.edu/~shi/psa.html>.

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