# V. Evolutionary Computing

## A. Genetic Algorithms



## Read Flake, ch. 20



## Genetic Algorithms

- Developed by John Holland in '60s
- Did not become popular until late '80s
- A simplified model of genetics and evolution by natural selection
- Most widely applied to optimization problems (maximize "fitness")

## Assumptions

- Existence of fitness function to quantify merit of potential solutions
  - This "fitness" is what the GA will maximize
- A mapping from bit-strings to potential solutions
  - best if each possible string generates a legal potential solution
  - choice of mapping is important
  - can use strings over other finite alphabets

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## Outline of Simplified GA

1. Random initial population $P(0)$
2. Repeat for $t = 0, \ldots, t_{\max}$ or until converges:
   a) create empty population $P(t + 1)$
   b) repeat until $P(t + 1)$ is full:
      1) select two individuals from $P(t)$ based on fitness
      2) optionally mate & replace with offspring
      3) optionally mutate offspring
      4) add two individuals to $P(t + 1)$

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## Fitness-Biased Selection

- Want the more "fit" to be more likely to reproduce
  - always selecting the best ⇒ premature convergence
  - probabilistic selection ⇒ better exploration
- Roulette-wheel selection: probability ∝ relative fitness:

$$\Pr\{i \text{ mates}\} = \frac{f_i}{\sum_{j=1}^{n} f_j}$$

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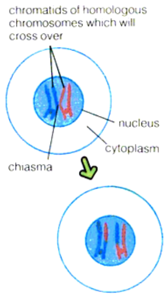
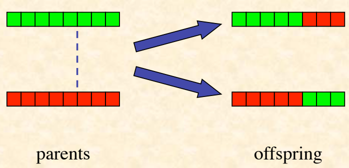
## Crossover: Biological Inspiration




- Occurs during meiosis, when haploid gametes are formed
- Randomly mixes genes from two parents
- Creates genetic variation in gametes

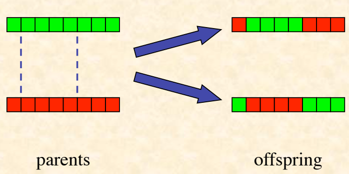
 (fig. from *B&N Thes. Biol.*) 

## GAs: One-point Crossover

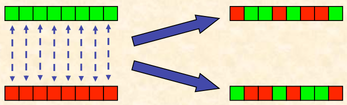





## GAs: Two-point Crossover



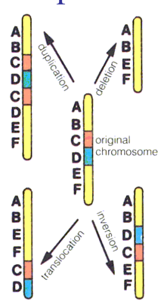
## GAs: *N*-point Crossover






## Mutation: Biological Inspiration

- Chromosome mutation ⇒
- Gene mutation: alteration of the DNA in a gene
  - inspiration for mutation in GAs
- In typical GA each bit has a low probability of changing
- Some GAs models rearrange bits




(fig. from *B&N Thes. Biol.*)



## The Red Queen Hypothesis

"Now, *here*, you see, it takes all the running *you* can do, to keep in the same place."
— *Through the Looking-Glass and What Alice Found There*

- *Observation*: a species probability of extinction is independent of time it has existed
- *Hypothesis*: species continually adapt to each other
- Extinction occurs with insufficient variability for further adaptation

## Demonstration of GA: Finding Maximum of Fitness Landscape

Run Genetic Algorithms — An Intuitive Introduction
by Pascal Glauser
<www.glauserweb.ch/gentore.htm>

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## Demonstration of GA: Evolving to Generate a Pre-specified Shape (Phenotype)

Run Genetic Algorithm Viewer
<www.rennard.org/alife/english/gavgb.html>

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## Demonstration of GA: Eaters Seeking Food

http://math.hws.edu/xJava/GA/

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## Morphology Project
by Michael "Flux" Chang

- Senior Independent Study project at UCLA
  - users.design.ucla.edu/~mflux/morphology
- Researched and programmed in 10 weeks
- Programmed in Processing language
  - www.processing.org

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## Genotype ⇒ Phenotype

- Cells are "grown," not specified individually
- Each gene specifies information such as:
  - angle
  - distance
  - type of cell
  - how many times to replicate
  - following gene
- Cells connected by "springs"
- Run phenome: <users.design.ucla.edu/~mflux/morphology/gallery/sketches/phenome>

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## Complete Creature

- Neural nets for control (blue)
  - integrate-and-fire neurons
- Muscles (red)
  - Decrease "spring length" when fire
- Sensors (green)
  - fire when exposed to "light"
- Structural elements (grey)
  - anchor other cells together
- Creature embedded in a fluid
- Run <users.design.ucla.edu/~mflux/morphology/gallery/sketches/creature>

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## Effects of Mutation

- Neural nets for control (blue)
- Muscles (red)
- Sensors (green)
- Structural elements (grey)
- Creature embedded in a fluid
- Run
  <users.design.ucla.edu/~mflux/morphology/gallery/sketches/creaturepack>

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## Evolution

- Population: 150–200
- Nonviable & nonresponsive creatures eliminated
- Fitness based on speed or light-following
- 30% of new pop. are mutated copies of best
- 70% are random
- No crossover

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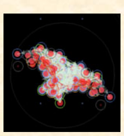
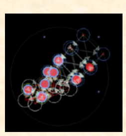
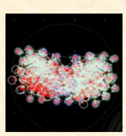
## Gallery of Evolved Creatures

- Selected for speed of movement
- Run
  <users.design.ucla.edu/~mflux/morphology/gallery/sketches/creaturegallery>

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## Karl Sims' Evolved Creatures

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## Why Does the GA Work?

The Schema Theorem

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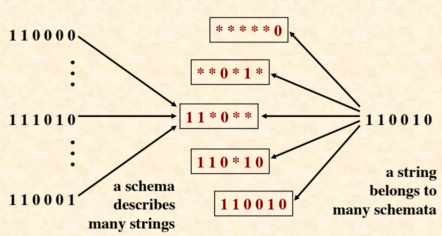
## Schemata

A schema is a description of certain patterns of bits in a genetic string

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## The Fitness of Schemata

- The schemata are the building blocks of solutions
- We would like to know the average fitness of all possible strings belonging to a schema
- We cannot, but the strings in a population that belong to a schema give an estimate of the fitness of that schema
- Each string in a population is giving information about all the schemata to which it belongs (implicit parallelism)

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## Effect of Selection

Let $n$ = size of population

Let $m(S,t)$ = number of instances of schema $S$ at time $t$

String $i$ gets picked with probability $\frac{f_i}{\sum_j f_j}$

Let $f(S)$ = avg fitness of instances of $S$ at time $t$

So expected $m(S,t+1) = m(S,t) \cdot n \cdot \frac{f(S)}{\sum_j f_j}$

Since $f_{\text{av}} = \frac{\sum_j f_j}{n}$, $m(S,t+1) = m(S,t)\frac{f(S)}{f_{\text{av}}}$

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## Exponential Growth

- We have discovered: $m(S, t+1) = m(S, t) \cdot f(S) / f_{\text{av}}$
- Suppose $f(S) = f_{\text{av}}(1 + c)$
- Then $m(S, t) = m(S, 0)(1 + c)^t$
- That is, exponential growth in above-average schemata

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## Effect of Crossover

**1 … 0***
|←δ→|

- Let λ = length of genetic strings
- Let δ(*S*) = defining length of schema *S*
- Probability {crossover destroys *S*}:
  $p_d \leq \delta(S) / (\lambda - 1)$
- Let $p_c$ = probability of crossover
- Probability schema survives:

$$p_s \geq 1 - p_c \frac{\delta(S)}{\lambda - 1}$$

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## Selection & Crossover Together

$$m(S,t+1) \geq m(S,t) \frac{f(S)}{f_{av}} \left[1 - p_c \frac{\delta(S)}{\lambda - 1}\right]$$

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## Effect of Mutation

- Let $p_m$ = probability of mutation
- So $1 - p_m$ = probability an allele survives
- Let $o(S)$ = number of fixed positions in *S*
- The probability they all survive is
  $(1 - p_m)^{o(S)}$
- If $p_m \ll 1$, $(1 - p_m)^{o(S)} \approx 1 - o(S)\, p_m$

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## Schema Theorem: "Fundamental Theorem of GAs"

$$m(S,t+1) \geq m(S,t)\frac{f(S)}{f_{\text{av}}}\left[1 - p_{\text{c}}\frac{\delta(S)}{\lambda - 1} - o(S)p_{\text{m}}\right]$$

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## The Bandit Problem

- Two-armed bandit:
  - random payoffs with (unknown) means $m_1$, $m_2$ and variances $\sigma_1^2$, $\sigma_2^2$
  - optimal strategy: allocate exponentially greater number of trials to apparently better lever
- $k$-armed bandit: similar analysis applies
- Analogous to allocation of population to schemata
- Suggests GA may allocate trials optimally

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## Goldberg's Analysis of Competent & Efficient GAs

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## Paradox of GAs

- Individually uninteresting operators:
  - selection, recombination, mutation
- Selection + mutation ⇒ continual improvement
- Selection + recombination ⇒ innovation
  - fundamental to invention: *generation* vs. *evaluation*
- Fundamental intuition of GAs: the three work well together



## Race Between Selection & Innovation: Takeover Time

- Takeover time $t^*$ = average time for most fit to take over population
- Transaction selection: population replaced by $s$ copies of top $1/s$
- $s$ quantifies selective pressure
- Estimate $t^* \approx \ln n / \ln s$



## Innovation Time

- Innovation time $t_\mathrm{i}$ = average time to get a better individual through crossover & mutation
- Let $p_\mathrm{i}$ = probability a single crossover produces a better individual
- Number of individuals undergoing crossover = $p_\mathrm{c}\, n$
- Number of probable improvements = $p_\mathrm{i}\, p_\mathrm{c}\, n$
- Estimate: $t_\mathrm{i} \approx 1 / (p_\mathrm{c}\, p_\mathrm{i}\, n)$

## Steady State Innovation

- Bad: $t^* < t_i$
  - because once you have takeover, crossover does no good
- Good: $t_i < t^*$
  - because each time a better individual is produced, the $t^*$ clock resets
  - *steady state innovation*
- Innovation number:

$$\text{Iv} = \frac{t^*}{t_i} = p_c p_i \frac{n \ln n}{\ln s} > 1$$

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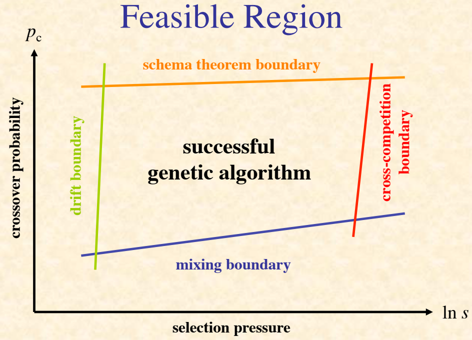
## Feasible Region

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## Other Algorithms Inspired by Genetics and Evolution

- Evolutionary Programming
  - natural representation, no crossover, time-varying continuous mutation
- Evolutionary Strategies
  - similar, but with a kind of recombination
- Genetic Programming
  - like GA, but program trees instead of strings
- Classifier Systems
  - GA + rules + bids/payments
- and many variants & combinations…

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## Additional Bibliography


1. Goldberg, D.E. *The Design of Innovation: Lessons from and for Competent Genetic Algorithms*. Kluwer, 2002.
2. Milner, R. *The Encyclopedia of Evolution*. Facts on File, 1990.


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