

Read Flake chs. 16–17

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Part B  
Ants (Natural and Artificial)

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Real Ants  
(especially the black garden ant,  
*Lasius niger*)

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### Adaptive Significance

- Selects most profitable from array of food sources
- Selects shortest route to it
  - longer paths abandoned within 1–2 hours
- Adjusts amount of exploration to quality of identified sources
- Collective decision making can be as accurate and effective as some vertebrate individuals

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### Observations on Trail Formation

- Two equal-length paths presented at same time: ants choose one at random
- Sometimes the longer path is initially chosen
- Ants may remain “trapped” on longer path, once established
- Or on path to lower quality source, if it’s discovered first
- But there may be advantages to sticking to paths
  - easier to follow
  - easier to protect trail & source
  - safer

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### Process of Trail Formation

1. Trail laying
2. Trail following

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### Trail Laying

- On discovering food, forager lays chemical trail while returning to nest
  - only ants who have found food deposit pheromone
- Others stimulated to leave nest by:
  - the trail
  - the recruiter exciting nestmates (sometimes)
- In addition to defining trail, pheromone:
  - serves as general orientation signal for ants outside nest
  - serves as arousal signal for ants inside

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### Additional Complexities

- Some ants begin marking on return from discovering food
- Others on their first return trip to food
- Others not at all, or variable behavior
- Probability of trail laying decreases with number of trips

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### Frequency of Trail Marking

- Ants modulate frequency of trail marking
- May reflect quality of source
  - hence more exploration if source is poor
- May reflect orientation to nest
  - ants keep track of general direction to nest
  - and of general direction to food source
  - trail laying is less intense if the angle to homeward direction is large

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### Trail Following

- Ants preferentially follow stronger of two trails
  - show no preference for path they used previously
- Ant may double back, because of:
  - decrease of pheromone concentration
  - unattractive orientation

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### Probability of Choosing One of Two Branches

- Let  $C_L$  and  $C_R$  be units of pheromone deposited on left & right branches
- Let  $P_L$  and  $P_R$  be probabilities of choosing them
- Then:

$$P_L = \frac{(C_L + 6)^2}{(C_L + 6)^2 + (C_R + 6)^2}$$

- Nonlinearity amplifies probability

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### Additional Adaptations

- If a source is crowded, ants may return to nest or explore for other sources
- New food sources are preferred if they are near to existing sources
- Foraging trails may rotate systematically around a nest

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**Pheromone Evaporation**

- Trails can persist from several hours to several months
- Pheromone has mean lifetime of 30–60 min.
- But remains detectable for many times this
- Long persistence of pheromone prevents switching to shorter trail
- Artificial ant colony systems rely more heavily on evaporation

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**Resnick's Ants**

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**Environment**

- Nest emits *nest-scent*, which
  - diffuses uniformly
  - decays slowly
  - provides general orientation signal
  - by diffusing around barriers, shows possible paths around barriers
- Trail pheromone
  - emitted by ants carrying food
  - diffuses uniformly
  - decays quickly
- Food detected only by contact

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## Resnick Ant Behavior

1. Looking for food:  
if trail pheromone weak **then** wander  
else move toward increasing concentration
2. Acquiring food:  
if at food **then**  
pick it up, turn around, & begin depositing pheromone
3. Returning to nest:  
deposit pheromone & decrease amount available  
move toward increasing nest-scent
4. Depositing food:  
if at nest **then**  
deposit food, stop depositing pheromone, & turn around
5. Repeat forever

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## Demonstration of Resnick Ants

[Run Ants.nlogo](#)

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## Ant Colony Optimization (ACO)

Developed in 1991 by Dorigo (PhD dissertation) in collaboration with Coloni & Maniezzo

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### Basis of all Ant-Based Algorithms

- Positive feedback
- Negative feedback
- Cooperation

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### Positive Feedback

- To reinforce portions of good solutions that contribute to their goodness
- To reinforce good solutions directly
- Accomplished by *pheromone accumulation*

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### Reinforcement of Solution Components

Parts of good solutions *may* produce better solutions

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### Negative Reinforcement of Non-solution Components

Parts not in good solutions *tend* to be forgotten

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### Negative Feedback

- To avoid premature convergence (*stagnation*)
- Accomplished by *pheromone evaporation*

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### Cooperation

- For simultaneous exploration of different solutions
- Accomplished by:
  - *multiple ants* exploring solution space
  - *pheromone trail* reflecting multiple perspectives on solution space

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## Traveling Salesman Problem

- Given the travel distances between  $N$  cities
  - may be symmetric or not
- Find the shortest route visiting each city exactly once and returning to the starting point
- NP-hard
- Typical combinatorial optimization problem

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## Ant System for Traveling Salesman Problem (AS-TSP)

- During each iteration, each ant completes a tour
- During each tour, each ant maintains *tabu list* of cities already visited
- Each ant has access to
  - distance of current city to other cities
  - intensity of local pheromone trail
- Probability of next city depends on both

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## Transition Rule

- Let  $\eta_{ij} = 1/d_{ij}$  = “nearness” of city  $j$  to current city  $i$
- Let  $\tau_{ij}$  = strength of trail from  $i$  to  $j$
- Let  $J_i^k$  = list of cities ant  $k$  still has to visit after city  $i$  in current tour
- Then transition probability for ant  $k$  going from  $i$  to  $j \in J_i^k$  in tour  $t$  is:

$$p_{ij}^k = \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in J_i^k} [\tau_{il}(t)]^\alpha [\eta_{il}]^\beta}$$

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### Pheromone Deposition

- Let  $T^k(t)$  be tour  $t$  of ant  $k$
- Let  $L^k(t)$  be the length of this tour
- After completion of a tour, each ant  $k$  contributes:

$$\Delta\tau_{ij}^k = \begin{cases} Q/L^k(t) & \text{if } (i,j) \in T^k(t) \\ 0 & \text{if } (i,j) \notin T^k(t) \end{cases}$$

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### Pheromone Decay

- Define total pheromone deposition for tour  $t$ :

$$\Delta\tau_{ij}(t) = \sum_{k=1}^m \Delta\tau_{ij}^k(t)$$

- Let  $\rho$  be decay coefficient
- Define trail intensity for next round of tours:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t)$$

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### Number of Ants is Critical

- Too many:
  - suboptimal trails quickly reinforced
  - $\therefore$  early convergence to suboptimal solution
- Too few:
  - don't get cooperation before pheromone decays
- Good tradeoff:
  - number of ants = number of cities
  - $(m = n)$

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### Improvement: “Elitist” Ants

- Add a few ( $e \approx 5$ ) “elitist” ants to population
- Let  $T^+$  be best tour so far
- Let  $L^+$  be its length
- Each “elitist” ant reinforces edges in  $T^+$  by  $Q/L^+$
- Add  $e$  more “elitist” ants
- This applies accelerating positive feedback to best tour

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### Time Complexity

- Let  $t$  be number of tours
- Time is  $\mathcal{O}(tn^2m)$
- If  $m = n$  then  $\mathcal{O}(tn^3)$ 
  - that is, cubic in number of cities

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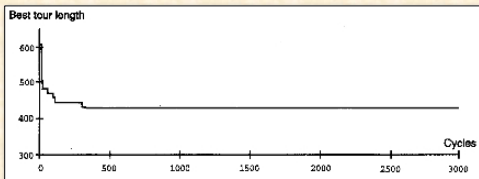
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### Convergence



- 30 cities (“Oliver30”)
- Best tour length
- Converged to optimum in 300 cycles

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fig. < Dorigo et al. (1996)

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### Evaluation

- Both “very interesting and disappointing”
- For 30-cities:
  - beat genetic algorithm
  - matched or beat tabu search & simulated annealing
- For 50 & 75 cities and 3000 iterations
  - did not achieve optimum
  - but quickly found good solutions
- I.e., does not scale up well
- Like all general-purpose algorithms, it is outperformed by special purpose algorithms

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### Improving Network Routing

1. Nodes periodically send *forward ants* to some recently recorded destinations
2. Collect information on way
3. Die if reach already visited node
4. When reaches destination, estimates time and turns into *backward ant*
5. Returns by same route, updating routing tables

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### Some Applications of ACO

- Routing in telephone networks
- Vehicle routing
- Job-shop scheduling
- Constructing evolutionary trees from nucleotide sequences
- Various classic NP-hard problems
  - shortest common supersequence, graph coloring, quadratic assignment, ...

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### Improvements as Optimizer

- Can be improved in many ways
- E.g., combine local search with ant-based methods
- As method of stochastic combinatorial optimization, performance is promising, comparable with best heuristic methods
- Much ongoing research in ACO
- But optimization is not a principal topic of this course

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### Nonconvergence

- Standard deviation of tour lengths
- Optimum = 420

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fig. < Dorigo et al. (1996)

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### Average Node Branching Number

- Branching number = number of edges leaving a node with pheromone > threshold
- Branching number = 2 for fully converged solution

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fig. < Dorigo et al. (1996)

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### The Nonconvergence Issue

- AS often does not converge to single solution
- Population maintains high diversity
- A bug or a feature?
- Potential advantages of nonconvergence:
  - avoids getting trapped in local optima
  - promising for dynamic applications
- Flexibility & robustness are more important than optimality in natural computation

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### Natural Computation

*Natural computation* is computation that occurs in nature or is inspired by computation occurring in nature

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### Optimization in Natural Computation

- Good, but suboptimal solutions may be preferable to optima if:
  - suboptima can be obtained more quickly
  - suboptima can be adapted more quickly
  - suboptima are more robust
  - an ill-defined suboptimum may be better than a sharp optimum
- “The best is the enemy of the good”  
*(Le mieux est l'ennemi du bien. – Voltaire)*

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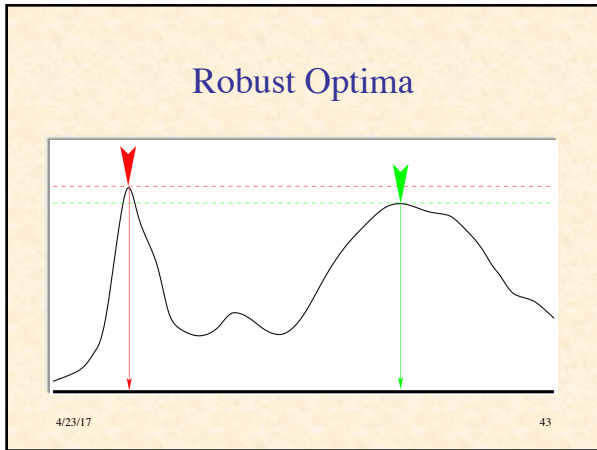
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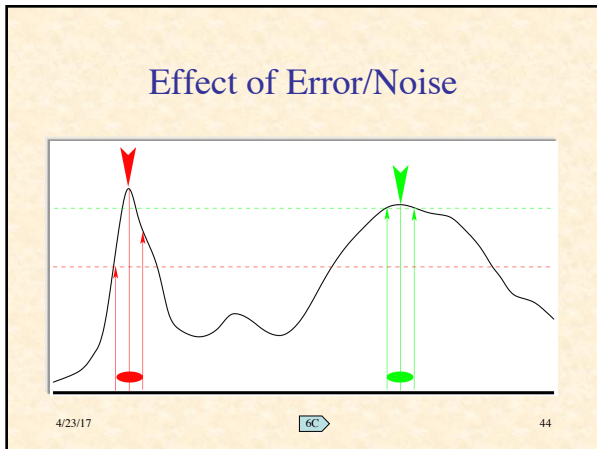
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