II. Cellular Automata

Cellular Automata (CAs)

- Invented by von Neumann in 1940s to study reproduction
- He succeeded in constructing a self-reproducing CA
- Have been used as:
  - massively parallel computer architecture
  - model of physical phenomena (Wolfram)
- Currently being investigated as model of quantum computation (QCAs)

Structure

- Discrete space (lattice) of regular *cells*
  - 1D, 2D, 3D, …
  - rectangular, hexagonal, …
- At each unit of time a cell changes state in response to:
  - its own previous state
  - states of neighbors (within some "radius")
- All cells obey same state update rule
  - an FSA
- Synchronous updating

Example: Conway's Game of Life

- Invented by Conway in late 1960s
- A simple CA capable of universal computation
- Structure:
  - 2D space
  - rectangular lattice of cells
  - binary states (alive/dead)
  - neighborhood of 8 surrounding cells (& self)
  - simple population-oriented rule

State Transition Rule

- Live cell has 2 or 3 live neighbors
  □ stays as is (stasis)
- Live cell has <2 live neighbors
  □ dies (loneliness)
- Live cell has >3 live neighbors
  □ dies (overcrowding)
- Empty cell has 3 live neighbors
  □ comes to life (reproduction)

Demonstration of Life

Go to CBN
Online Experimentation Center

Some Observations About Life

1. Long, chaotic-looking initial transient
   – unless initial density too low or high
2. Intermediate phase
   – isolated islands of complex behavior
   – matrix of static structures & "blinkers"
   – gliders creating long-range interactions
3. Cyclic attractor
   – typically short period

From Life to CAs in General

- What gives Life this very rich behavior?
- Is there some simple, general way of characterizing CAs with rich behavior?
- It belongs to Wolfram's Class IV

The four classes of feedback behaviour

(a) Fixed points

(b) Simple periodic orbits

(c) Period-n orbit

(d) Chaos

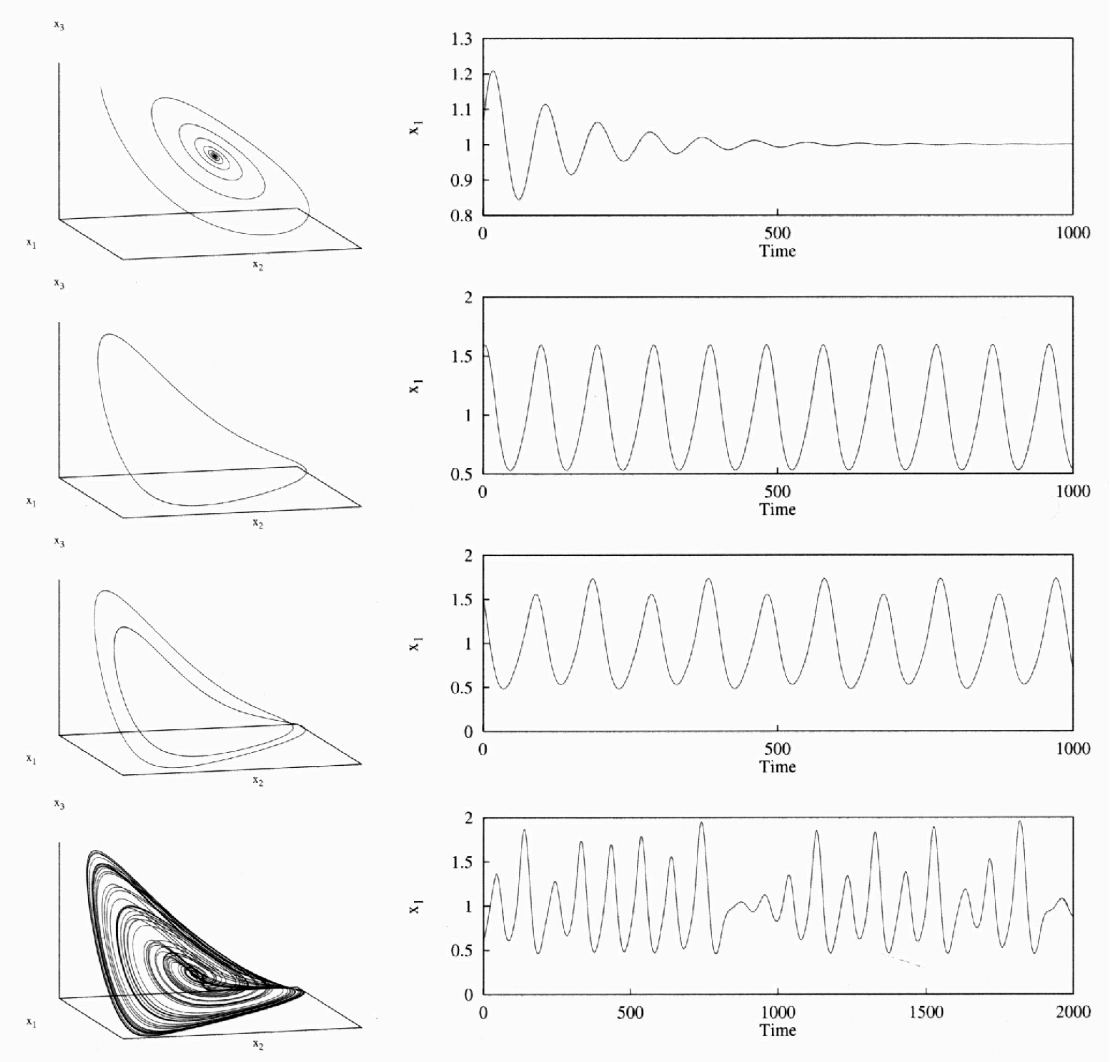


fig. from Flake via EVALife

Wolfram's Classification

- Class I: evolve to fixed, homogeneous state
  ~ limit point
- Class II: evolve to simple separated periodic structures
  ~ limit cycle
- Class III: yield chaotic aperiodic patterns
  ~ strange attractor (chaotic behavior)
- Class IV: complex patterns of localized structure
  ~ long transients, no analog in dynamical systems

Langton's Investigation

Under what conditions can we expect a complex dynamics of information to emerge spontaneously and come to dominate the behavior of a CA?

Approach

- Investigate 1D CAs with:
  - random transition rules
  - starting in random initial states
- Systematically vary a simple parameter characterizing the rule
- Evaluate qualitative behavior (Wolfram class)

Assumptions

- Periodic boundary conditions
  - no special place
- Strong quiescence:
  - if all the states in the neighborhood are the same, then the new state will be the same
  - persistence of uniformity
- Spatial isotropy:
  - all rotations of neighborhood state result in same new state
  - no special direction
- Totalistic [not used by Langton]:
  - depend only on sum of states in neighborhood
  - implies spatial isotropy

Langton's Lambda

- Designate one state to be quiescent state
- Let K = number of states
- Let $N = 2r + 1$ = area of neighborhood
- Let $T = K^N$ = number of entries in table
- Let n_q = number mapping to quiescent state
- Then

$$\lambda = \frac{T - n_q}{T}$$

Range of Lambda Parameter

- If *all* configurations map to quiescent state:
  $$\lambda = 0$$
- If *no* configurations map to quiescent state:
  $$\lambda = 1$$
  - If every state is represented *equally*:
    $$\lambda = 1 - 1/K$$
  - A sort of measure of "excitability"

Entropy

- Among other things, a way to measure the uniformity of a distribution

$$H = -\sum_{i} p_i \lg p_i$$

- Let n_k = number mapping into state k

$$H = \lg T - \frac{1}{T}\sum_{k=1}^{K} n_k \lg n_k$$

Entropy Range

- Maximum entropy:

  uniform as possible

  all $n_k = T/K$

  $H_{\max} = \lg K$

- Minimum entropy:

  nonuniform as possible

  one $n_s = T$

  all other $n_r = 0 \ (r \neq s)$

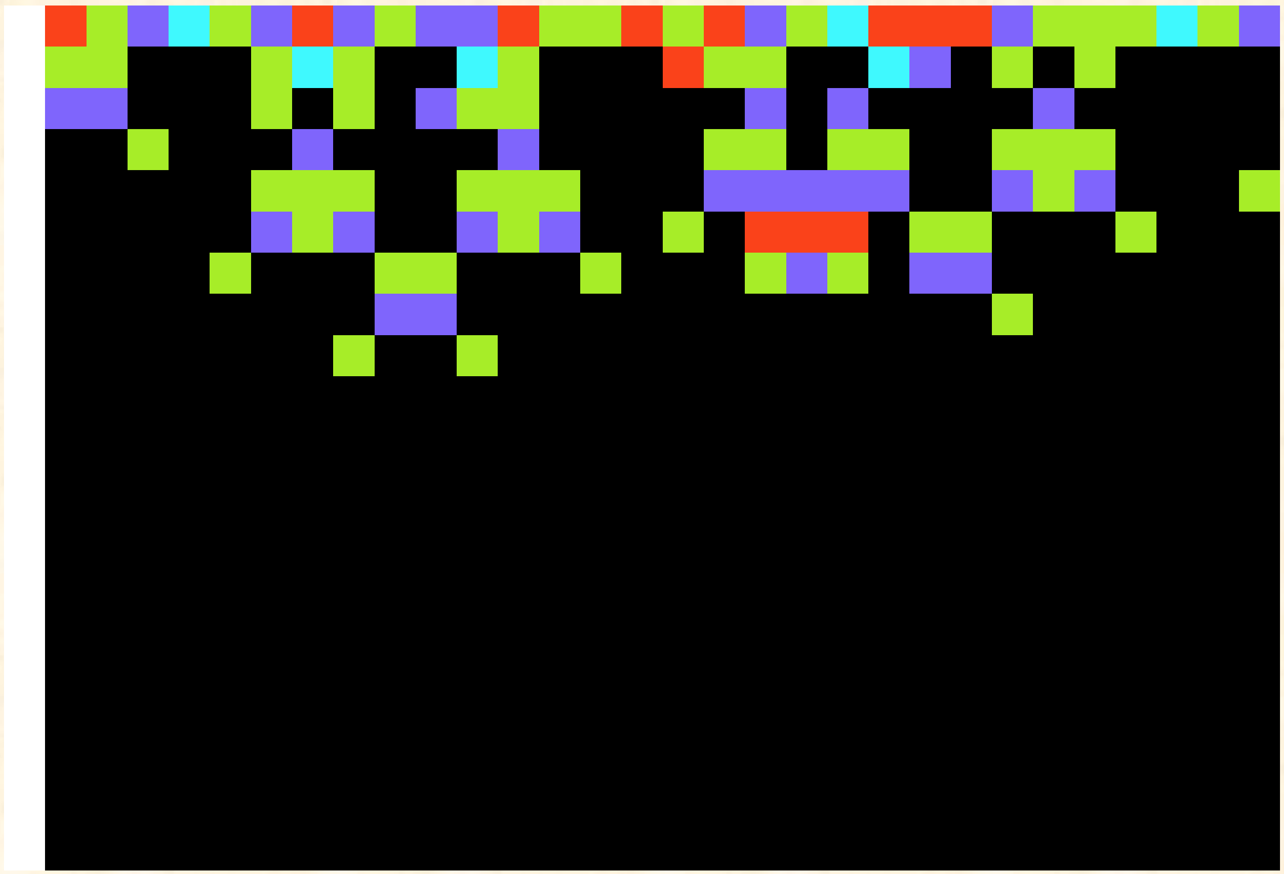
  $H_{\min} = 0$

Example

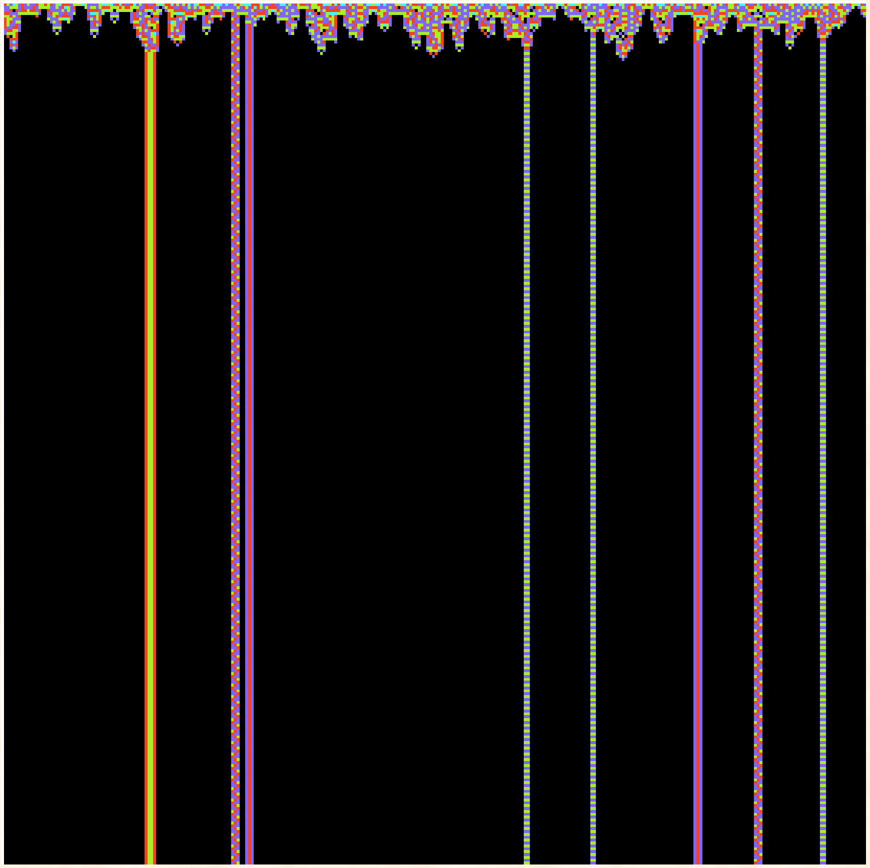
- States: $K = 5$
- Radius: $r = 1$
- Initial state: random
- Transition function: random (given □)

Class I (□ = 0.2)

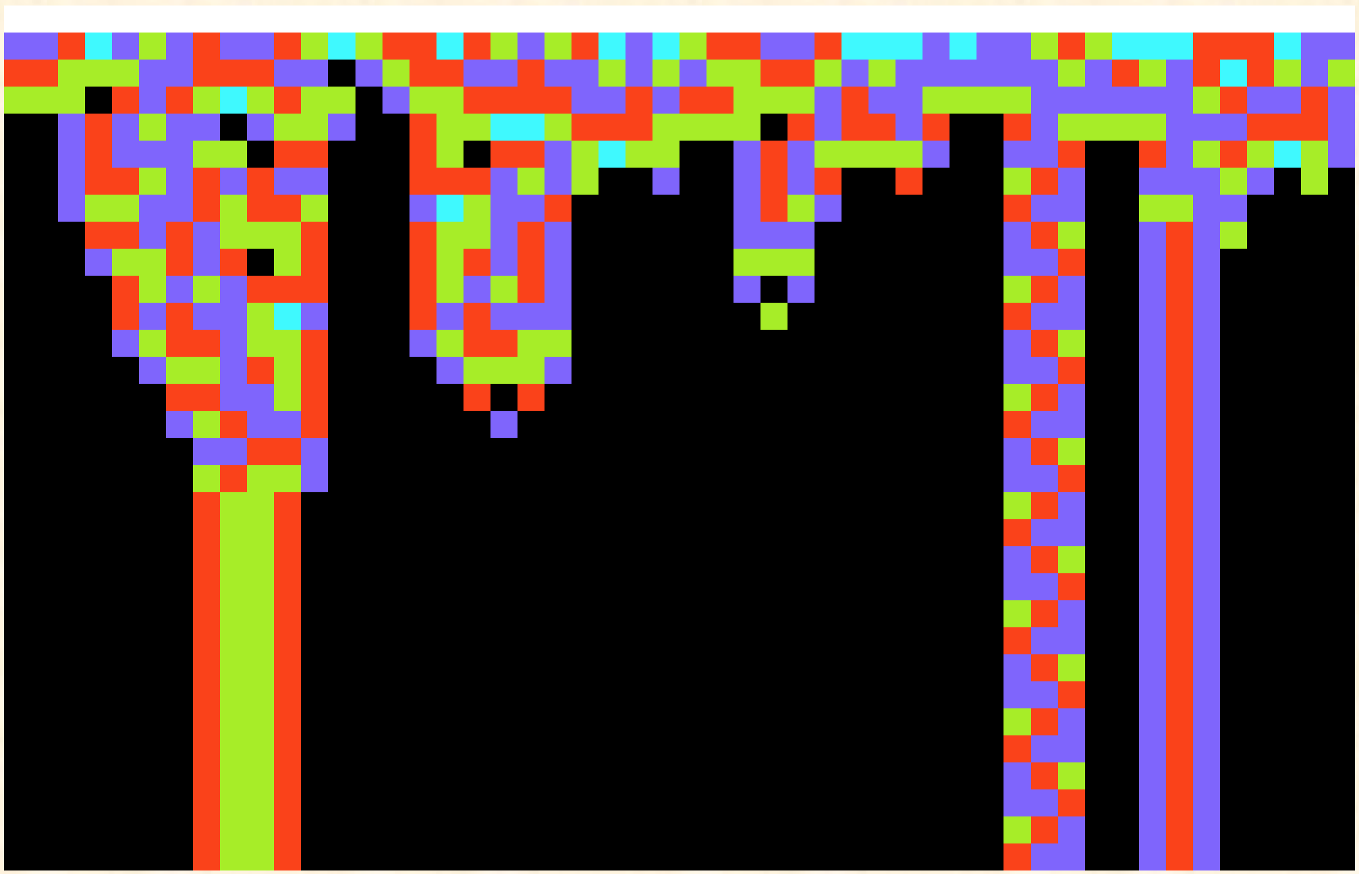
Class I (λ = 0.2) Closeup



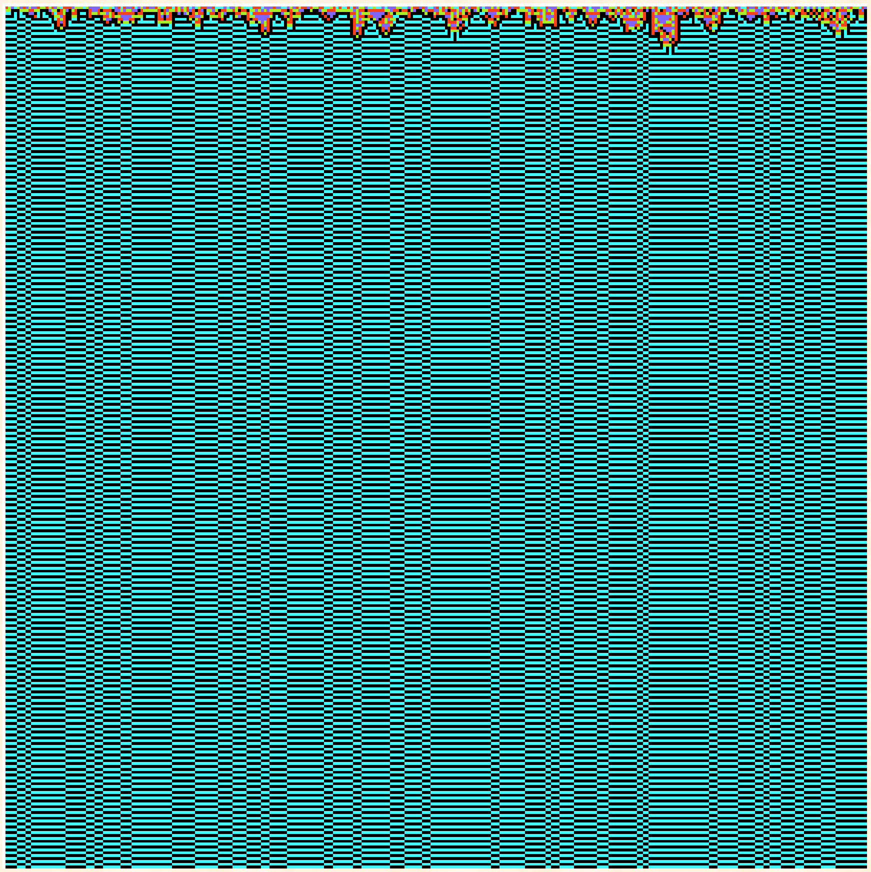
Class II (= 0.4)



Class II (λ = 0.4) Closeup

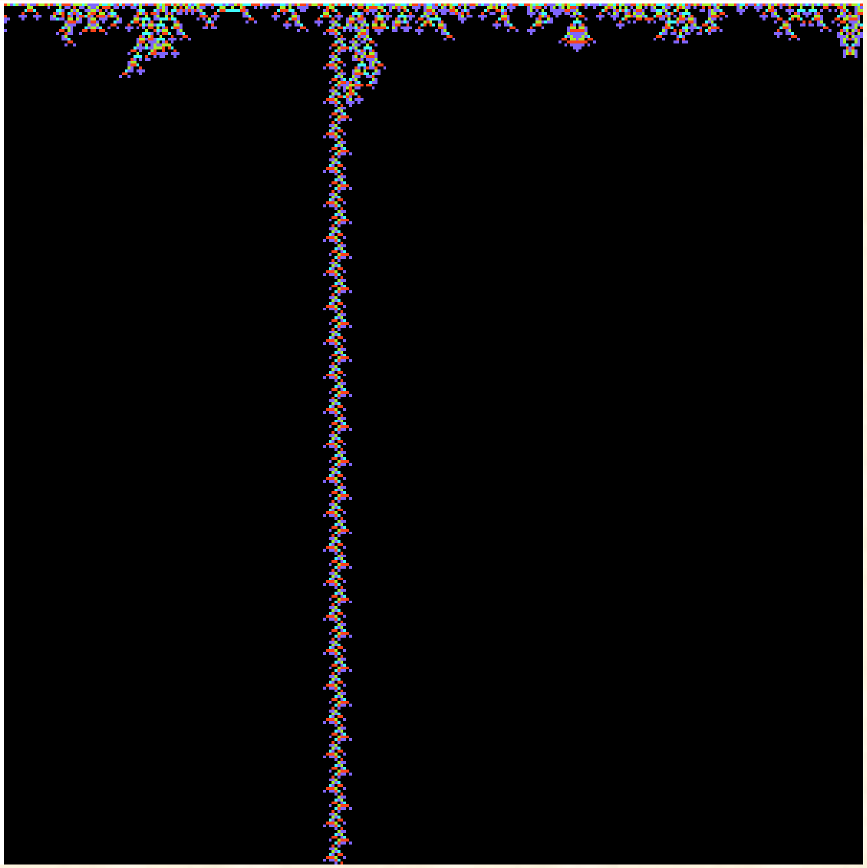


Class II ($\lambda = 0.31$)

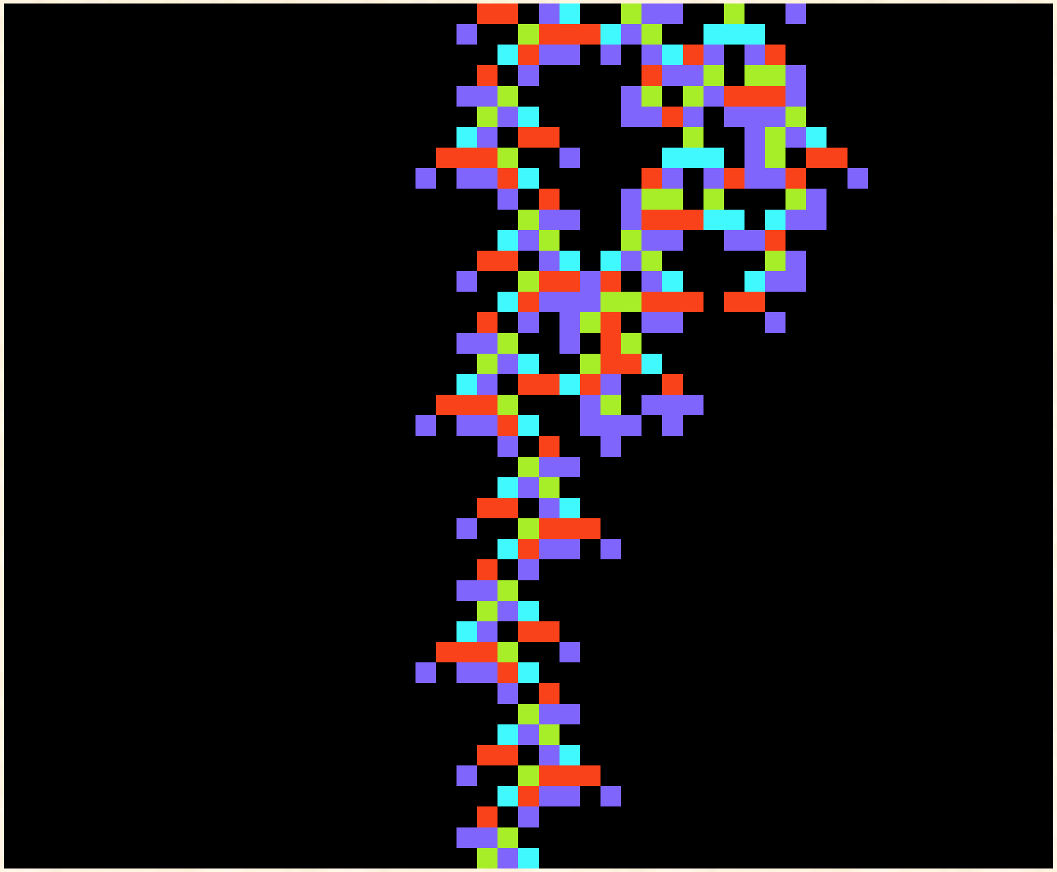


Class II (λ = 0.31) Closeup

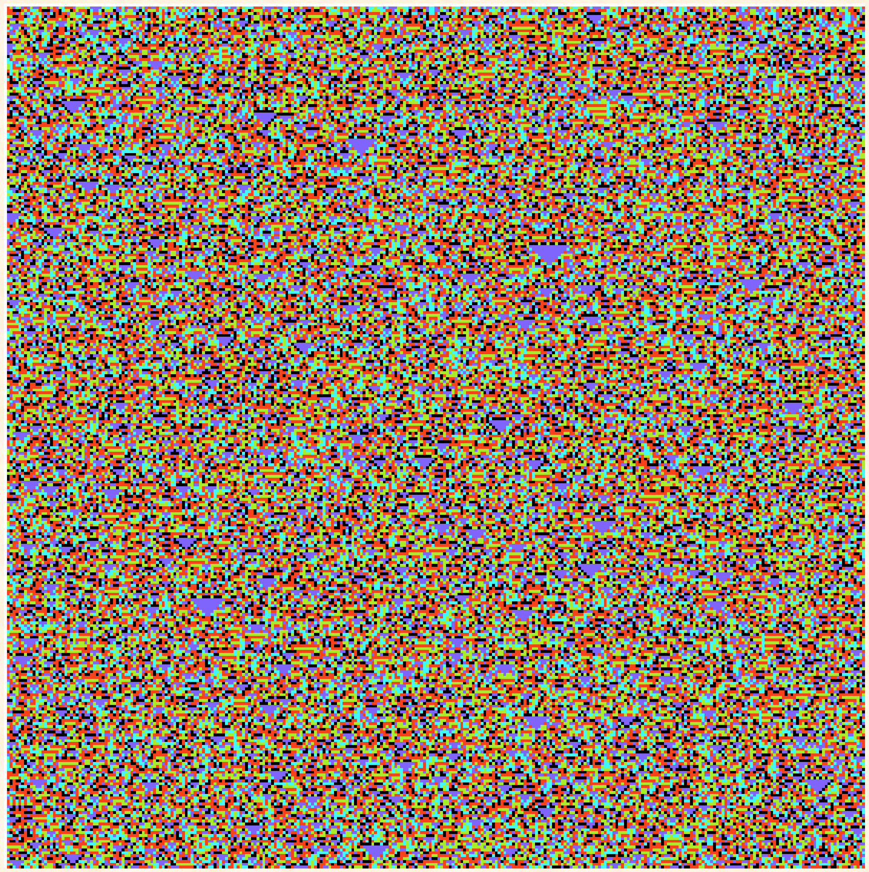
Class II (λ = 0.37)



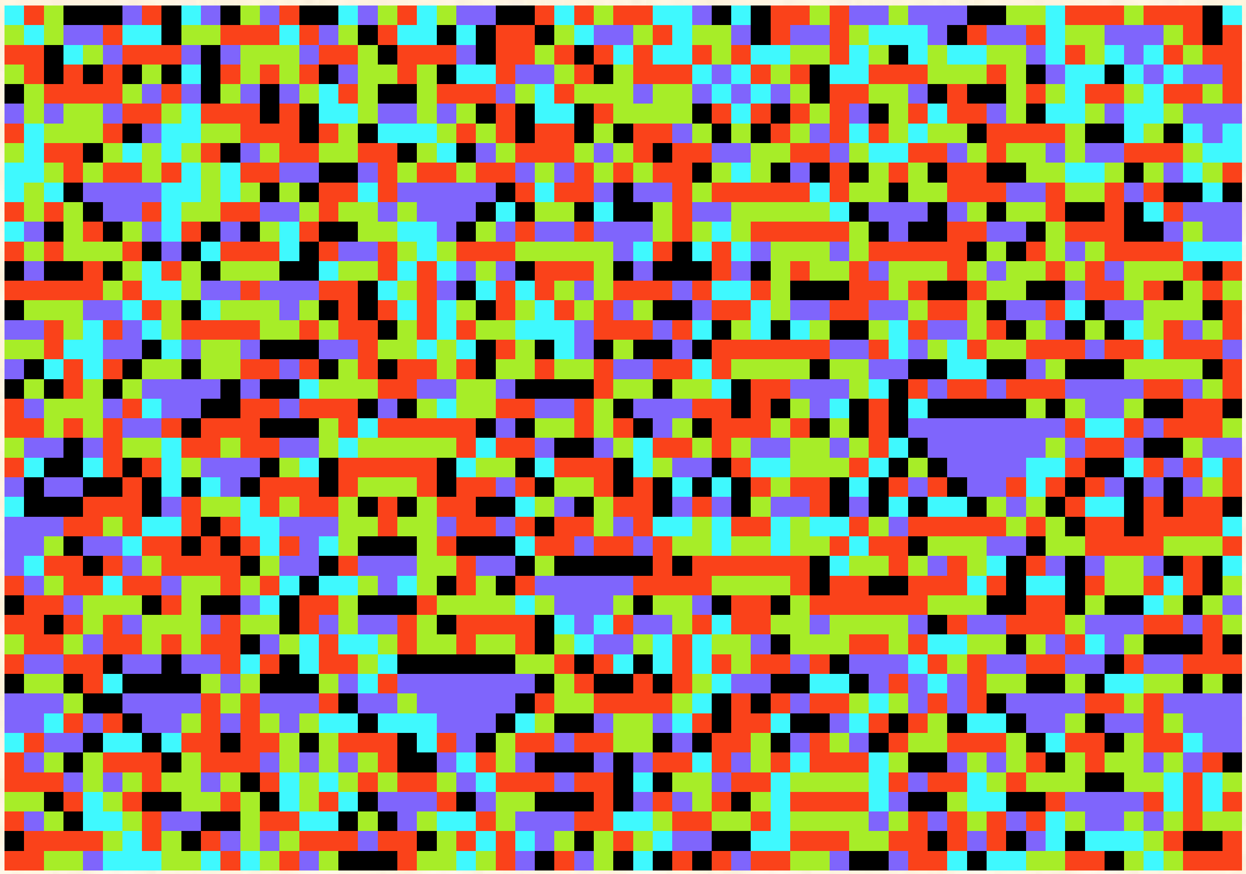
Class II (λ = 0.37) Closeup



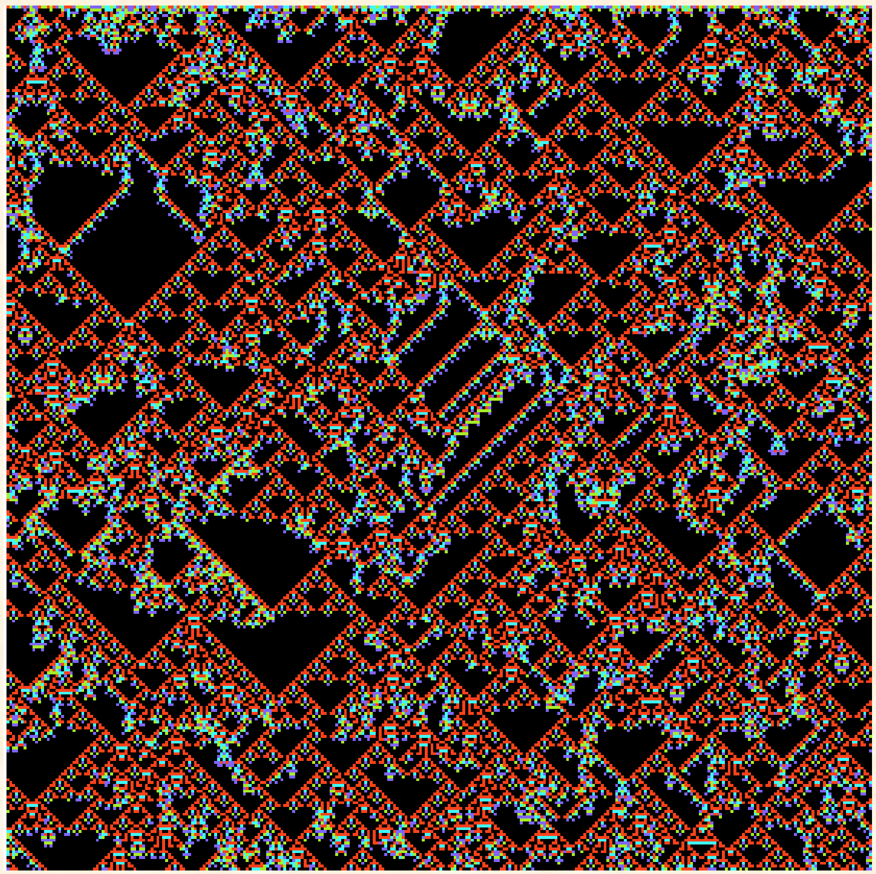
Class III ($\lambda = 0.5$)



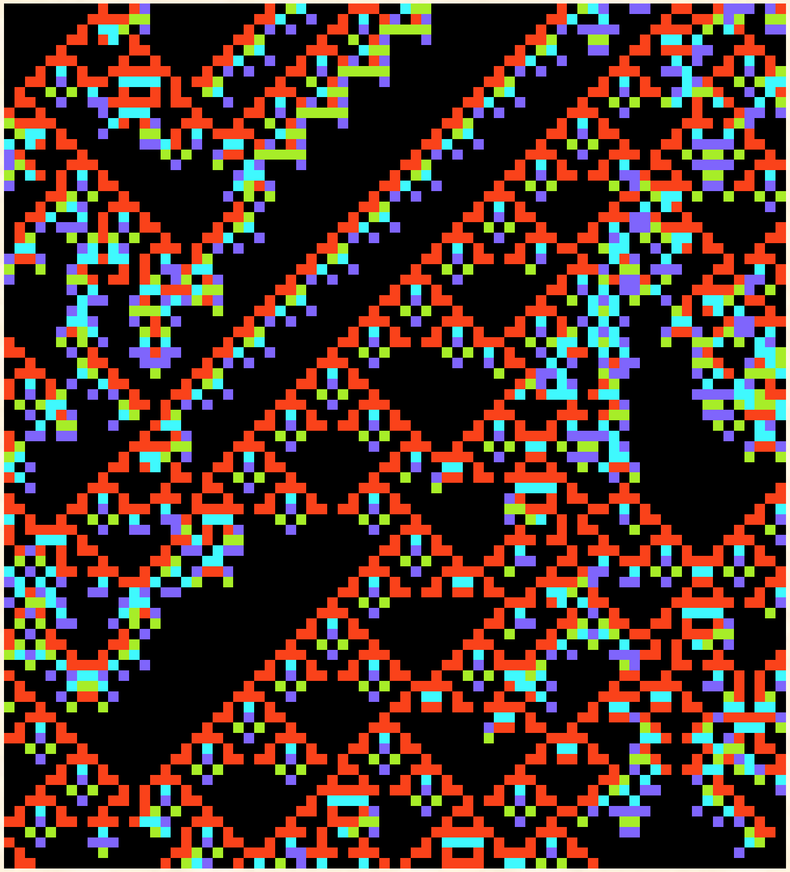
Class III (□ = 0.5) Closeup



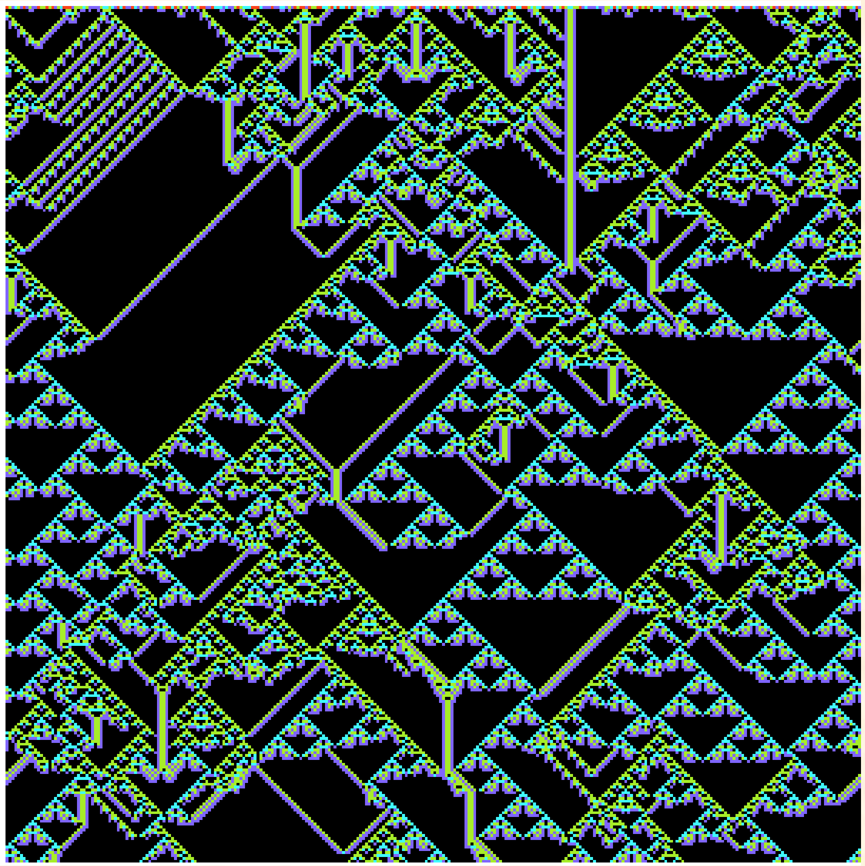
Class IV ($\lambda = 0.35$)

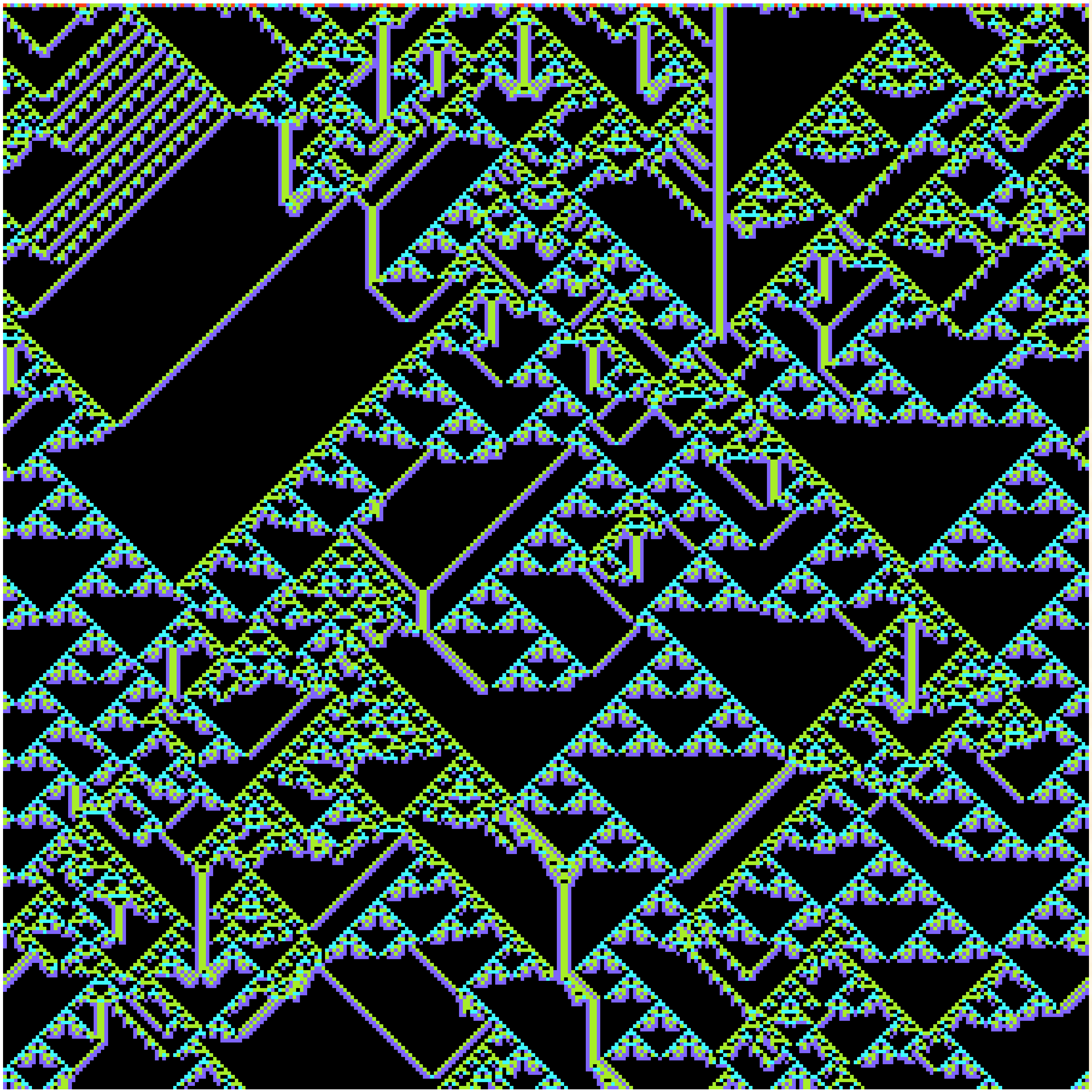


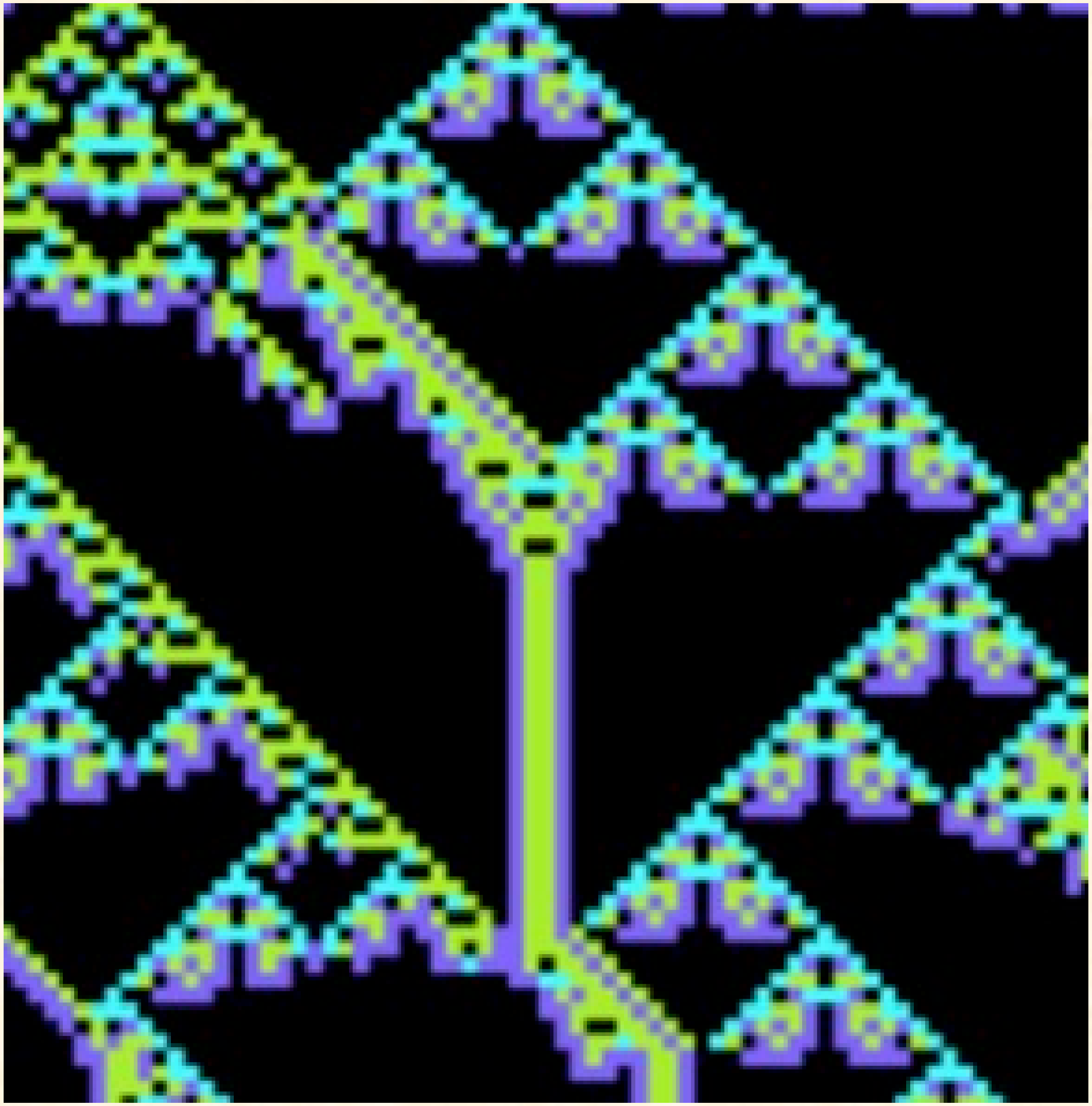
Class IV (λ = 0.35) Closeup

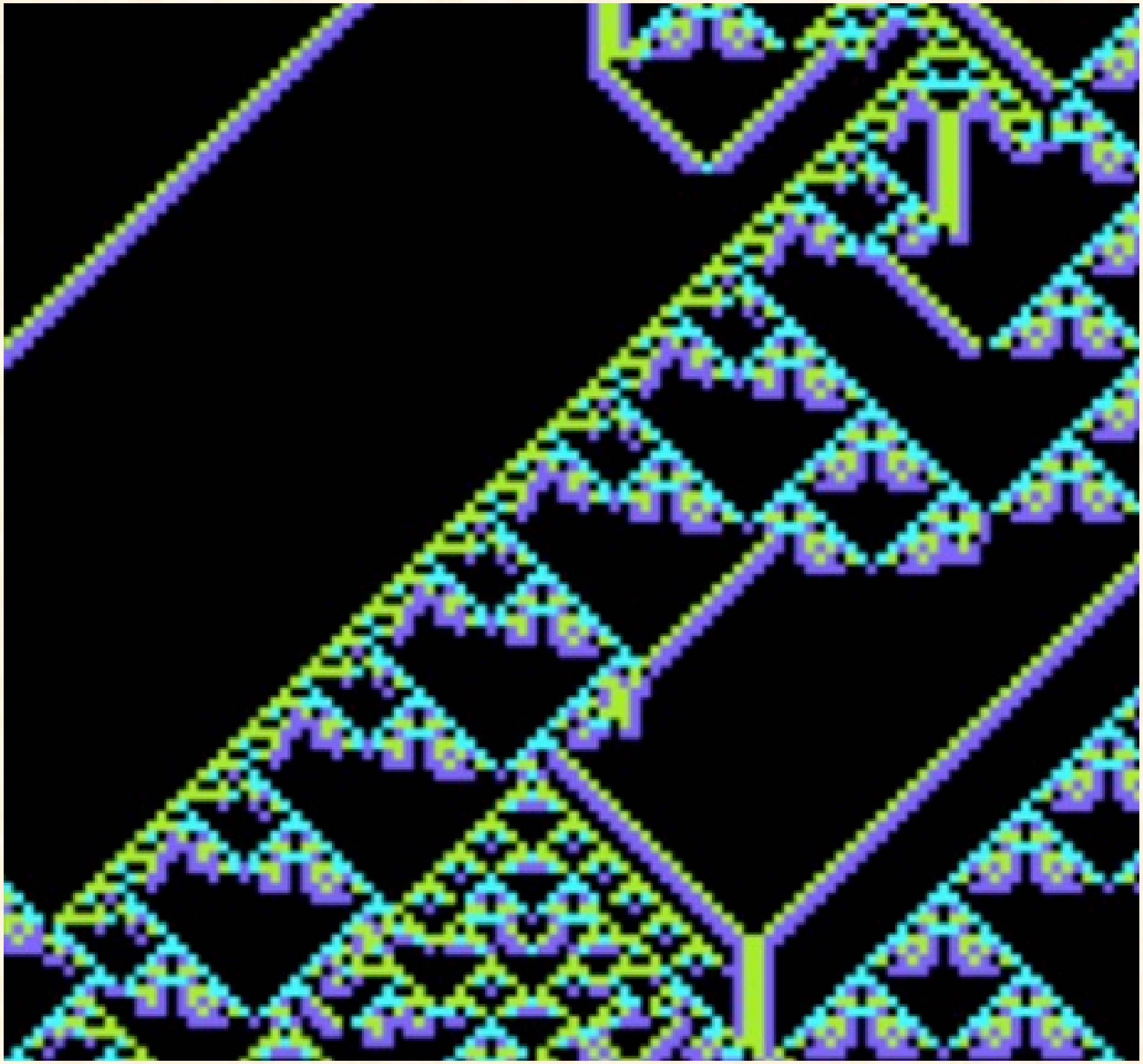


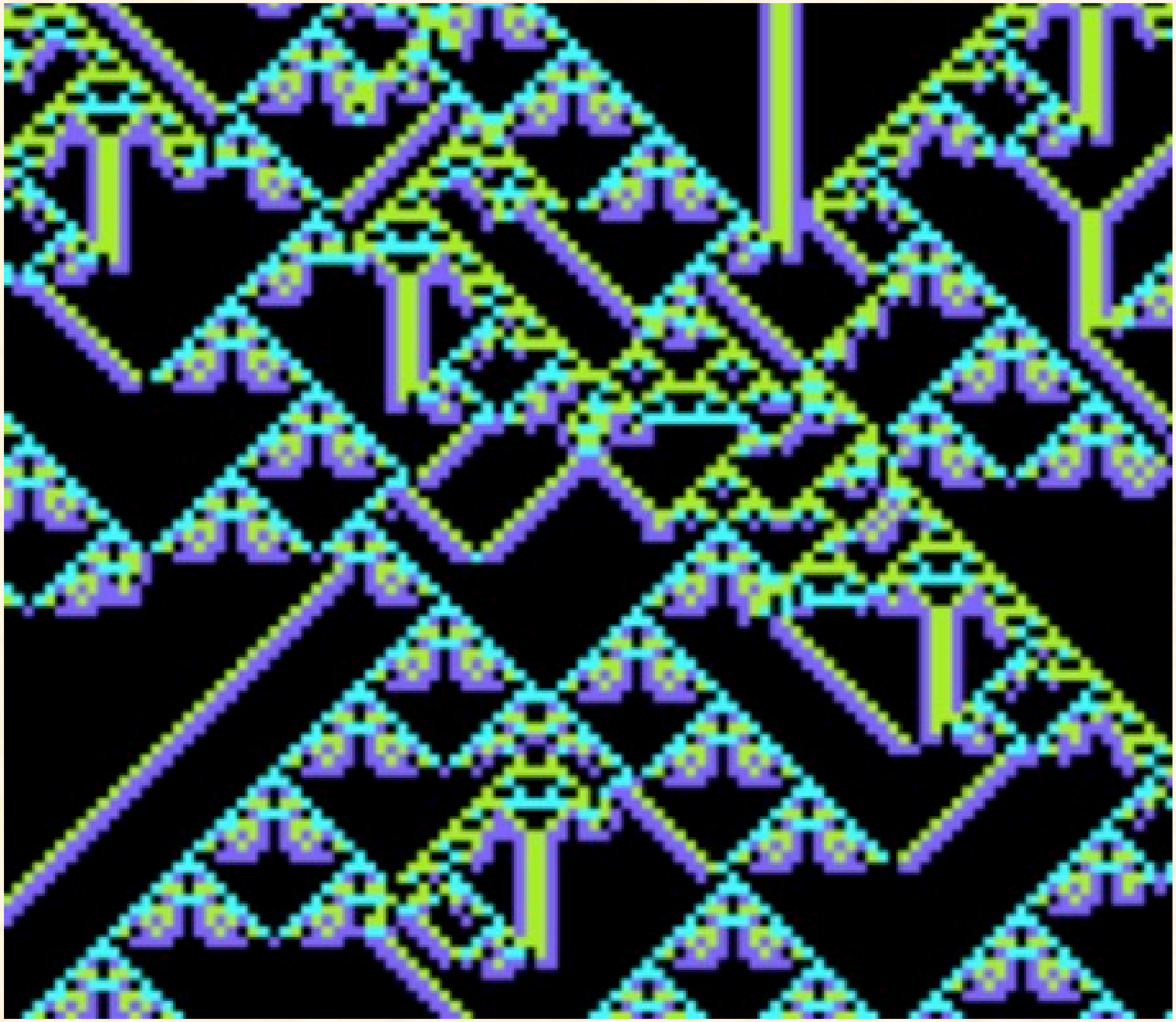
Class IV (λ = 0.34)











Class IV Shows Some of the Characteristics of Computation

- Persistent, but not perpetual storage
- Terminating cyclic activity
- Global transfer of control/information

□ of Life

- For Life, □ ≈ 0.273
- which is near the critical region for CAs with:

  $K = 2$

  $N = 9$