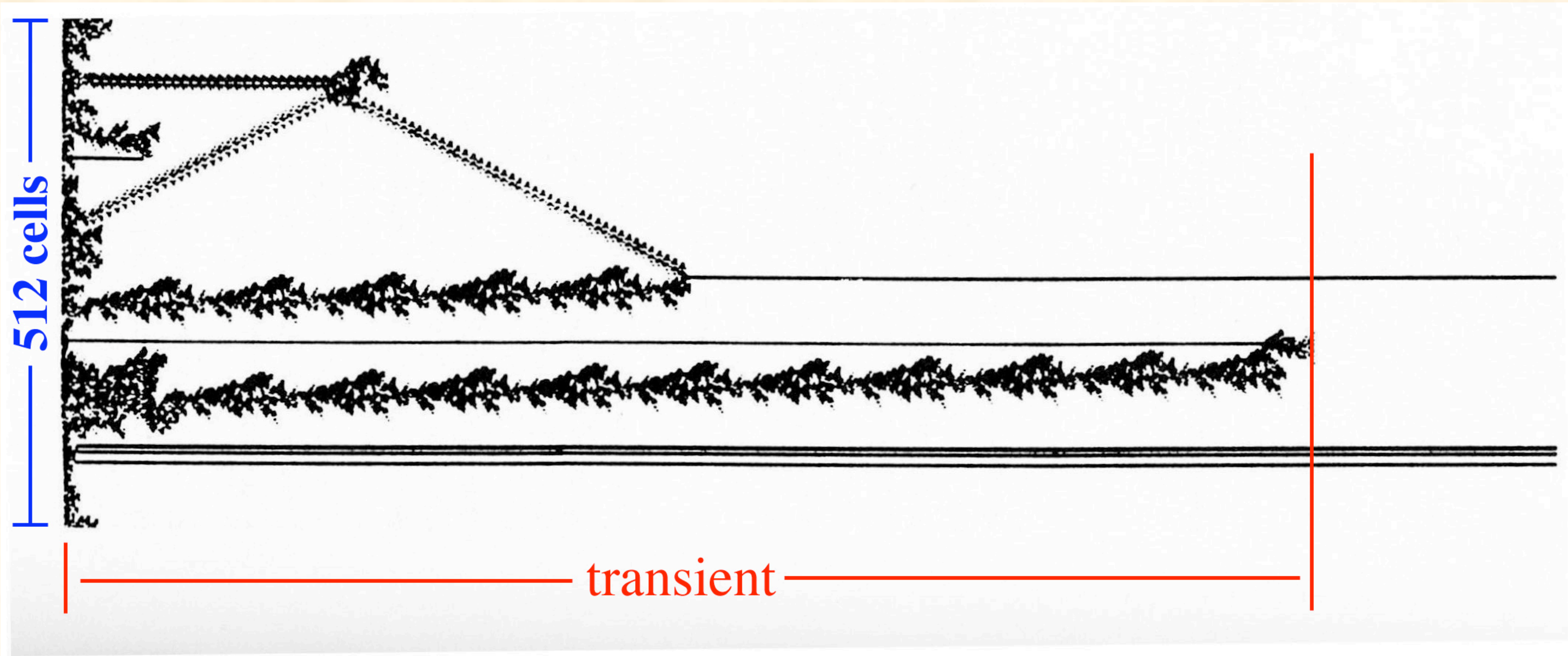
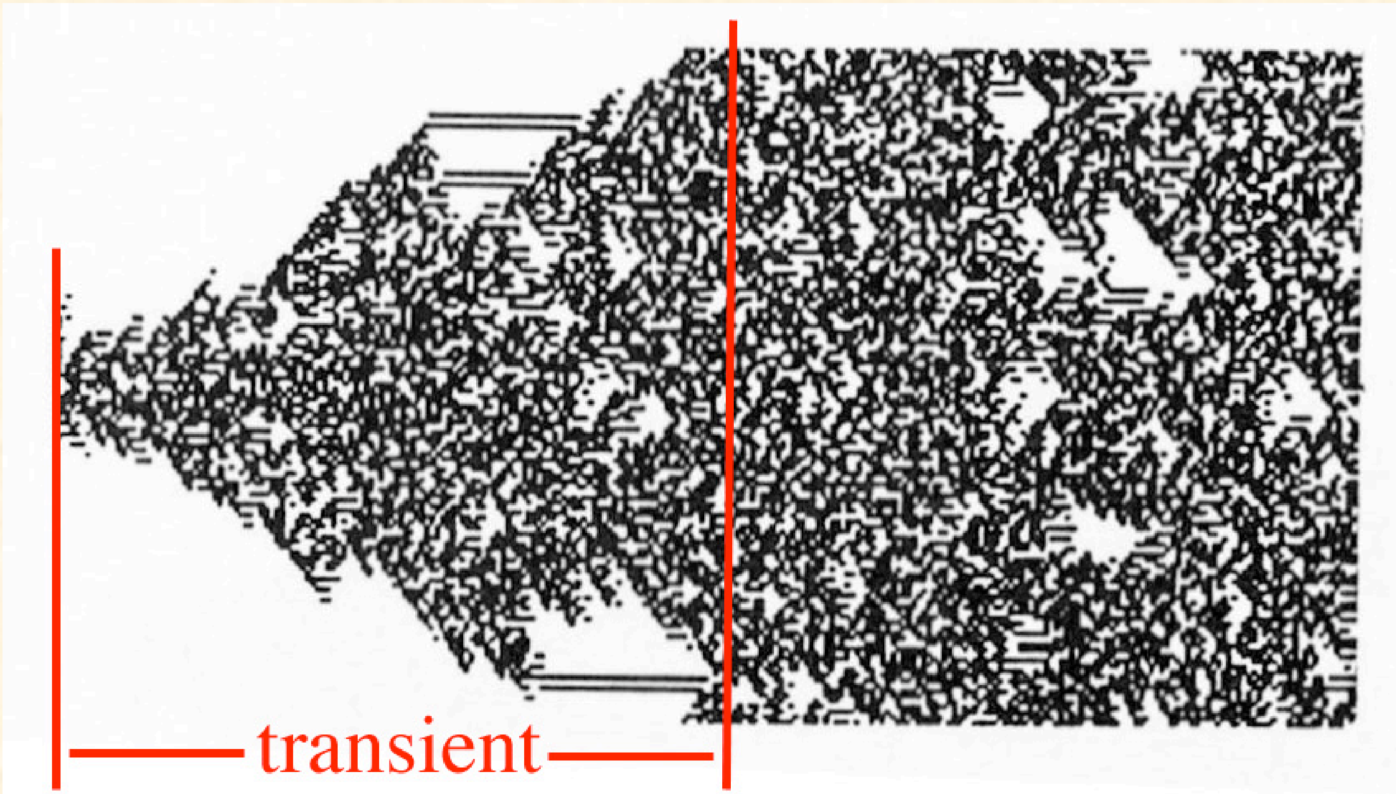
Further Investigations by Langton

- 2-D CAs
- K = 8
- N = 5
- 64 × 64 lattice
- periodic boundary conditions

Transient Length (I, II)



Transient Length (III)



Shannon Information
(very briefly!)

- Information varies directly with surprise
- Information varies inversely with probability
- Information is additive
- □ The information content of a message is proportional to the negative log of its probability

$$I\{s\} = -\lg \Pr\{s\}$$

Entropy

- Suppose have source S of symbols from ensemble $\{s_1, s_2, \ldots, s_N\}$
- Average information per symbol:

$$\sum_{k=1}^{N} \Pr\{s_k\} I\{s_k\} = \sum_{k=1}^{N} \Pr\{s_k\}\left(-\lg \Pr\{s_k\}\right)$$

- This is the *entropy* of the source:

$$H\{S\} = -\sum_{k=1}^{N} \Pr\{s_k\} \lg \Pr\{s_k\}$$

Maximum and Minimum Entropy

- Maximum entropy is achieved when all signals are equally likely

  No ability to guess; maximum surprise

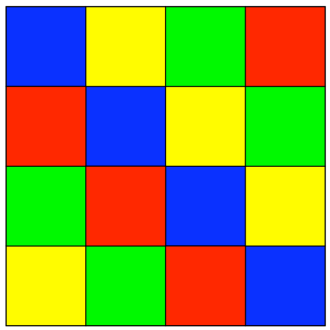
  $H_{\max} = \lg N$

- Minimum entropy occurs when one symbol is certain and the others are impossible

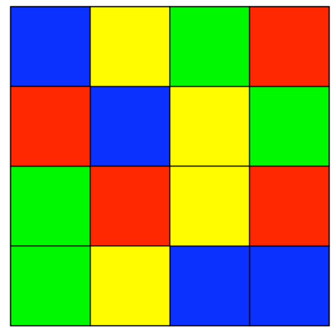
  No uncertainty; no surprise

  $H_{\min} = 0$

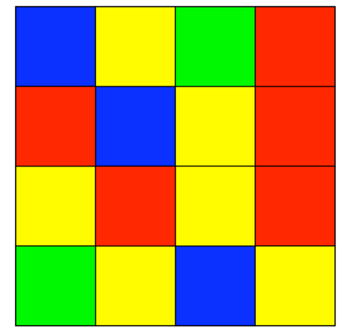
Entropy Examples



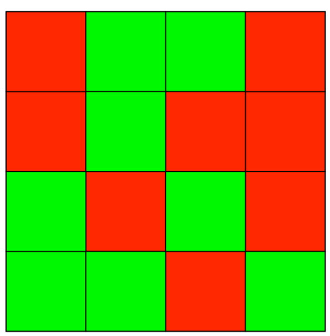
H = 2.0 bits



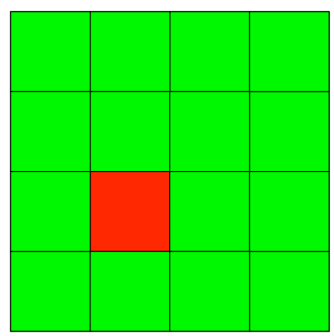
H = 2.0 bits



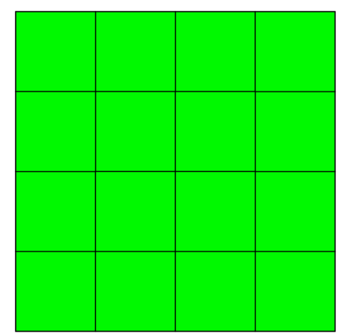
H = 1.9 bits



H = 1.0 bits

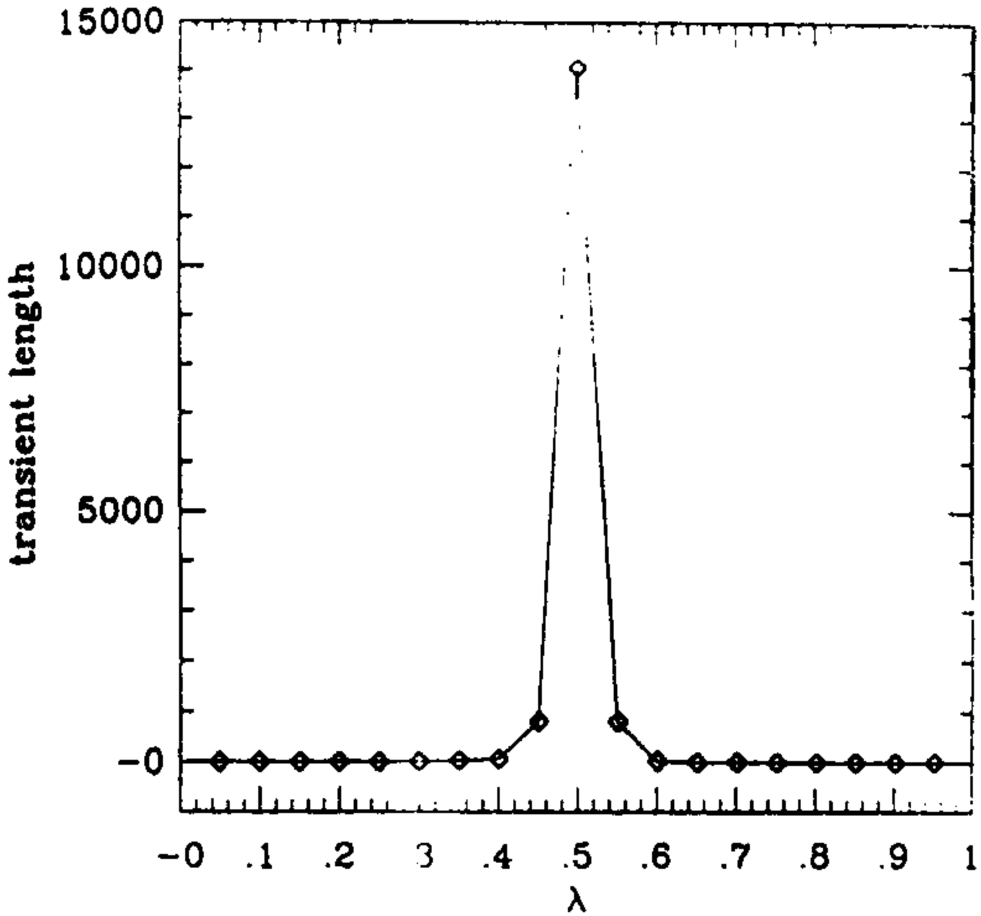


H = 0.3 bits

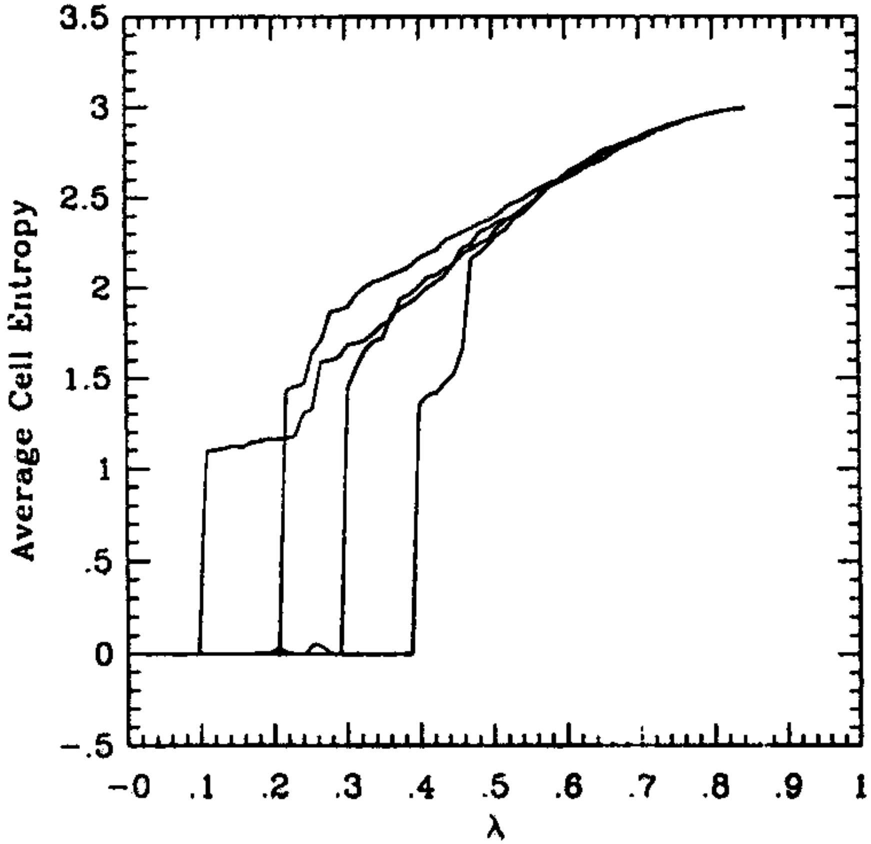


H = 0.0 bits

Avg. Transient Length vs. λ
(K=4, N=5)

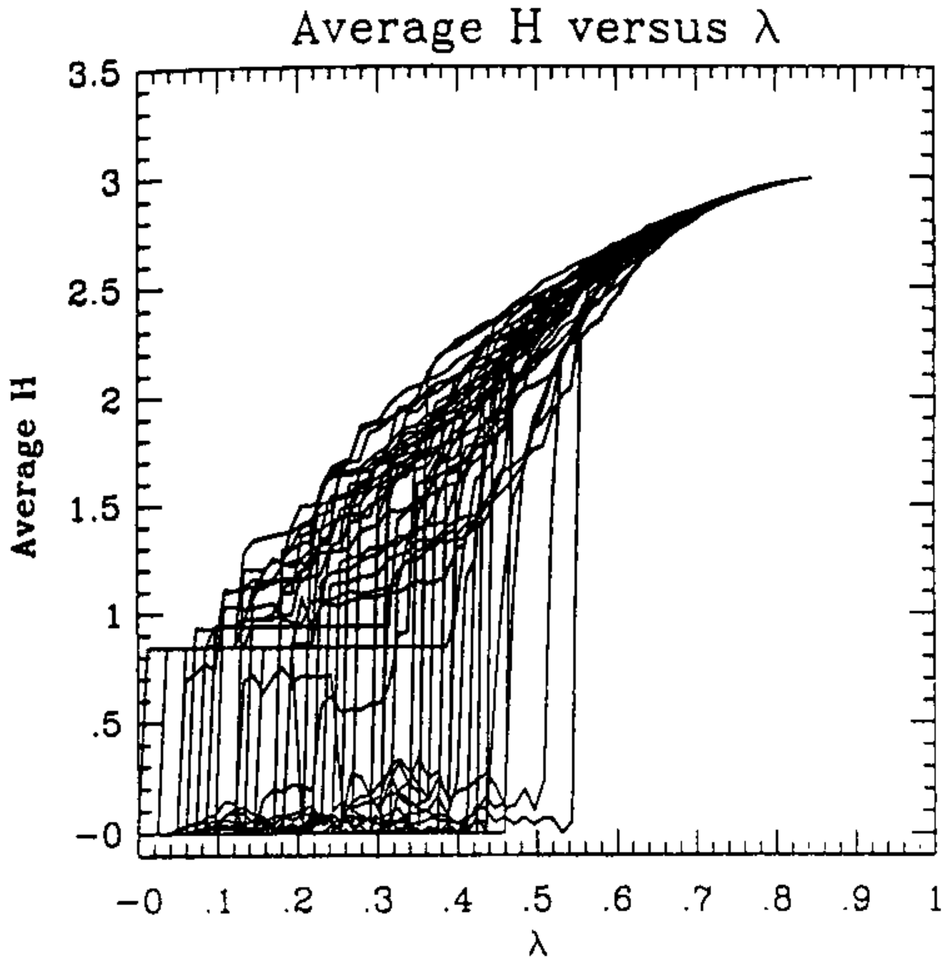


Avg. Cell Entropy vs. □ (K=4, N=5)

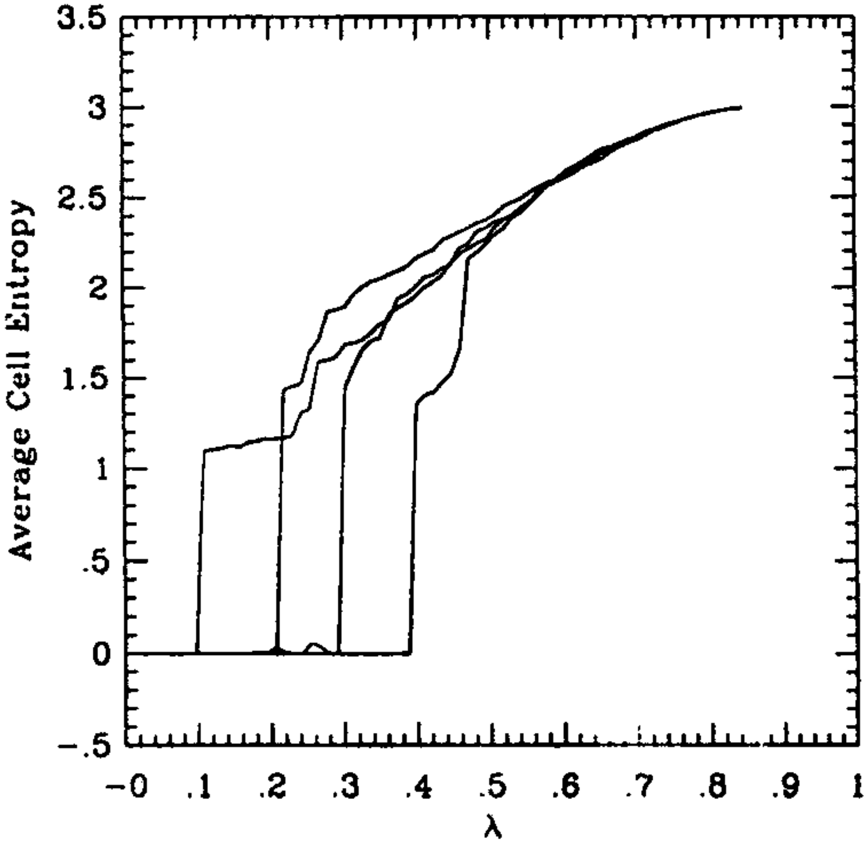


$$H(A) = \sum_{k=1}^{K} p_k \lg p_k$$

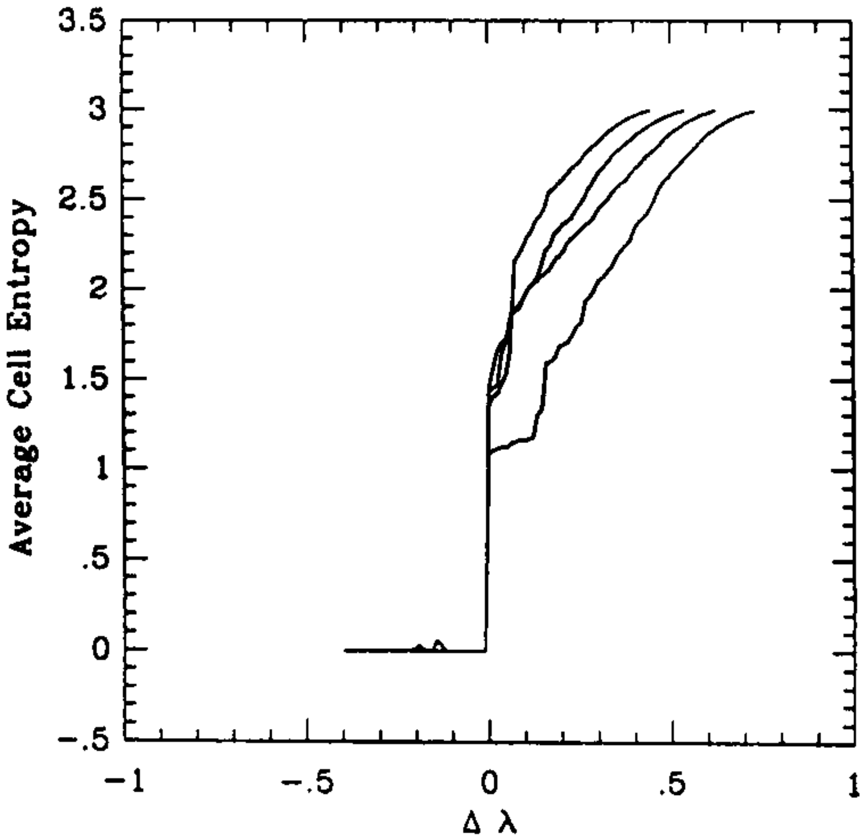
Avg. Cell Entropy vs.
(K=4, N=5)



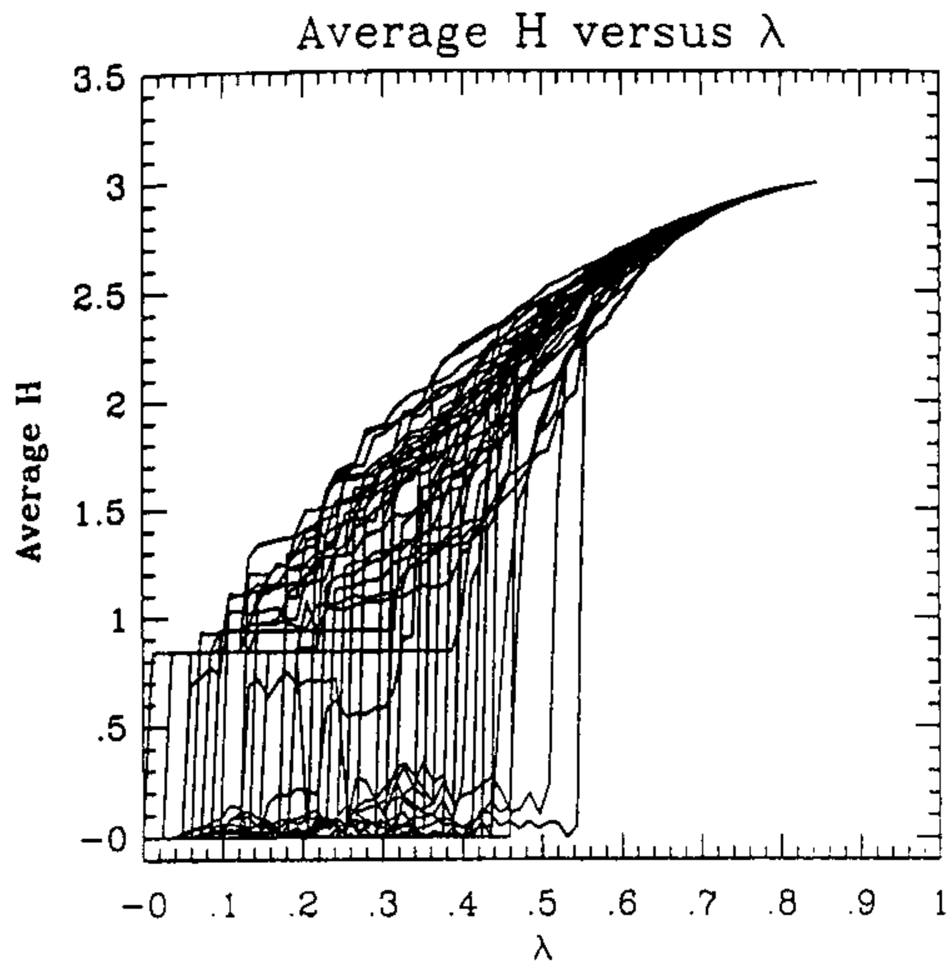
Avg. Cell Entropy vs. λ
(K=4, N=5)



Avg. Cell Entropy vs. □□
(*K*=4, *N*=5)

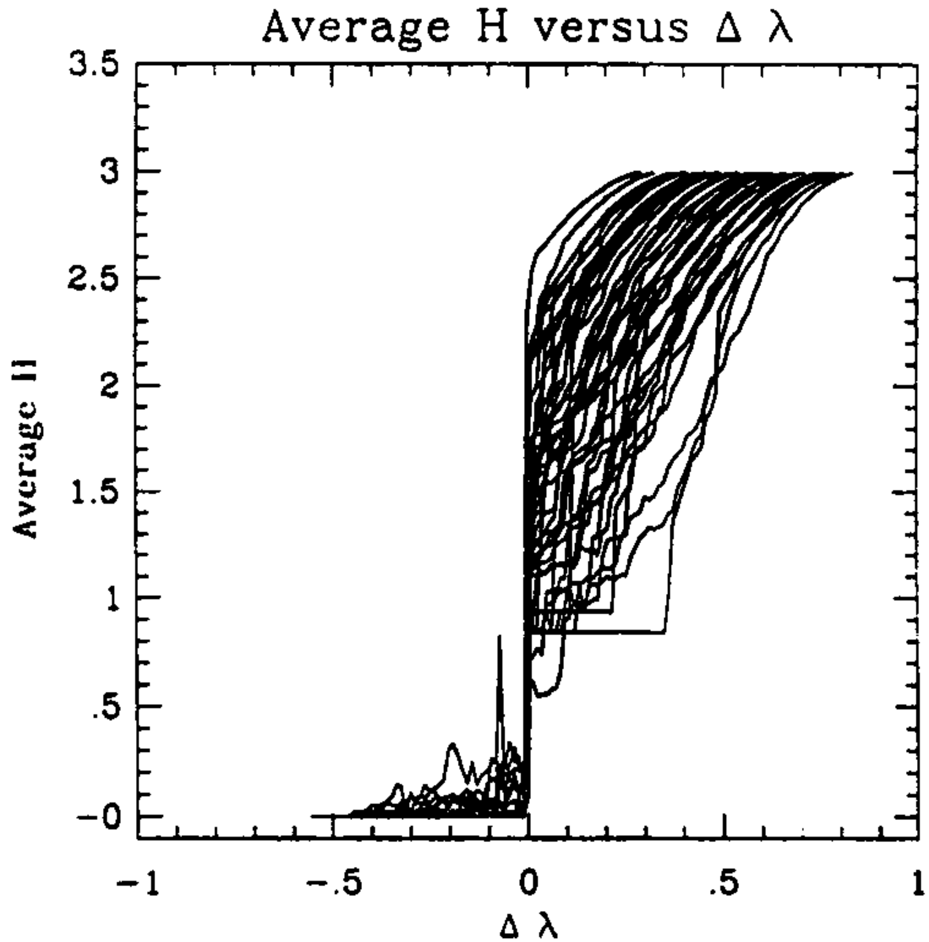


Avg. Cell Entropy vs. □ (K=4, N=5)



Avg. Cell Entropy vs.

(K=4, N=5)



Entropy of Independent Systems

- Suppose sources *A* and *B* are independent
- Let $p_j = \Pr\{a_j\}$ and $q_k = \Pr\{b_k\}$
- Then $\Pr\{a_j, b_k\} = \Pr\{a_j\} \Pr\{b_k\} = p_j q_k$

$$H(A,B) = \sum_{j,k} \Pr(a_j, b_k) \lg \Pr(a_j, b_k)$$

$$= \sum_{j,k} p_j q_k \lg(p_j q_k) = \sum_{j,k} p_j q_k (\lg p_j + \lg q_k)$$

$$= \sum_{j} p_j \lg p_j + \sum_{k} q_k \lg q_k = H(A) + H(B)$$

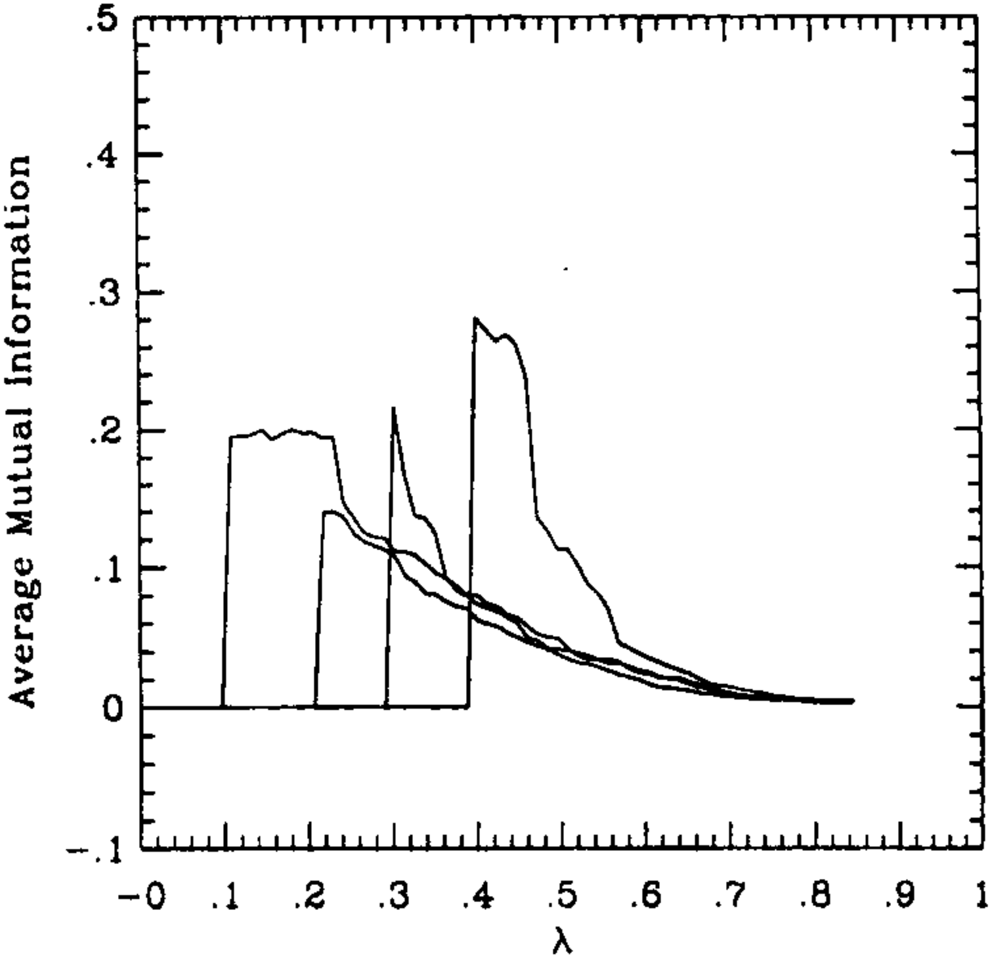
Mutual Information

- *Mutual information* measures the degree to which two sources are not independent
- A measure of their correlation

$$I(A,B) = H(A) + H(B) - H(A,B)$$

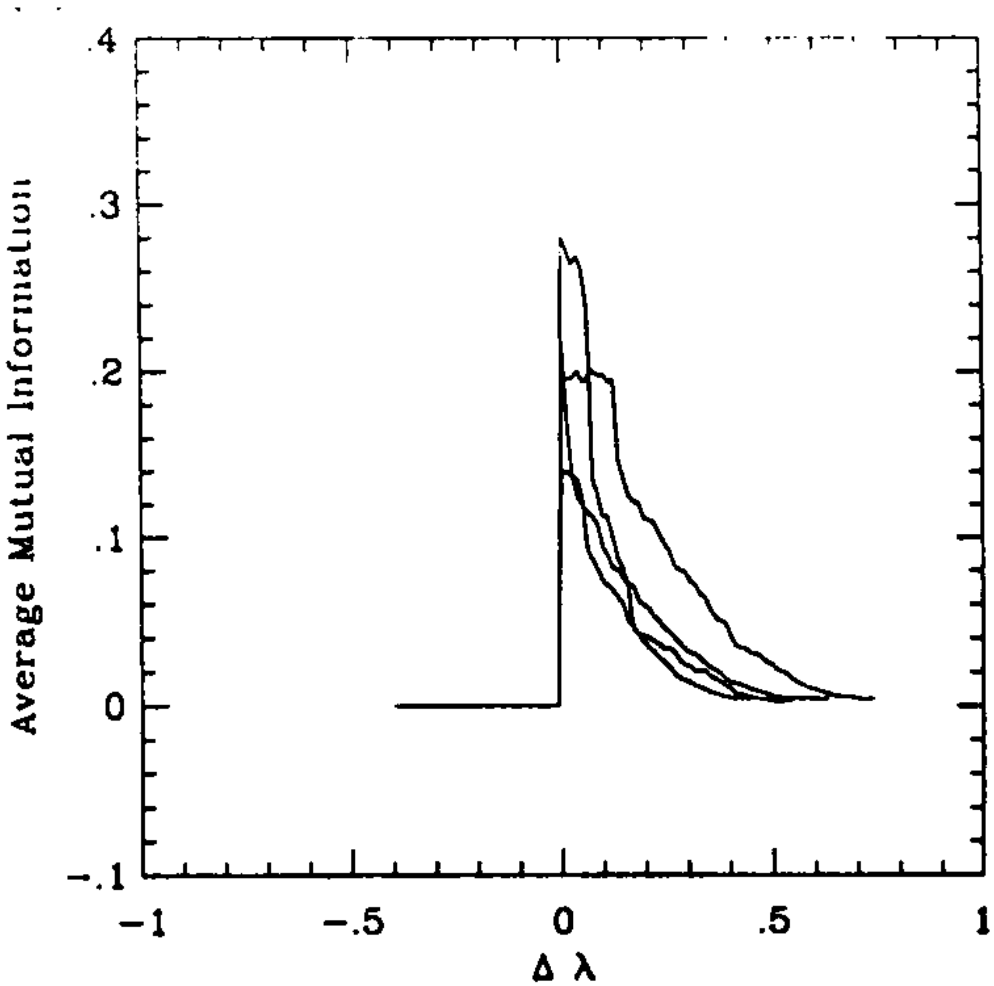
- $I(A,B) = 0$ for completely independent sources
- $I(A,B) = H(A) = H(B)$ for completely correlated sources

Avg. Mutual Info vs.
(K=4, N=5)

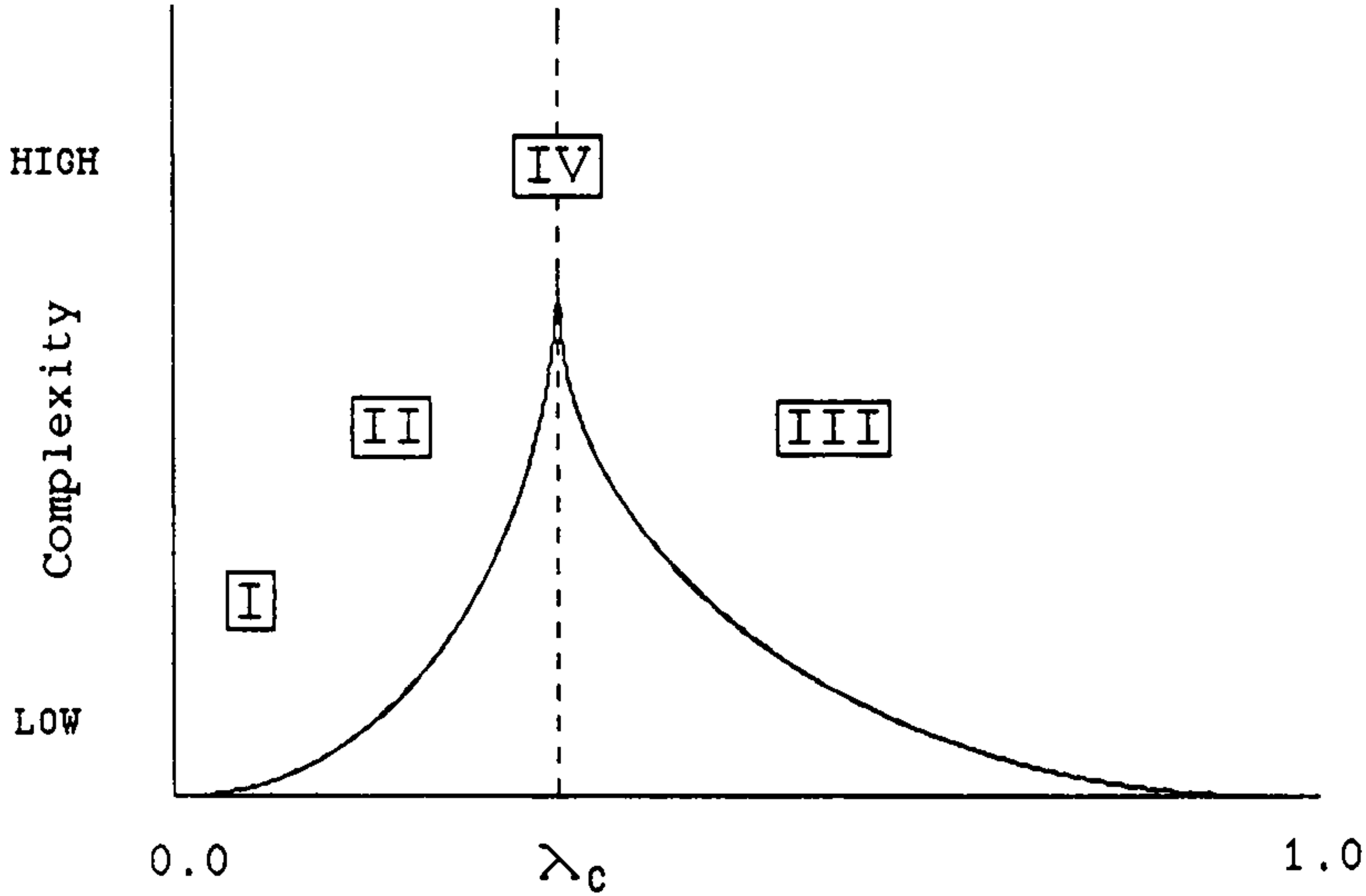


$I(A,B) =$
$H(A) + H(B)$
$- H(A,B)$

Avg. Mutual Info vs. □□ (*K*=4, *N*=5)



Complexity vs.



Schematic of CA Rule Space vs.

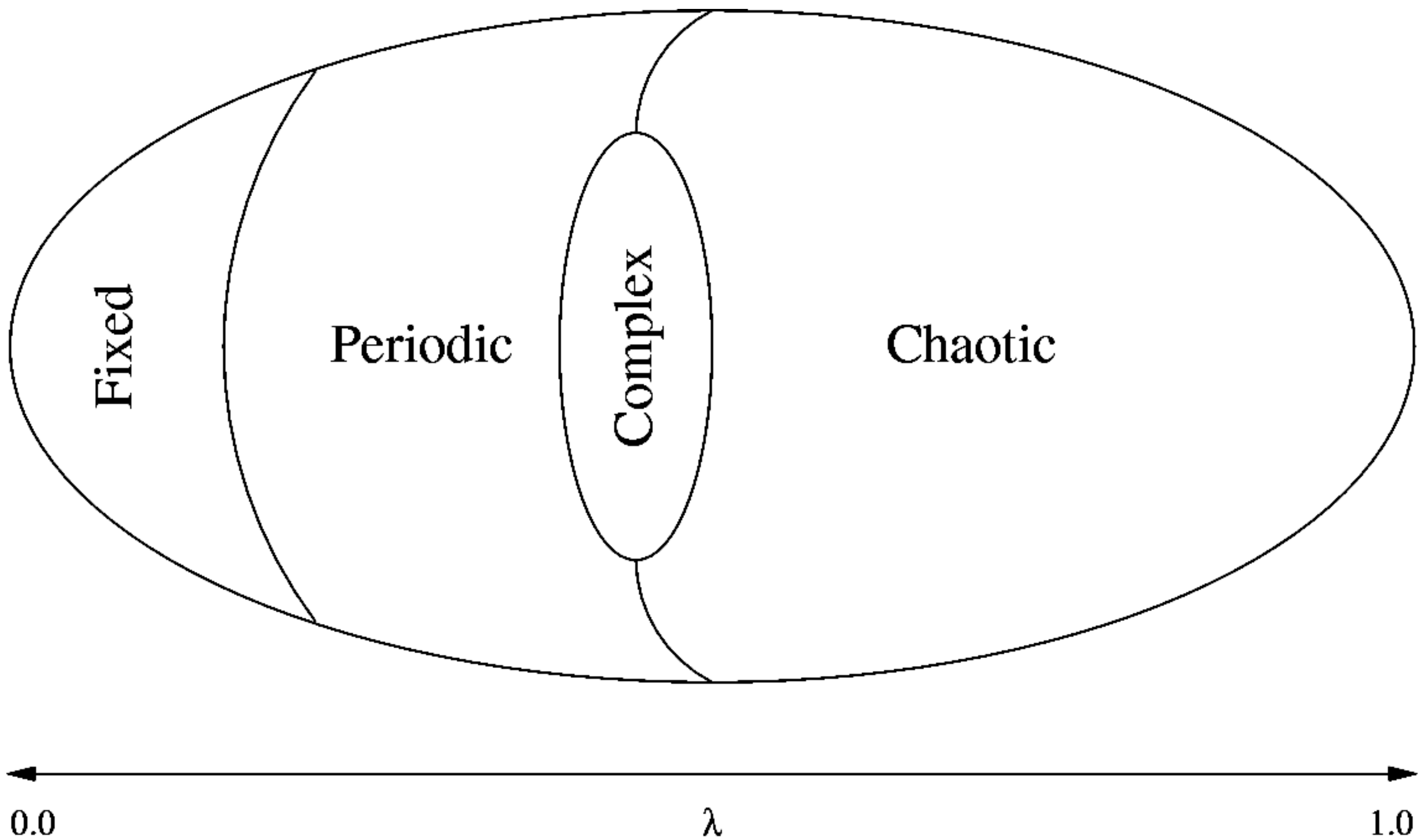


Fig. from Langton, “Life at Edge of Chaos”

Additional Bibliography

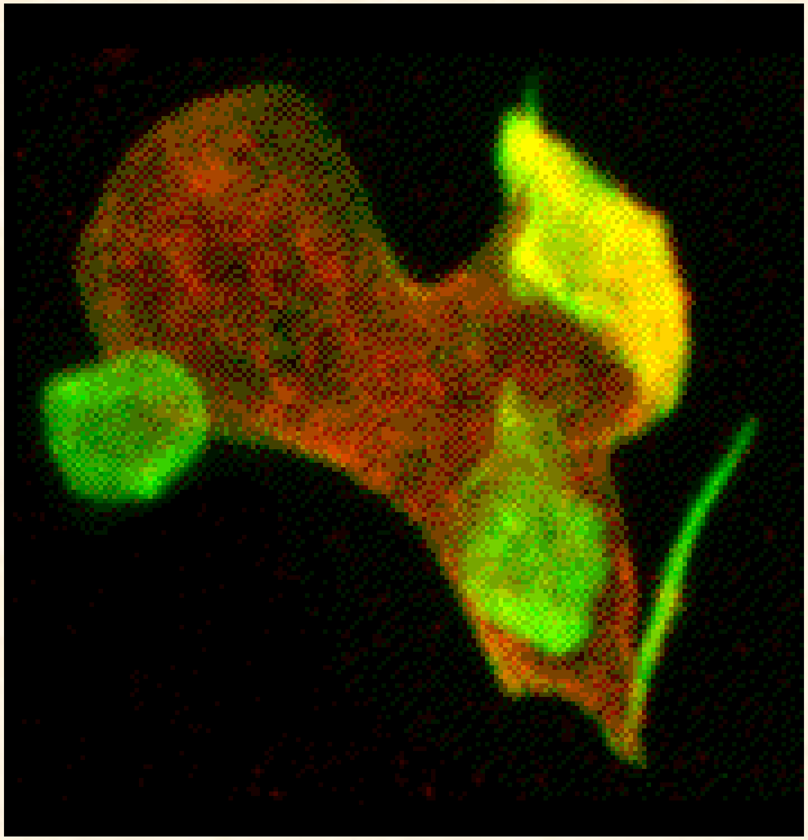
1. Langton, Christopher G. "Life at the Edge of Chaos," in *Artificial Life II*, ed. Langton et al. Addison-Wesley, 1992.
2. Emmeche, Claus. *The Garden in the Machine: The Emerging Science of Artificial Life*. Princeton, 1994.

Slime Mold

(*Dictyostelium discoideum*)

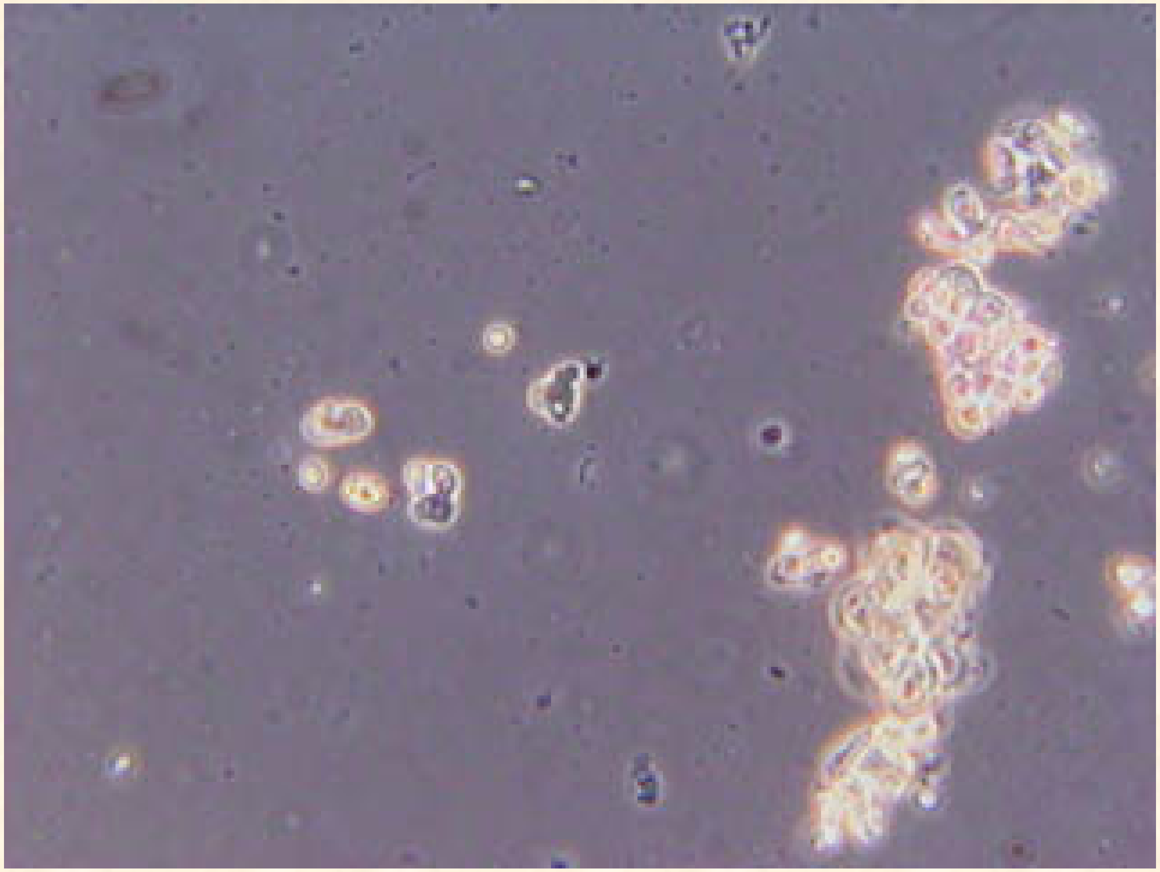
“Dicty”

Amoeba Stage



- Single cell
- Lives in soil
- Free moving
- Engulfs food (bacteria)
- Divides asexually

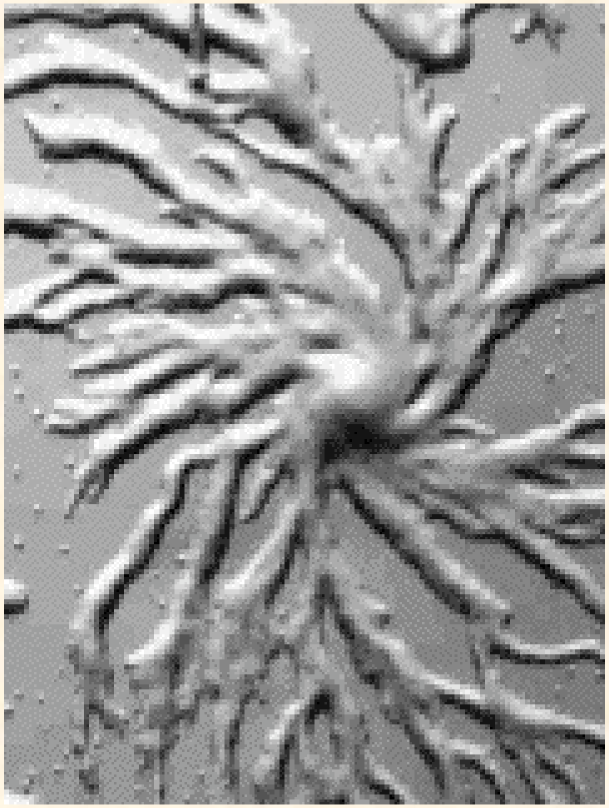
Amoebas



Aggregation Stage

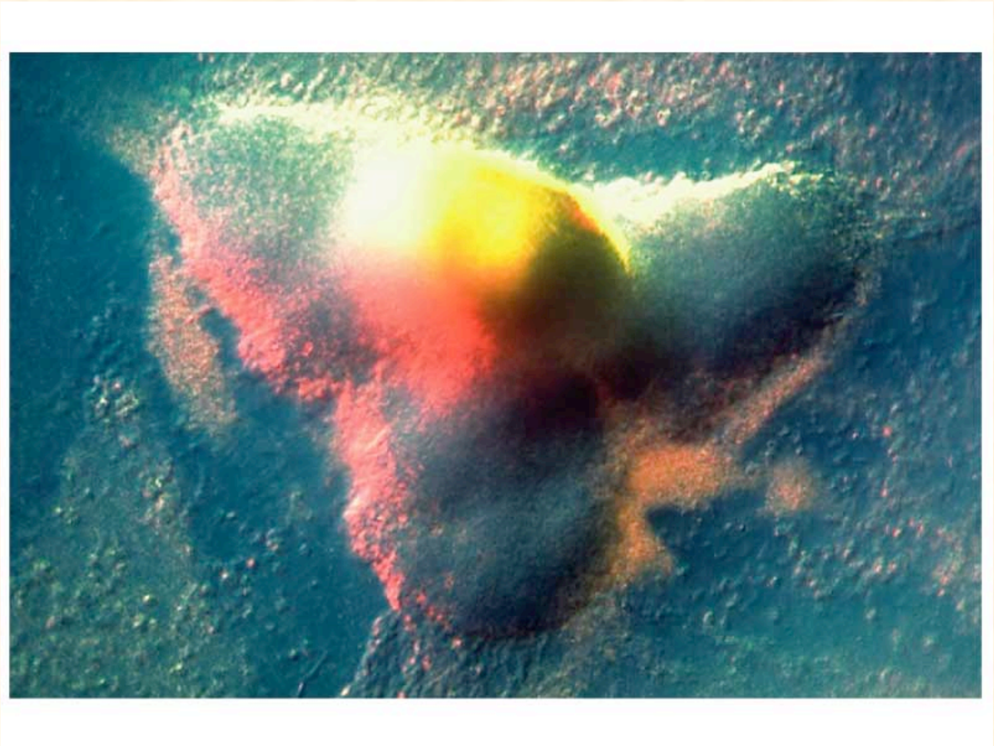
- Triggered by exhaustion of food
- Aggregate by *chemotaxis*
- Form expanding concentric rings and spirals
- Up to 125 000 individuals

Stream Formation Stage



- As density increases, begin to adhere
- Begin to form *mound*

Mound Stage



- Cells differentiate
- Some form an elongated finger

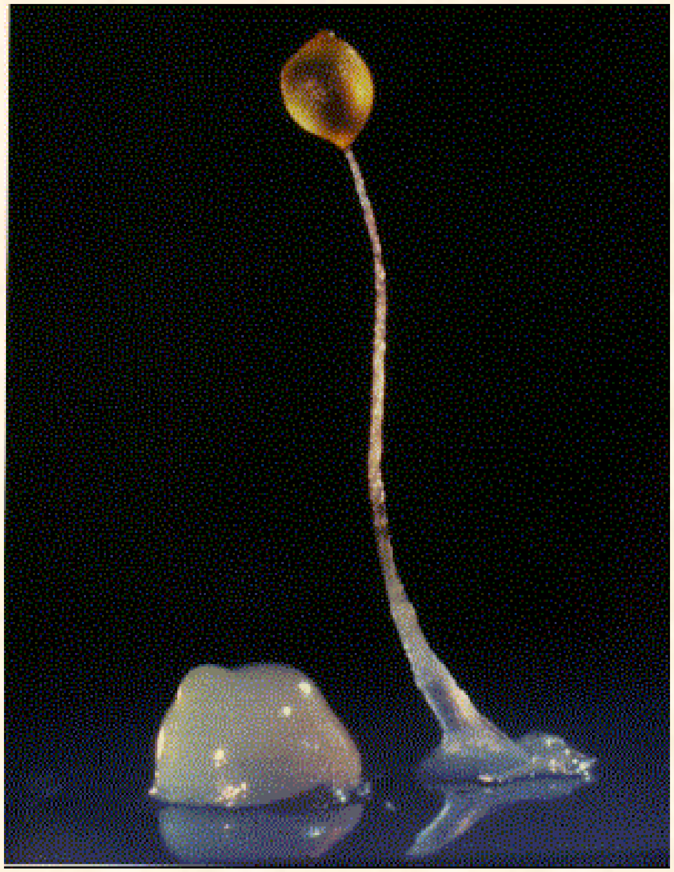
Slug Stage

- Prestalk elongates, topples, to form slug
- Behaves as single organism with 10^5 cells
- Migrates; seeks light; seeks or avoids heat
- No brain or nervous system

Culmination Stage

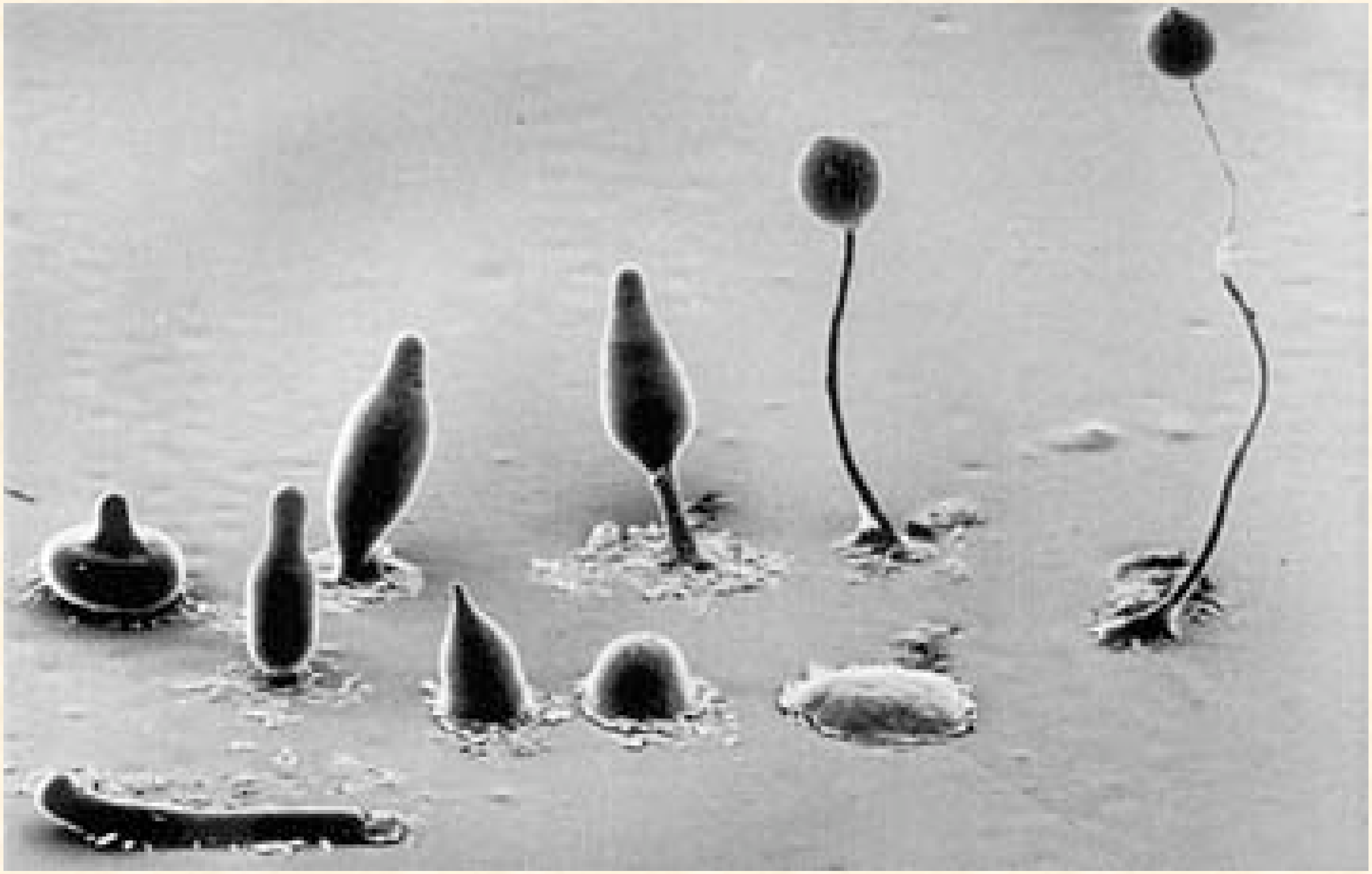
- Cells differentiate into base, stalk, and spores
- Prestalk cells form rigid bundles of cellulose & die
- Prespore cells (at end) cover selves with cellulose & become dormant

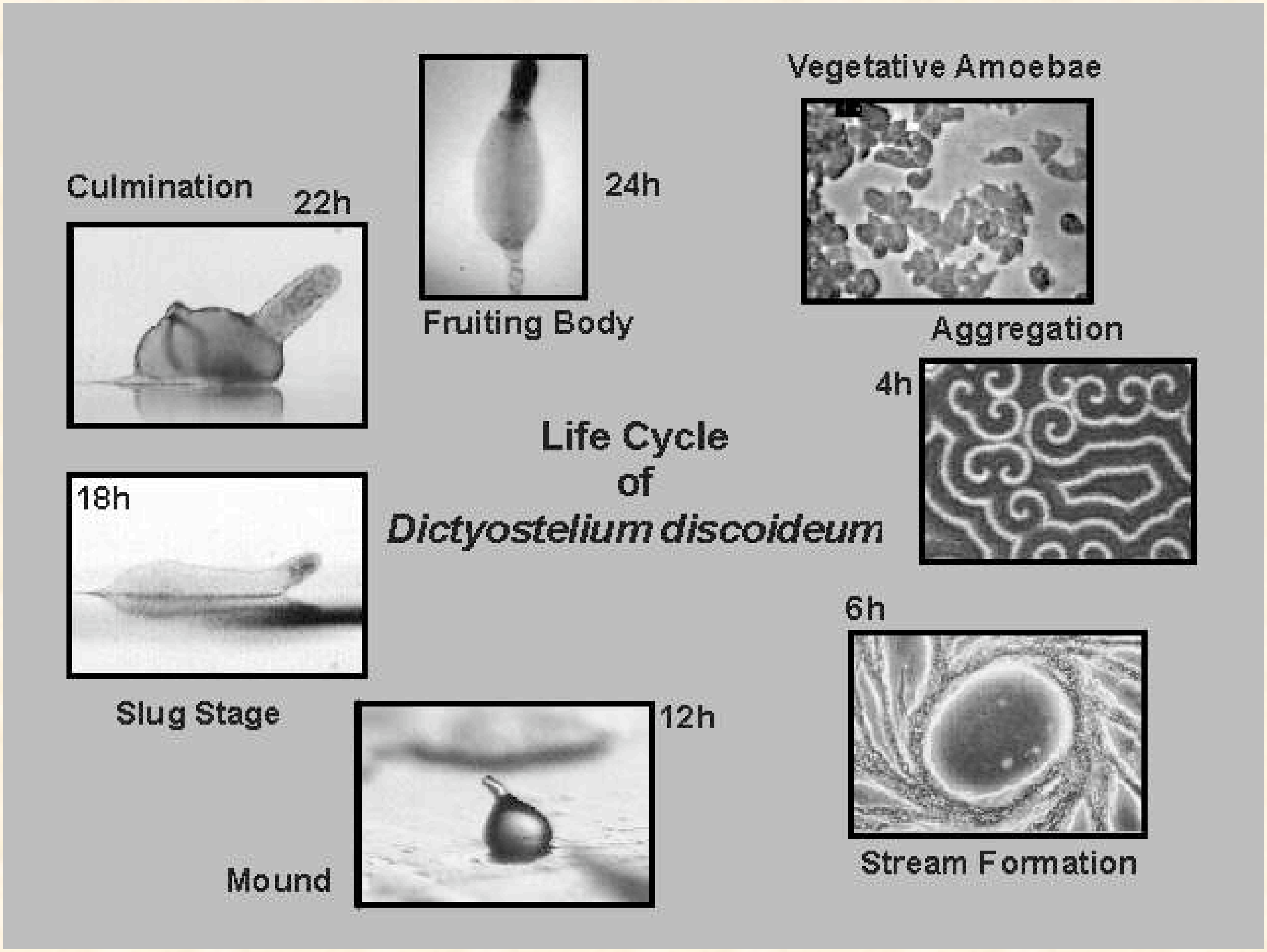
Fruiting Body Stage



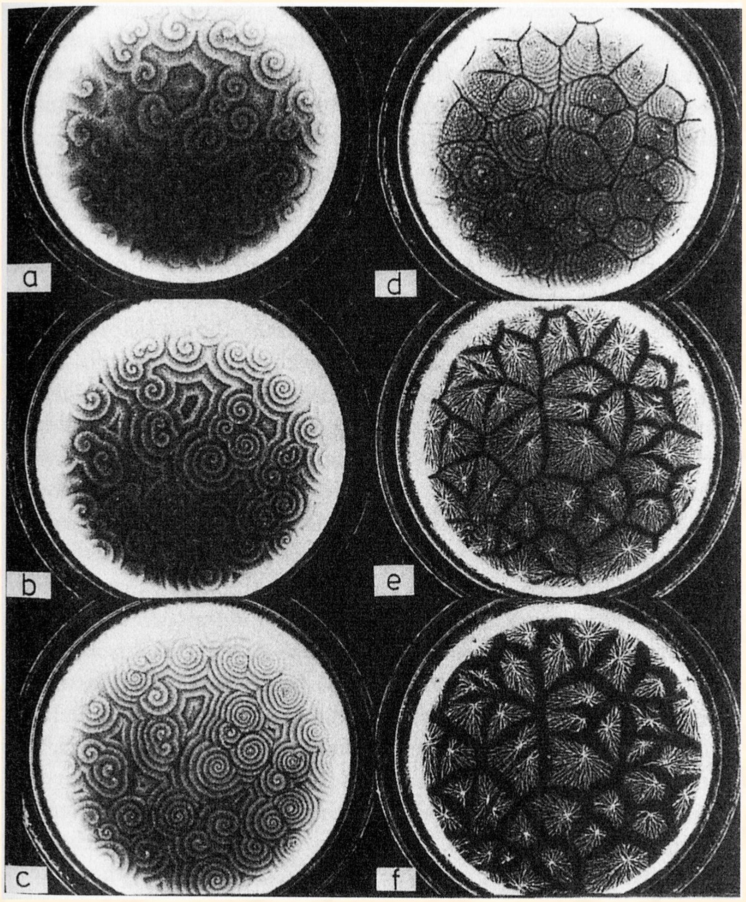
- Spores are dispersed
- Wind or animals carry spores to new territory
- If sufficient moisture, spores germinate, release amoebas
- Cycle begins again

Complete Life Cycle




Vegetative Amoebae
Culmination
22h
24h
Fruiting Body
Aggregation
4h
Life Cycle
of
Dictyostelium discoideum
18h
6h
Slug Stage
12h
Stream Formation
Mound

Emergent Patterns During Aggregation

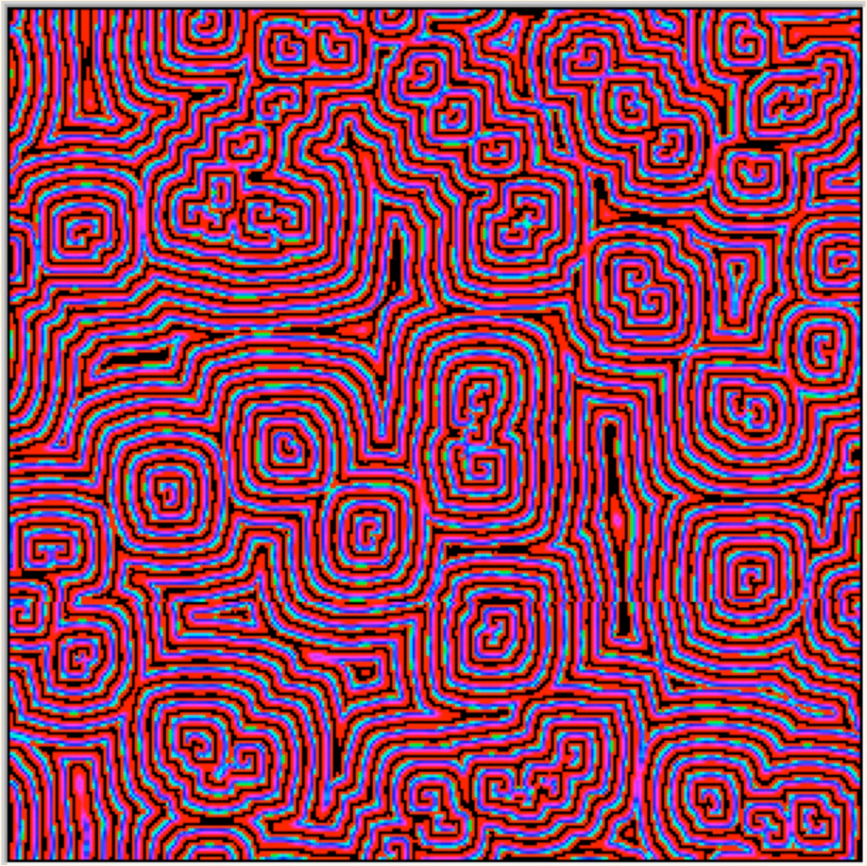


- a-c. As aggregate, wave lengths shorten
- d. Population divides into disjoint domains
- e-f. Domains contract into “fingers” (streaming stage)

fig. from Solé & Goodwin

Belousov-Zhabotinski Reaction

Hodgepodge Machine



Demonstration of Hodgepodge Machine

Go to hodgepodge machine

applets at CBN website

or unix program at course website

Universal Properties

- What leads to these expanding rings and spirals in very different systems?
- Under what conditions do these structures form?
- What causes the rotation?
- These are all examples of *excitable media*