Reading

- Everyone: Flake, ch. 16
- CS594: Bar-Yam, Sections 7.1, 7.2.1-7.2.2 (pp. 621-48)

Universal Properties

- What leads to these expanding rings and spirals in very different systems?
- Under what conditions do these structures form?
- What causes the rotation?
- These are all examples of *excitable media*

Excitable Media

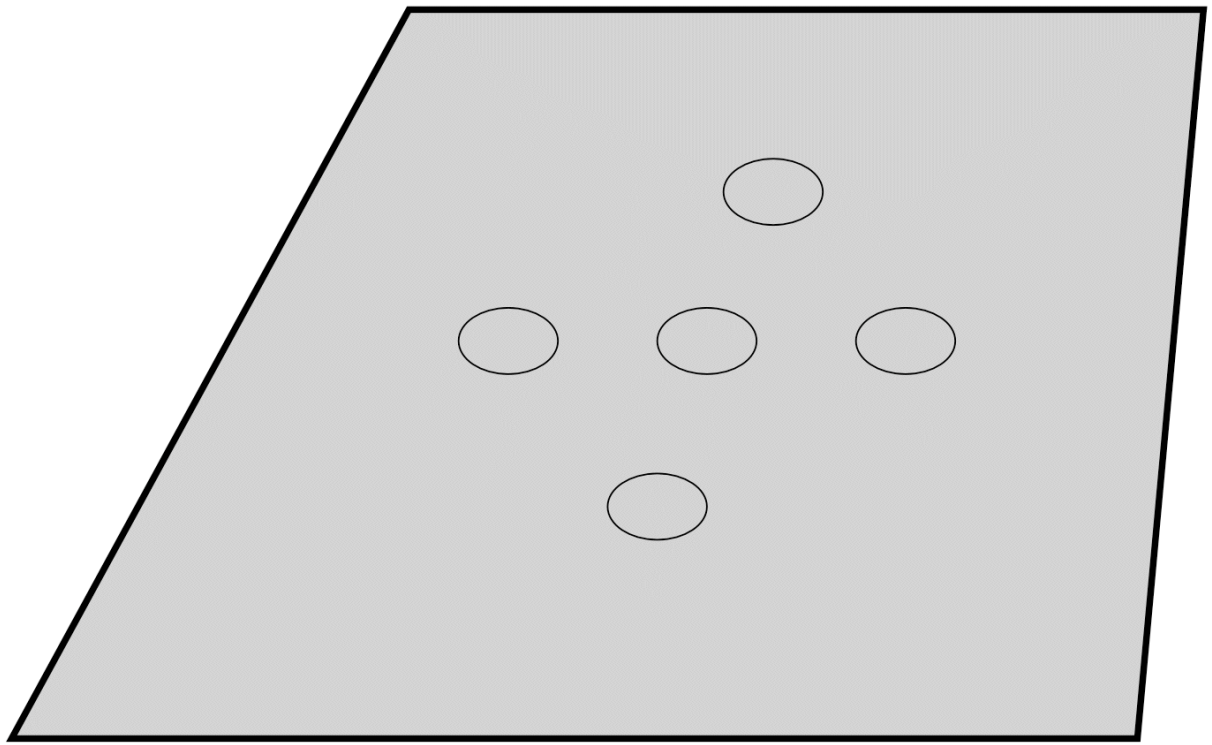
Examples of Excitable Media

- Slime mold amoebas
- Cardiac tissue (& other muscle tissue)
- Cortical tissue
- Certain chemical systems (e.g., BZ reaction)
- Hodgepodge machine

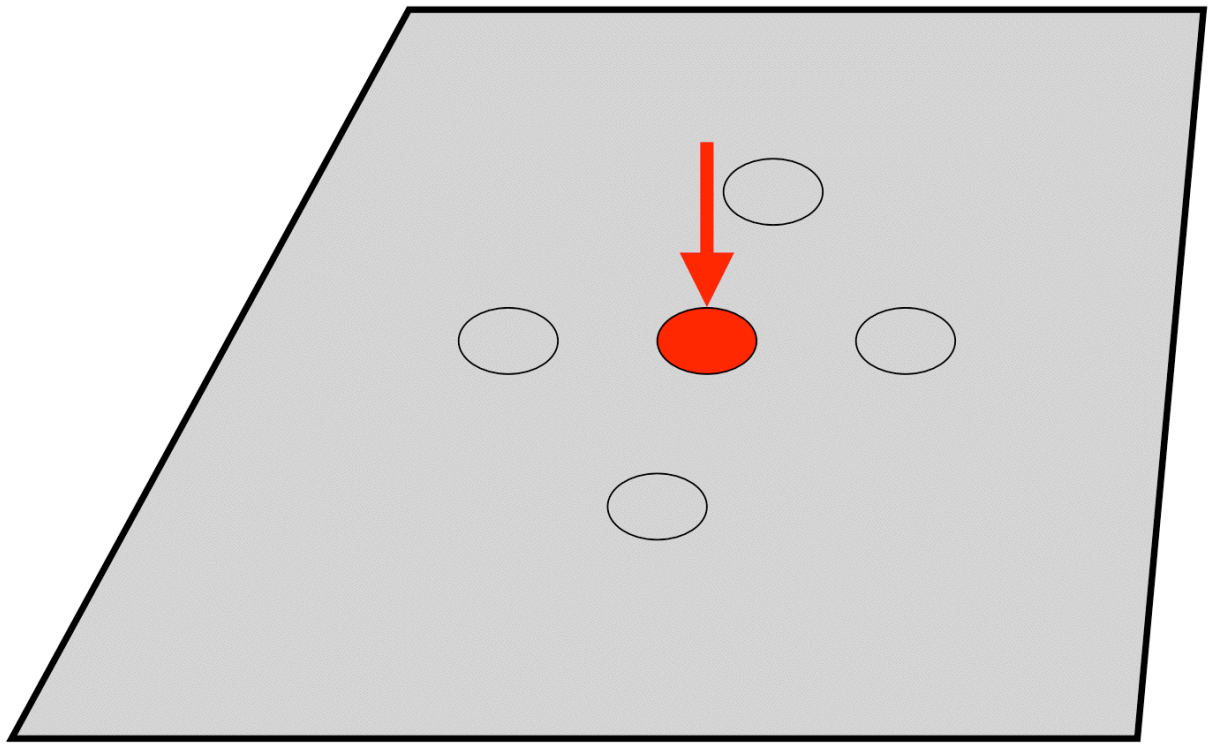
Characteristics of Excitable Media

- Local spread of excitation
  - for signal propagation
- Refractory period
  - for unidirectional propagation
- Decay of signal
  - avoid saturation of medium

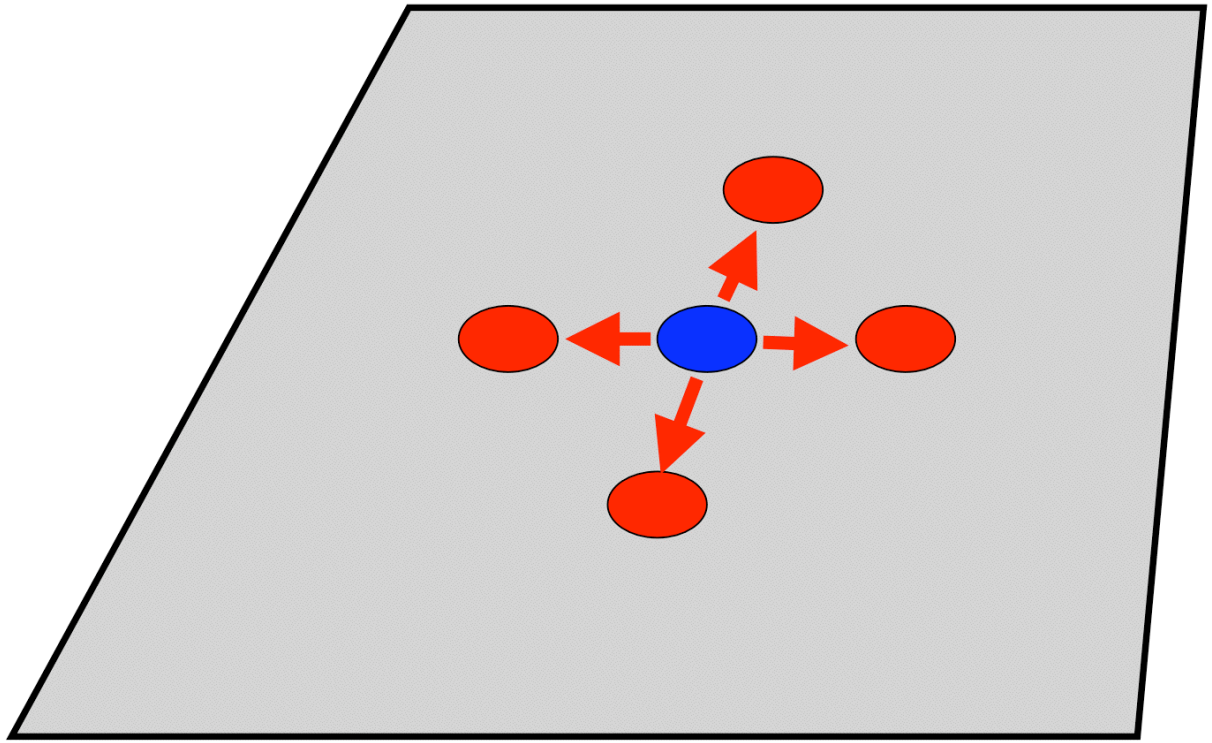
Behavior of Excitable Media



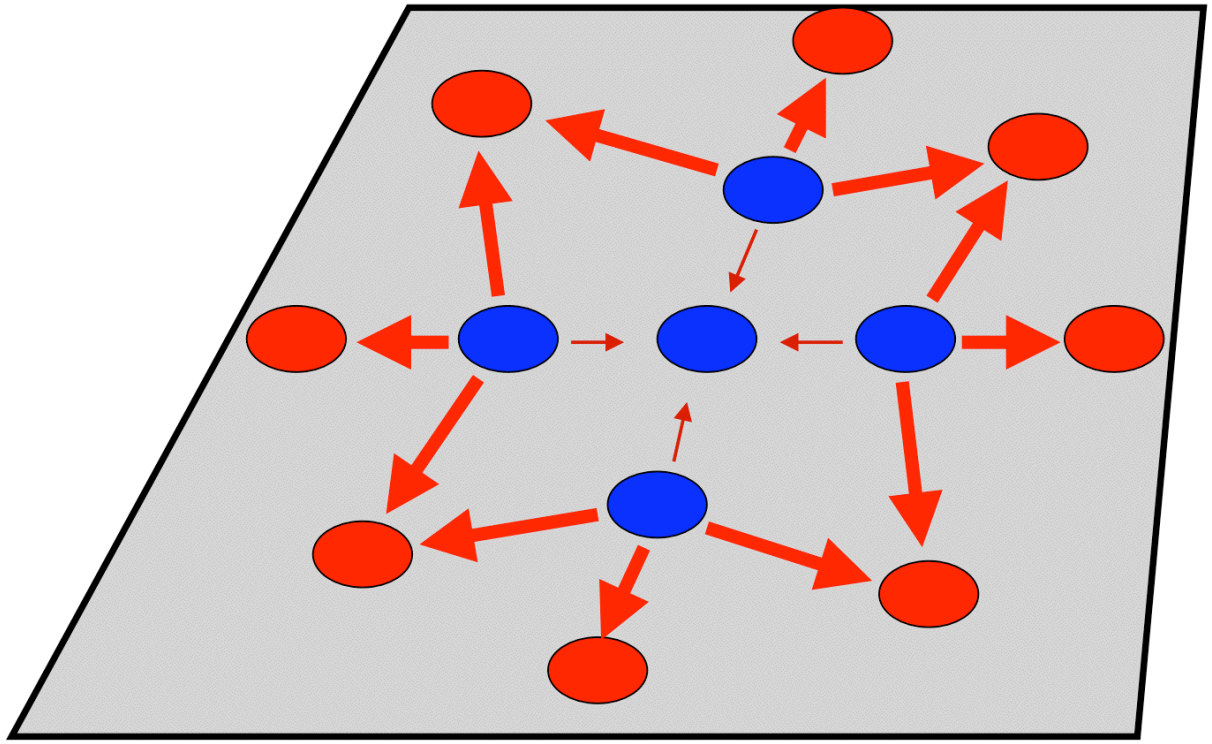
Stimulation



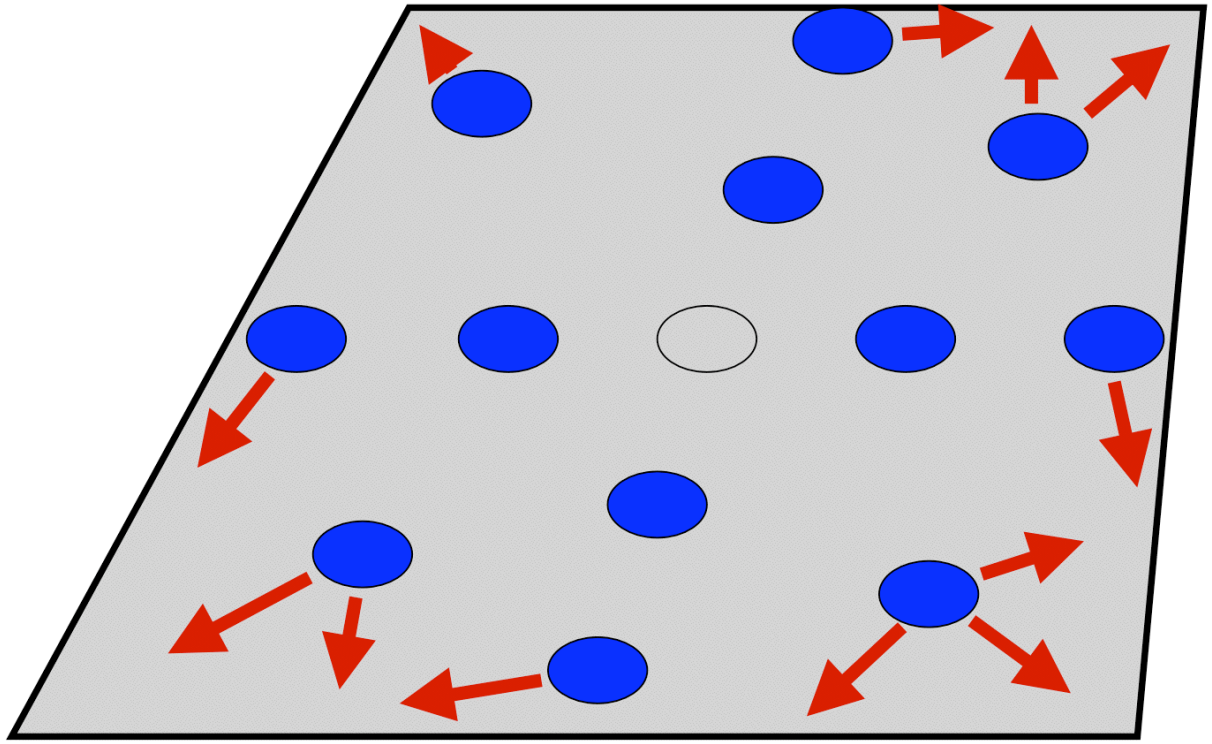
Relay (Spreading Excitation)



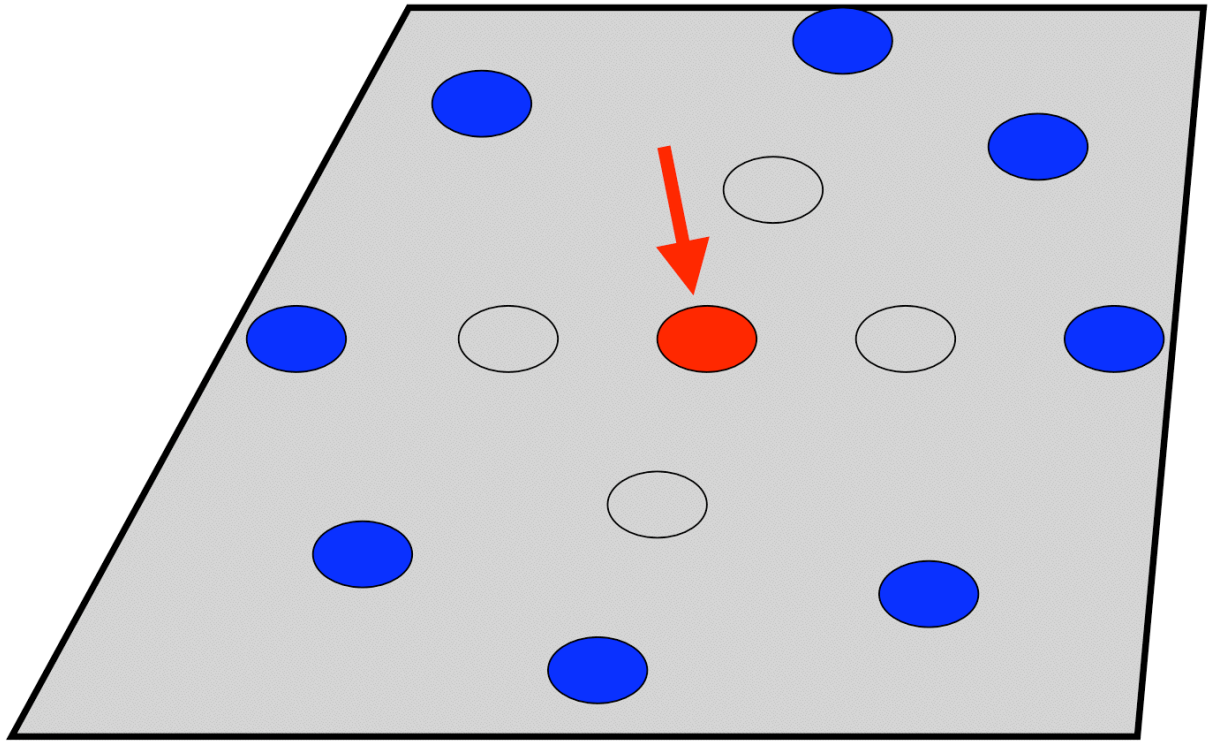
Continued Spreading



Recovery



Restimulation



Typical Equations for Excitable Medium (ignoring diffusion)

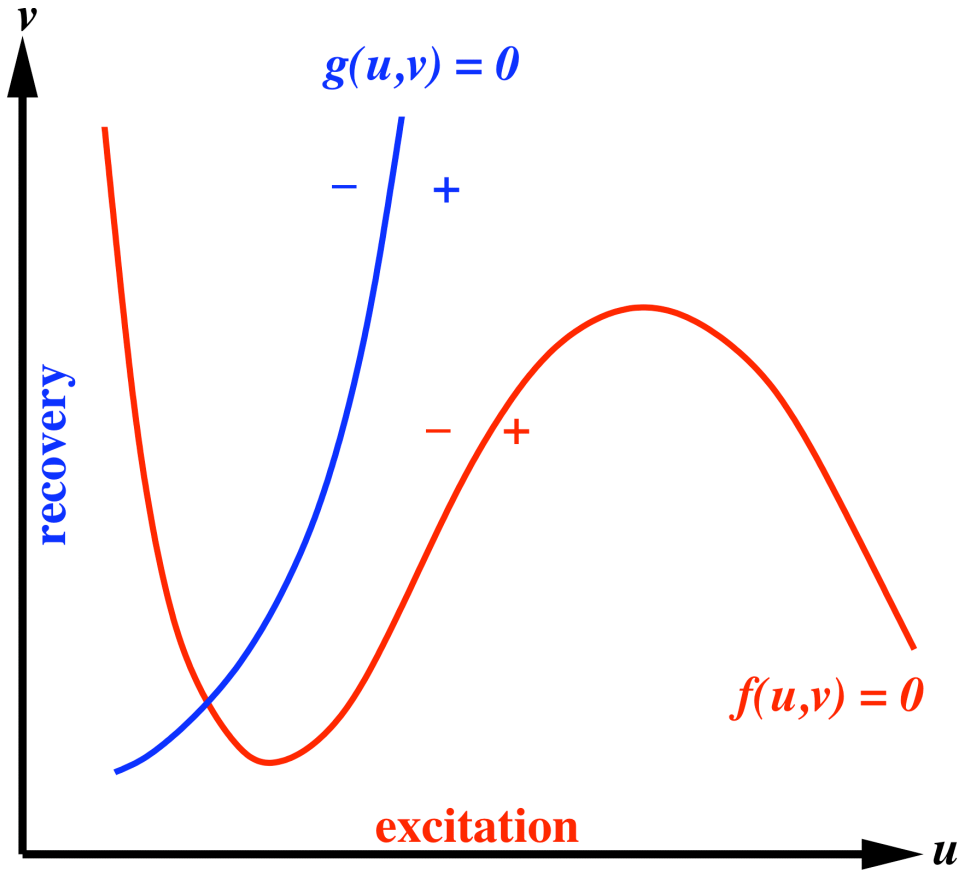
- Excitation variable:

$$\dot{u} = f(u,v)$$

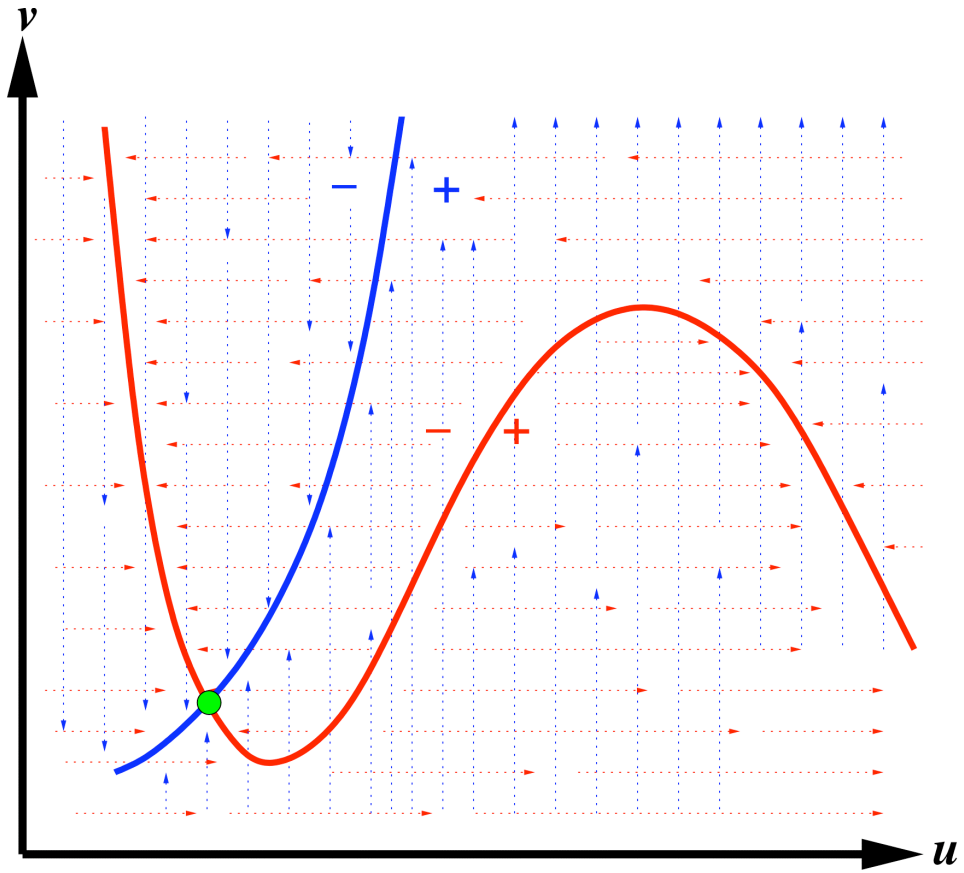
- Recovery variable:

$$\dot{v} = g(u,v)$$

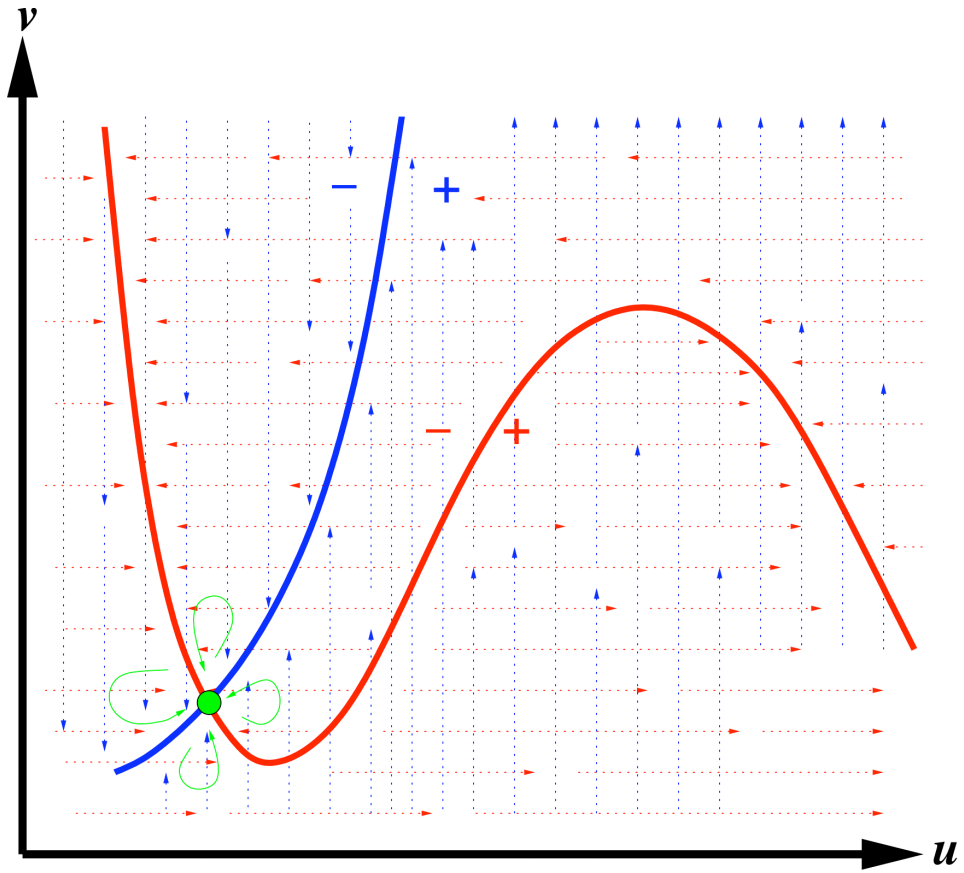
Nullclines



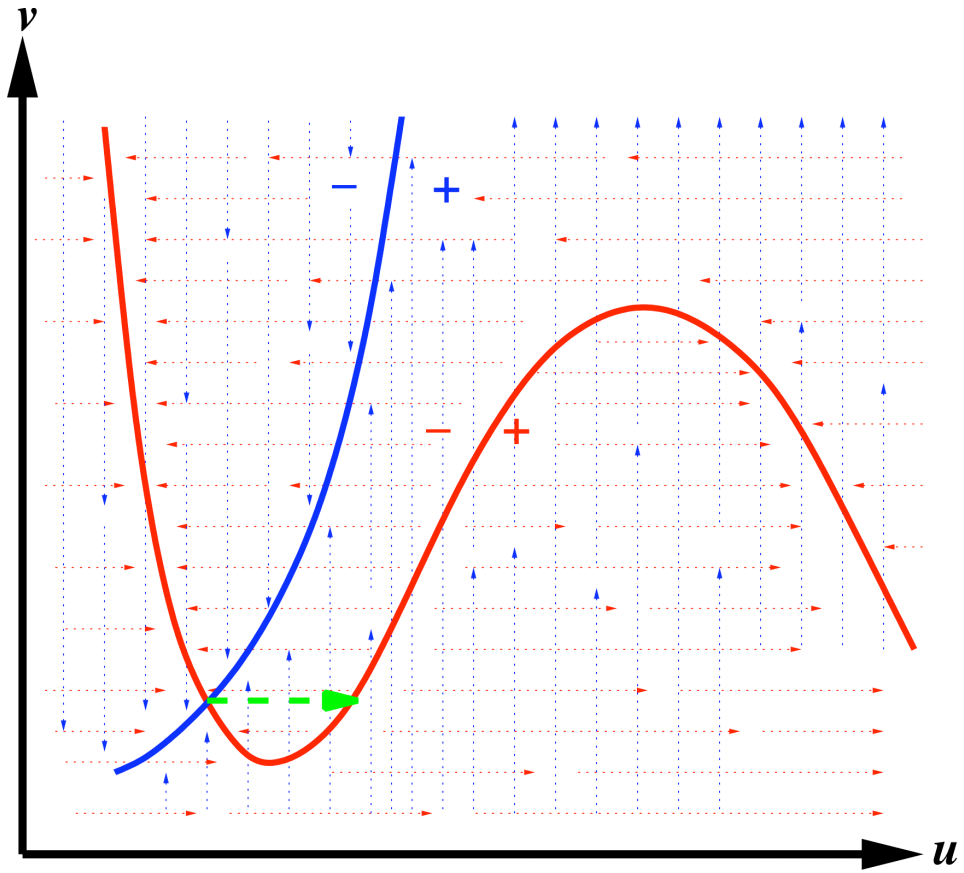
Rest State



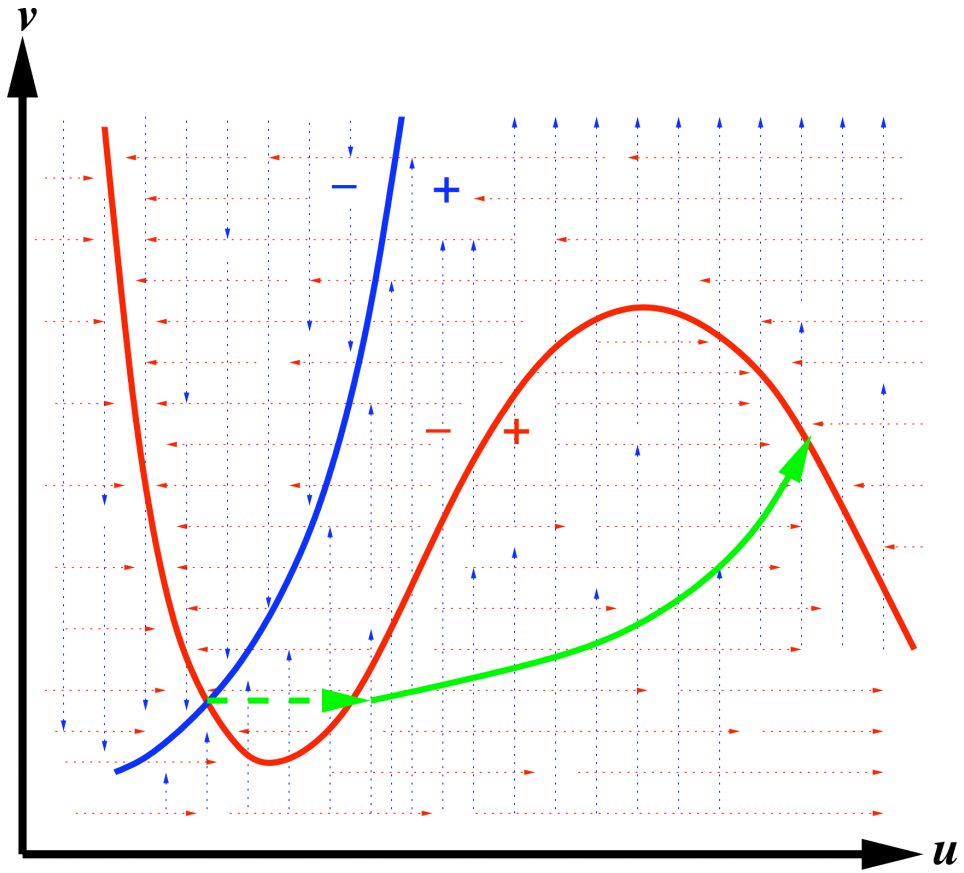
Stability



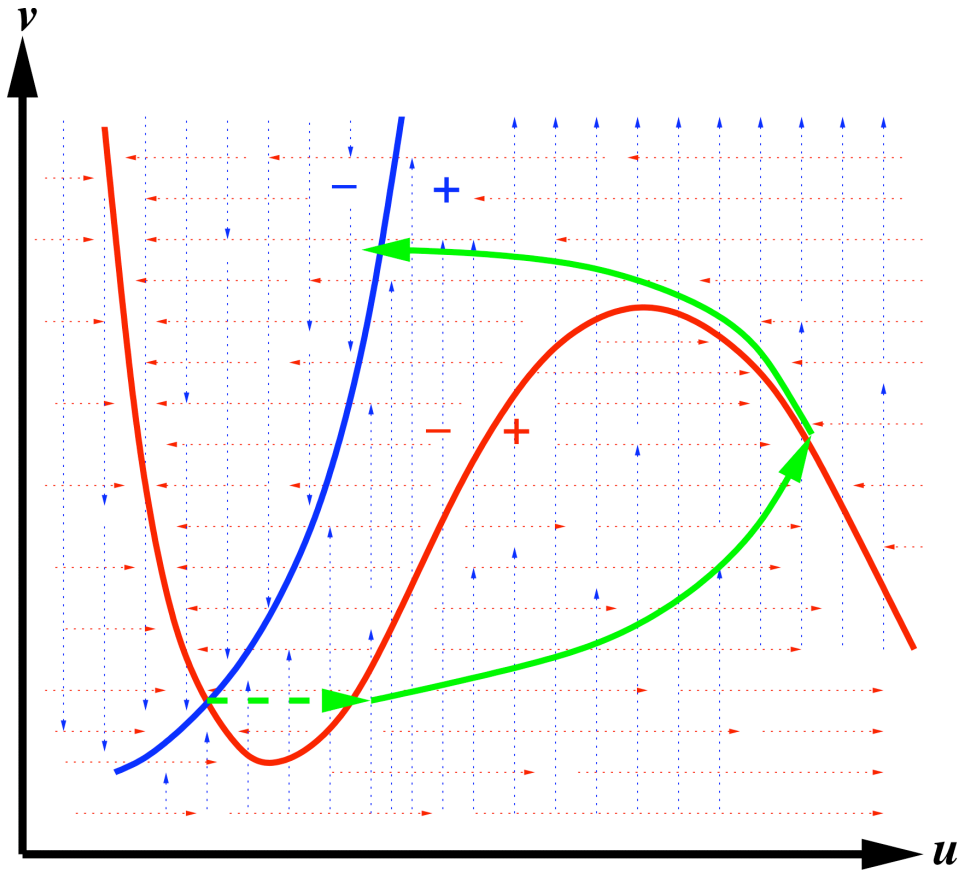
Super-threshold Excitation



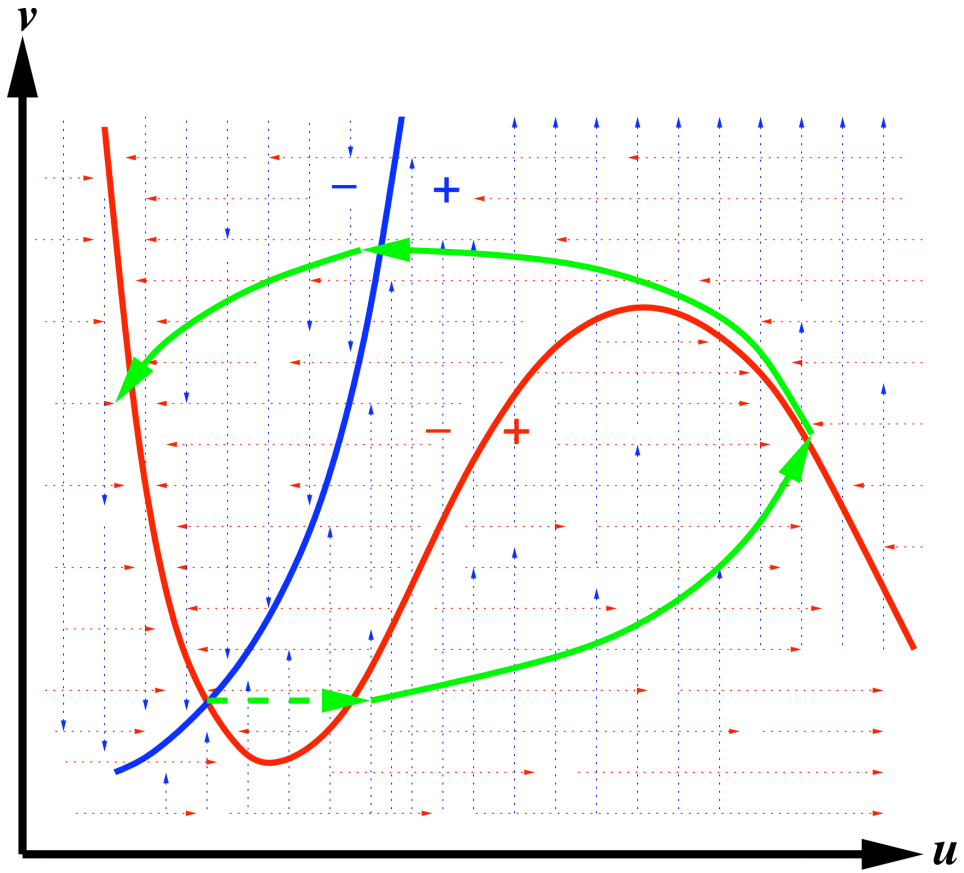
Phase 1: Increasing Excitation



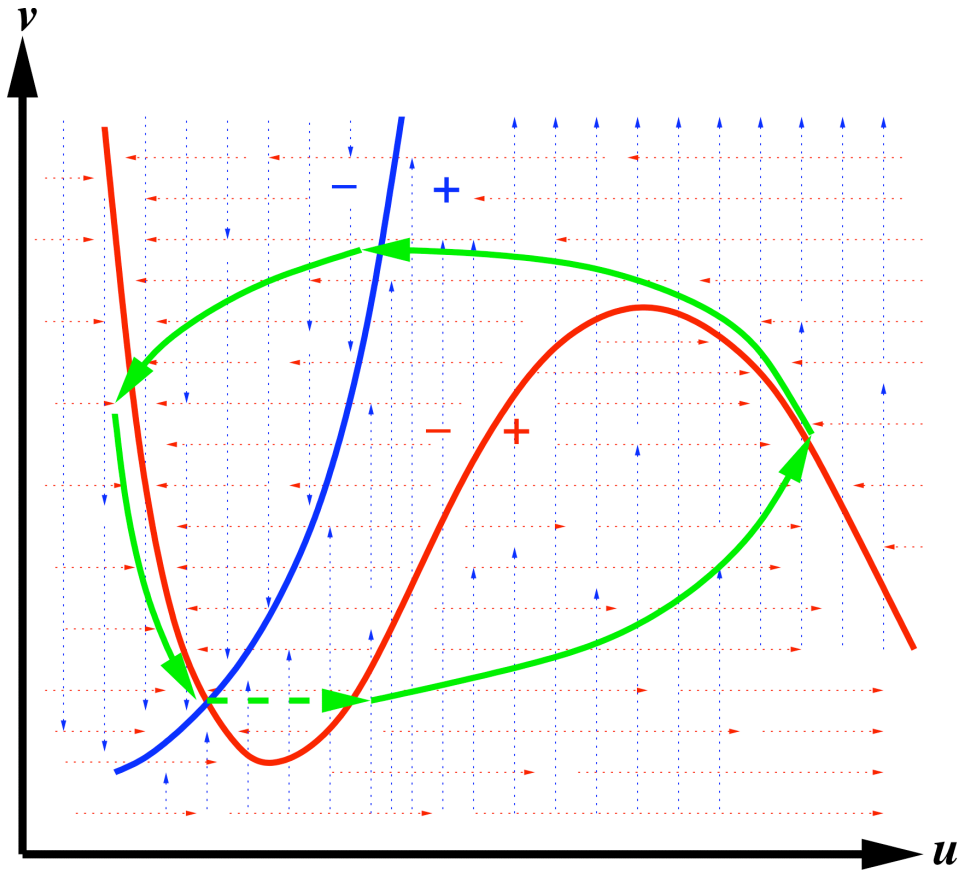
Phase 2: Start of Extinction



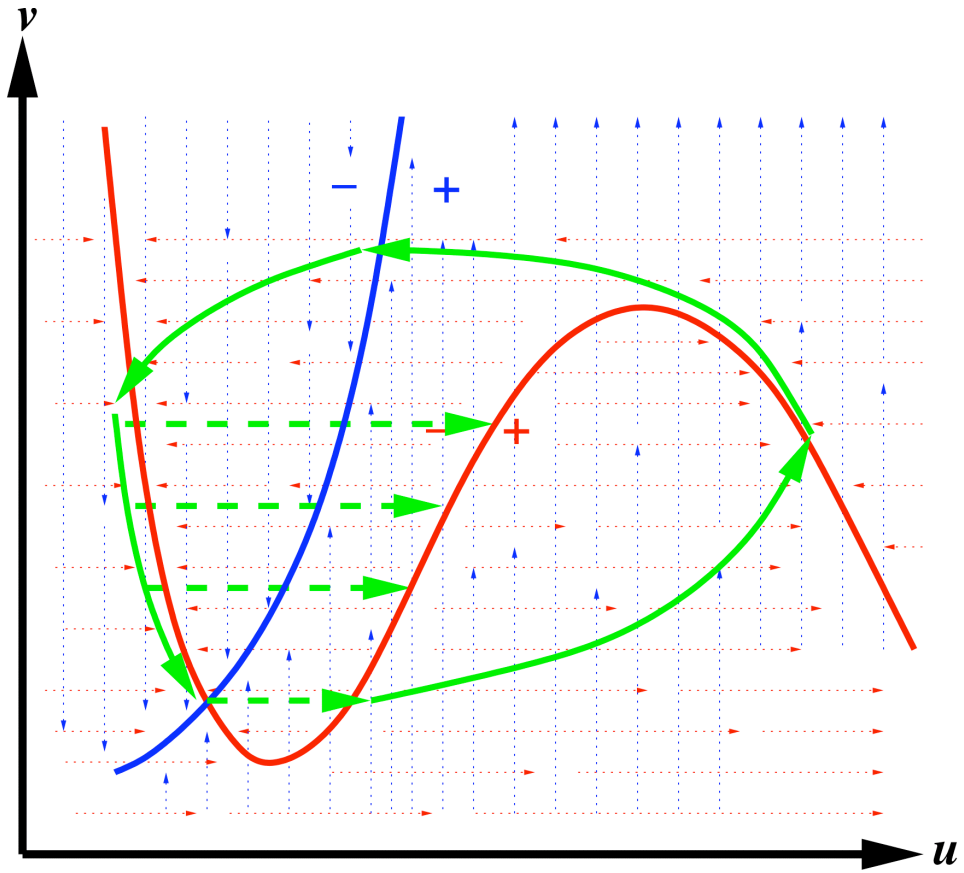
Phase 3: End of Extinction



Phase 4: Recovery



Elevated Thresholds During Recovery



Modified Martiel & Goldbeter Model for Dicty Signalling

Variables (functions of x, y, t):

$\square$ = intracellular concentration of cAMP

$\square$= extracellular concentration of cAMP

$\square$ = fraction of receptors in active state

Equations

$$\frac{d\beta(x,y,t)}{dt} = s\Phi(\rho,\gamma) - \beta k_i - \beta k_t \quad [1]$$

Rate of change in intracellular [cAMP] = Production of cAMP − Intracellular hydrolysis − Secretion of cAMP

$$\frac{d\gamma(x,y,t)}{dt} = \frac{k_t}{h}\beta - k_e\gamma + D\nabla^2\gamma \quad [2]$$

Rate of change in extracellular [cAMP] = Secretion of cAMP − Extracellular hydrolysis + Diffusion of cAMP

$$\frac{d\rho(x,y,t)}{dt} = f_2(\gamma)(1-\rho) - f_1(\gamma)\rho \quad [3]$$

Rate of change in fraction of active receptor = Dephosphorylation of receptor − Phosphorylation of receptor

Positive Feedback Loop

- Extracellular cAMP increases
  (□increases)
- □ Rate of synthesis of intracellular cAMP increases
  (□ increases)
- □ Intracellular cAMP increases
  (□ increases)
- □ Rate of secretion of cAMP increases
- (□ Extracellular cAMP increases)

See Equations

Negative Feedback Loop

- Extracellular cAMP increases
  (increases)
- cAMP receptors desensitize
  (f_1 increases, f_2 decreases, decreases)
- Rate of synthesis of intracellular cAMP decreases
  (decreases)
- Intracellular cAMP decreases
  (decreases)
- Rate of secretion of cAMP decreases
- Extracellular cAMP decreases
  (decreases)

See Equations

Dynamics of Model

- Unperturbed
  → cAMP concentration reaches steady state
- Small perturbation in extracellular cAMP
  → returns to steady state
- Perturbation > threshold
  → large transient in cAMP,
  then return to steady state
- Or oscillation (depending on model parameters)

Circular & Spiral Waves Observed in:

- Slime mold aggregation
- Chemical systems (e.g., BZ reaction)
- Neural tissue
- Retina of the eye
- Heart muscle
- Intracellular calcium flows
- Mitochondrial activity in oocytes

Cause of Concentric Circular Waves

- Excitability is not enough
- But at certain developmental stages, cells can operate as pacemakers
- When stimulated by cAMP, they begin emitting regular pulses of cAMP

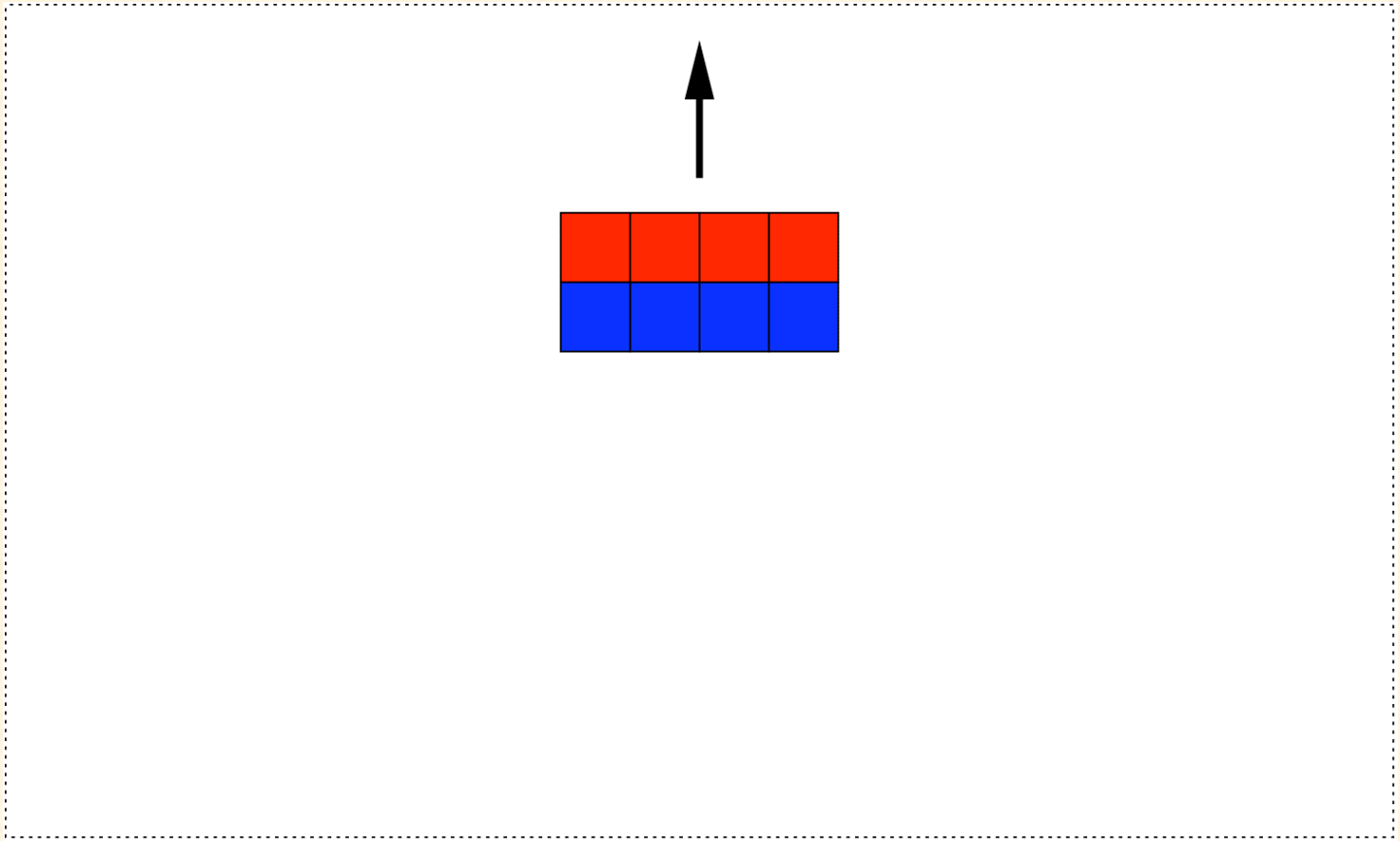
Spiral Waves

- Persistence & propagation of spiral waves explained analytically (Tyson & al., 1989)
- Rotate around a small core of of non-excitable cells
- Propagate at higher frequency than circular
- Therefore they dominate circular in collisions
- But how do the spirals form initially?

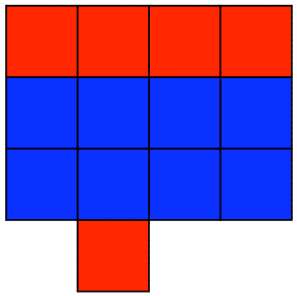
Some Explanations of Spiral Formation

- “the origin of spiral waves remains obscure” (1997)
- Traveling wave meets obstacle and is broken
- Desynchronization of cells in their developmental path
- Random pulse behind advancing wave front

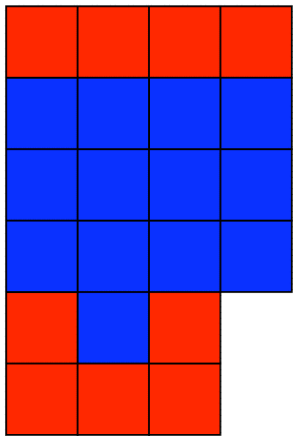
Step 0: Passing Wave Front



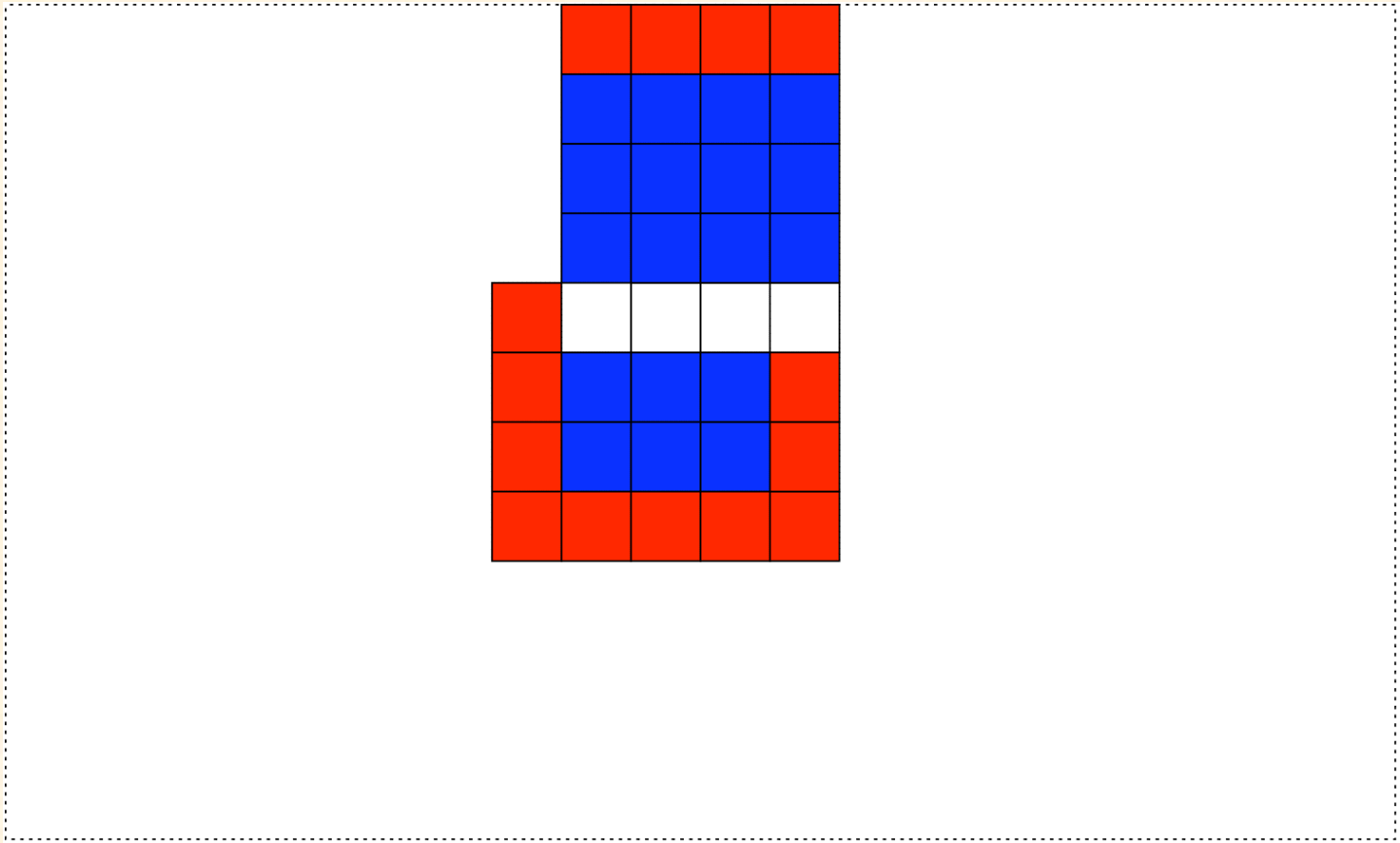
Step 1: Random Excitation



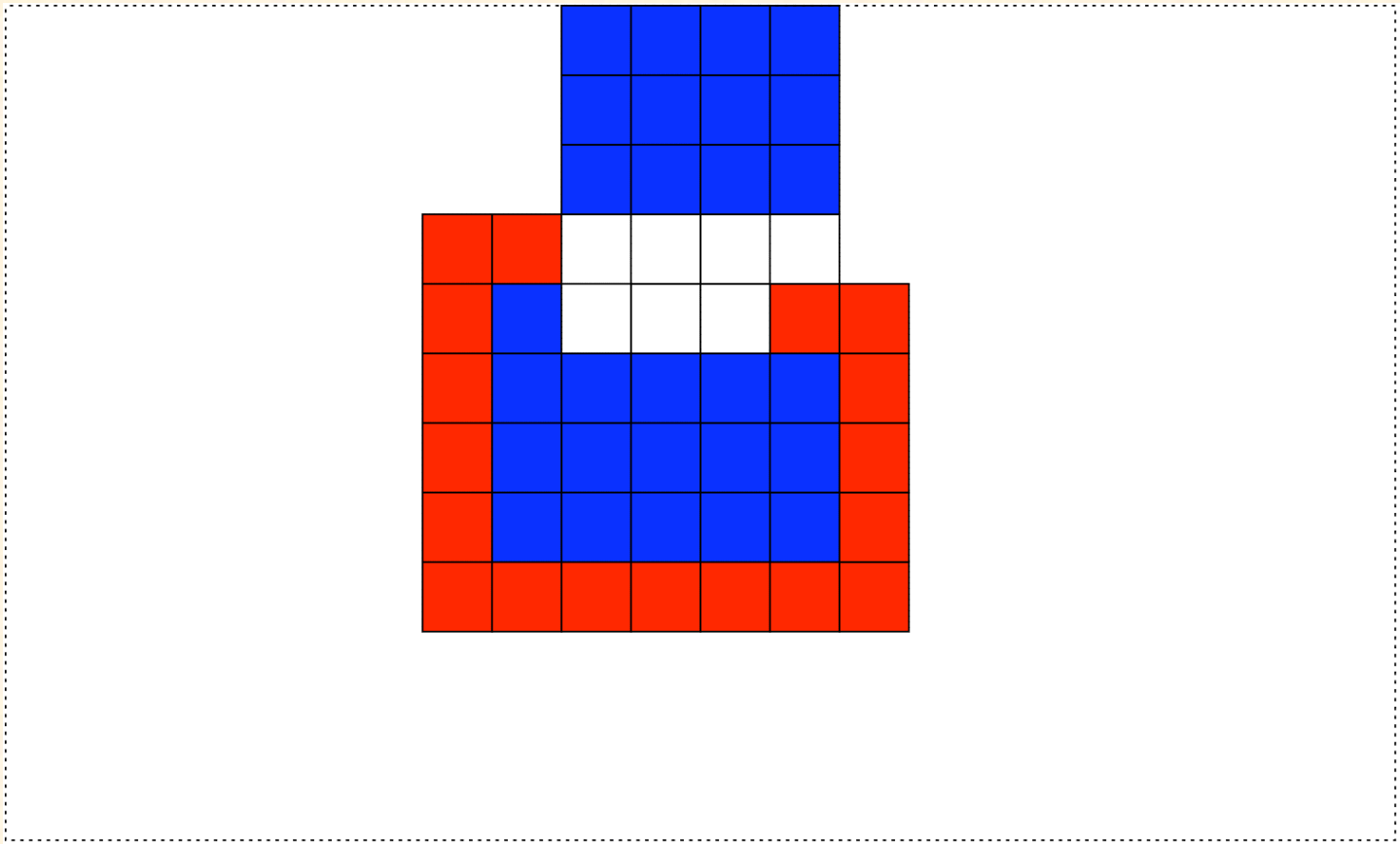
Step 2: Beginning of Spiral



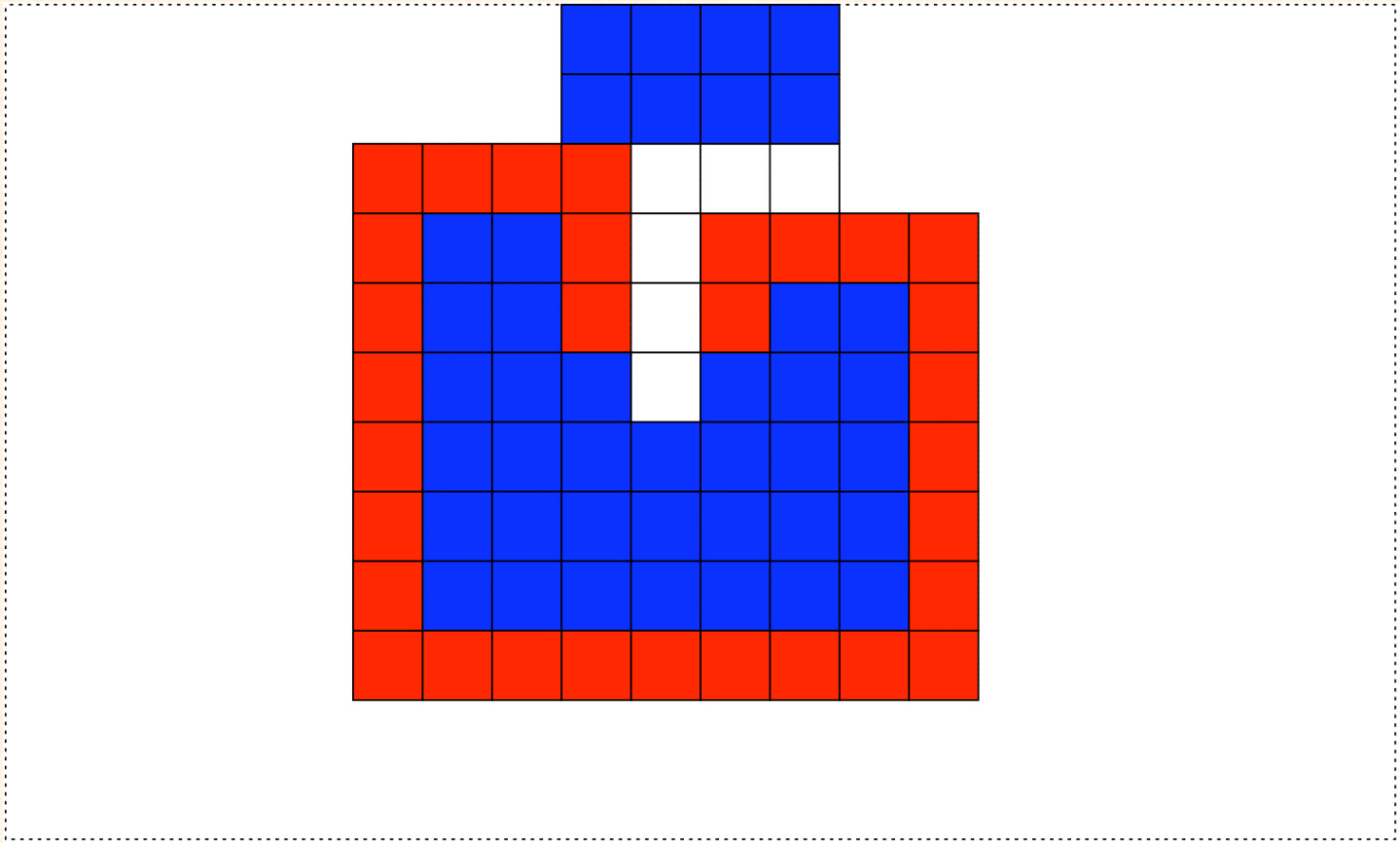
Step 3



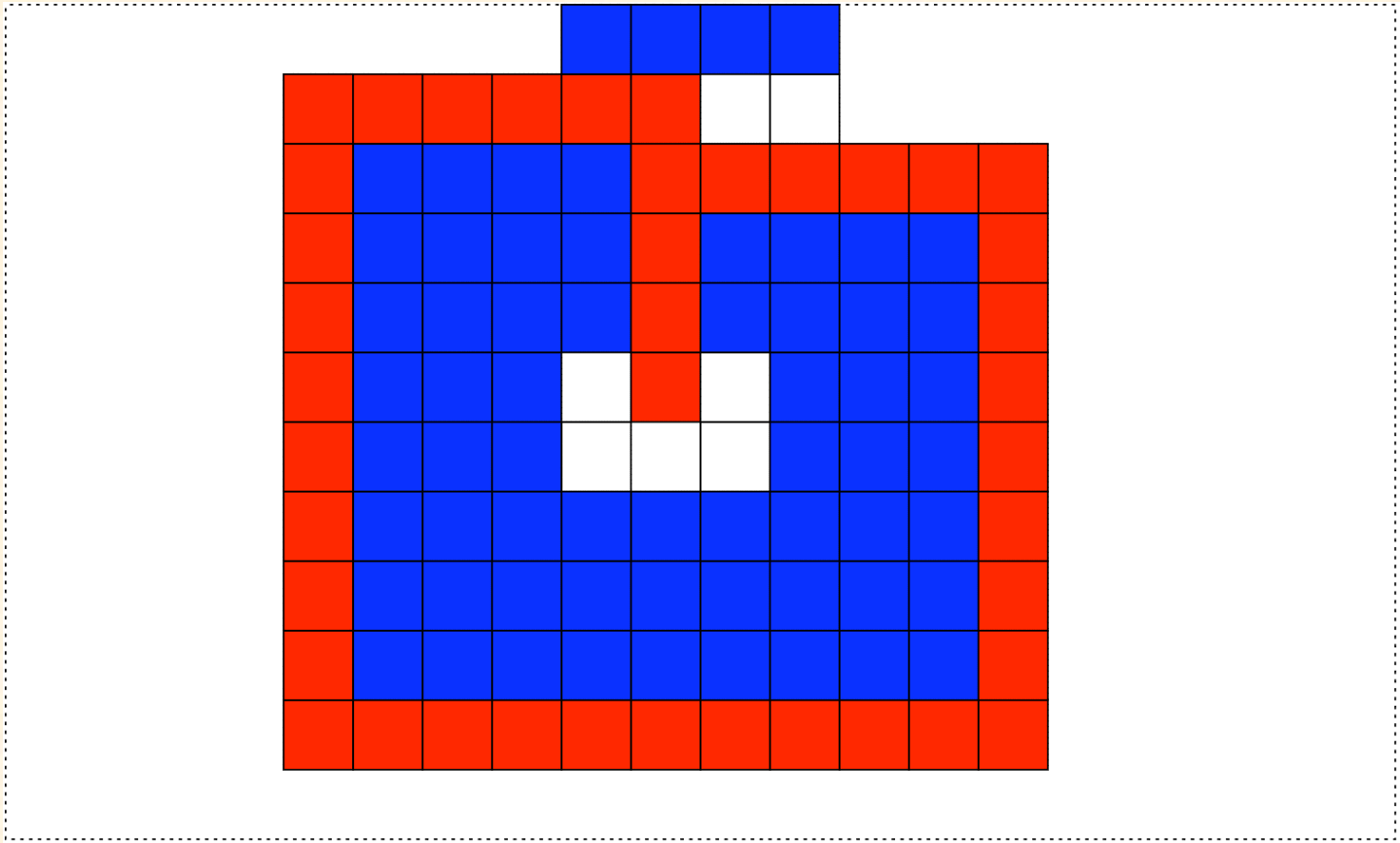
Step 4



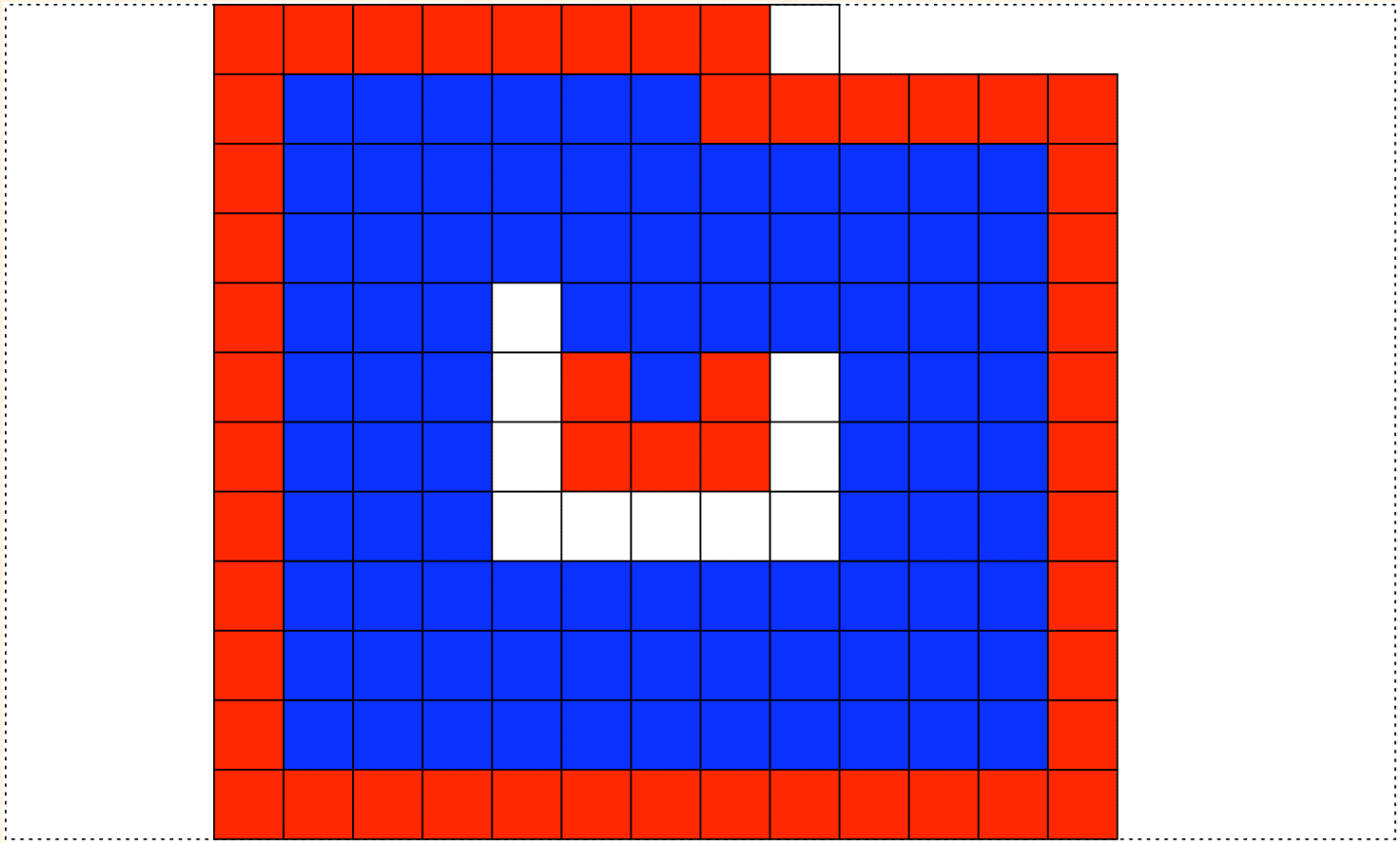
Step 5



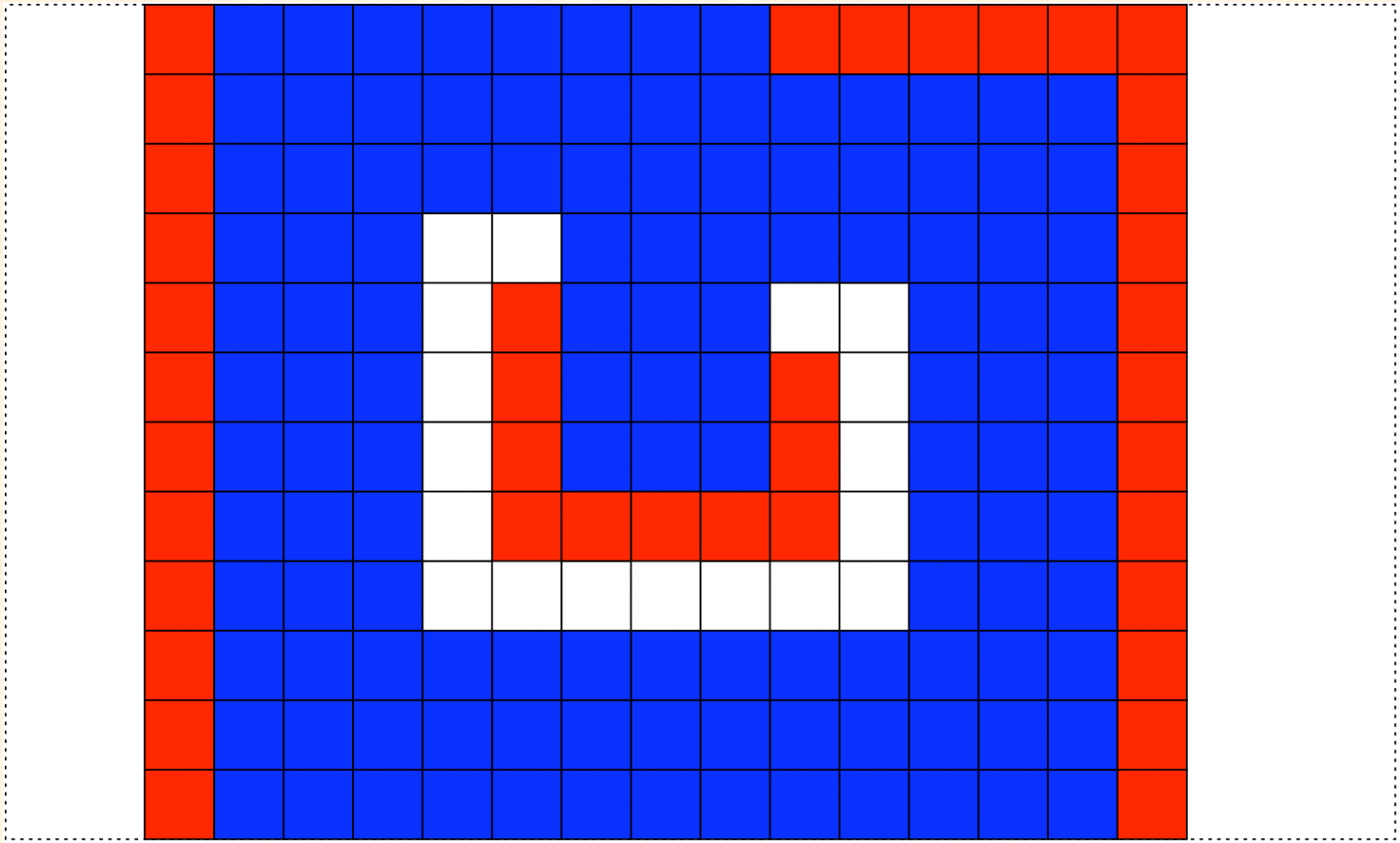
Step 6: Rejoining & Reinitiation



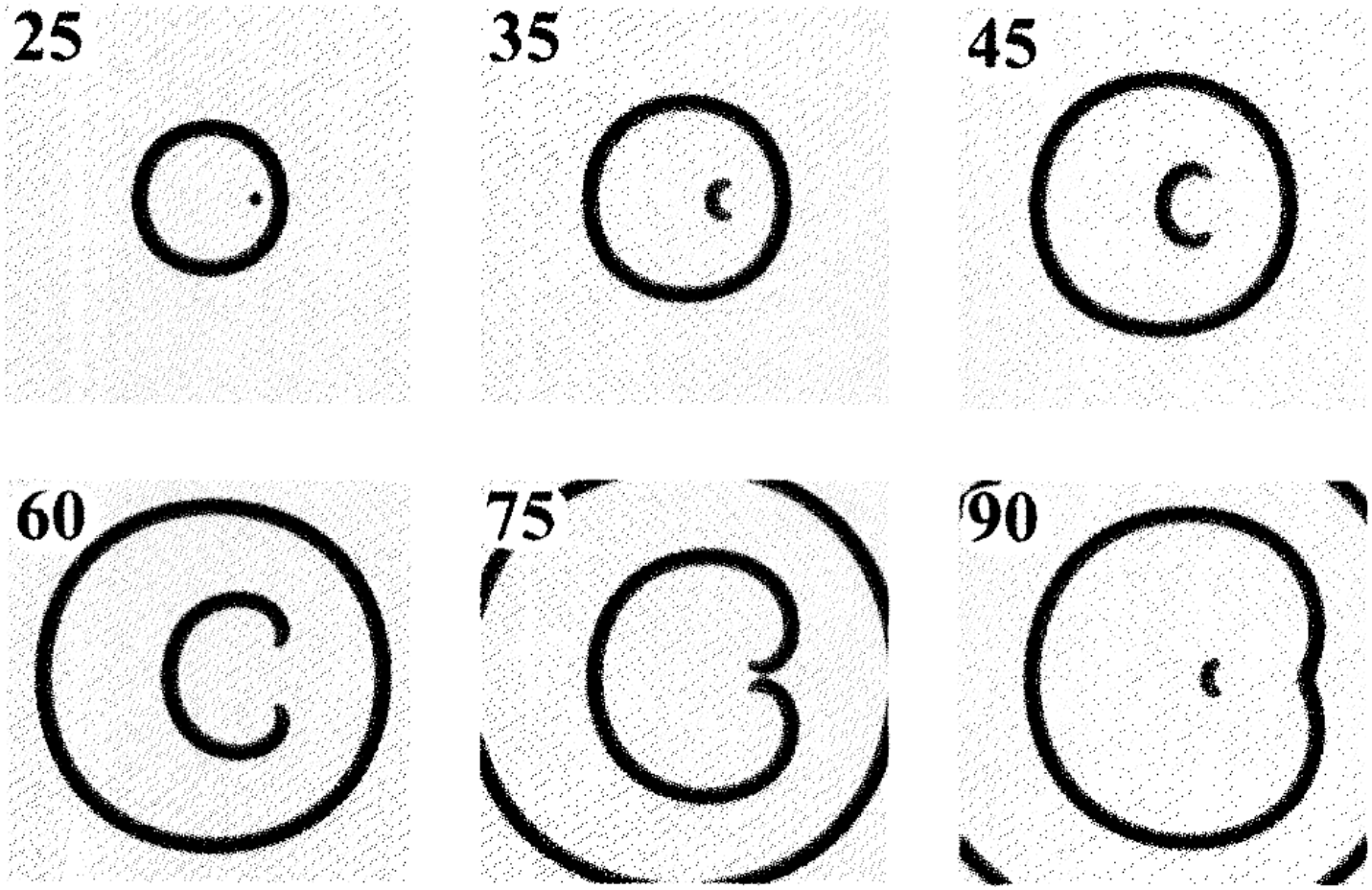
Step 7: Beginning of New Spiral



Step 8



Formation of Double Spiral



from Pálsson & Cox (1996)

StarLogo Simulation Of Spiral Formation

- Amoebas are immobile at timescale of wave movement
- A fraction of patches are inert (grey)
- A fraction of patches has initial concentration of cAMP
- At each time step:
  - chemical diffuses
  - each patch responds to local concentration

Response of Patch

if patch is not refractory (brown) **then**

  if local chemical > threshold **then**

    set refractory period

    produce pulse of chemical (red)

else

  decrement refractory period

  degrade chemical in local area

Demonstration of StarLogo Simulation of Spiral Formation

Run SlimeSpiral.slogo