A Simple Pair of Rules

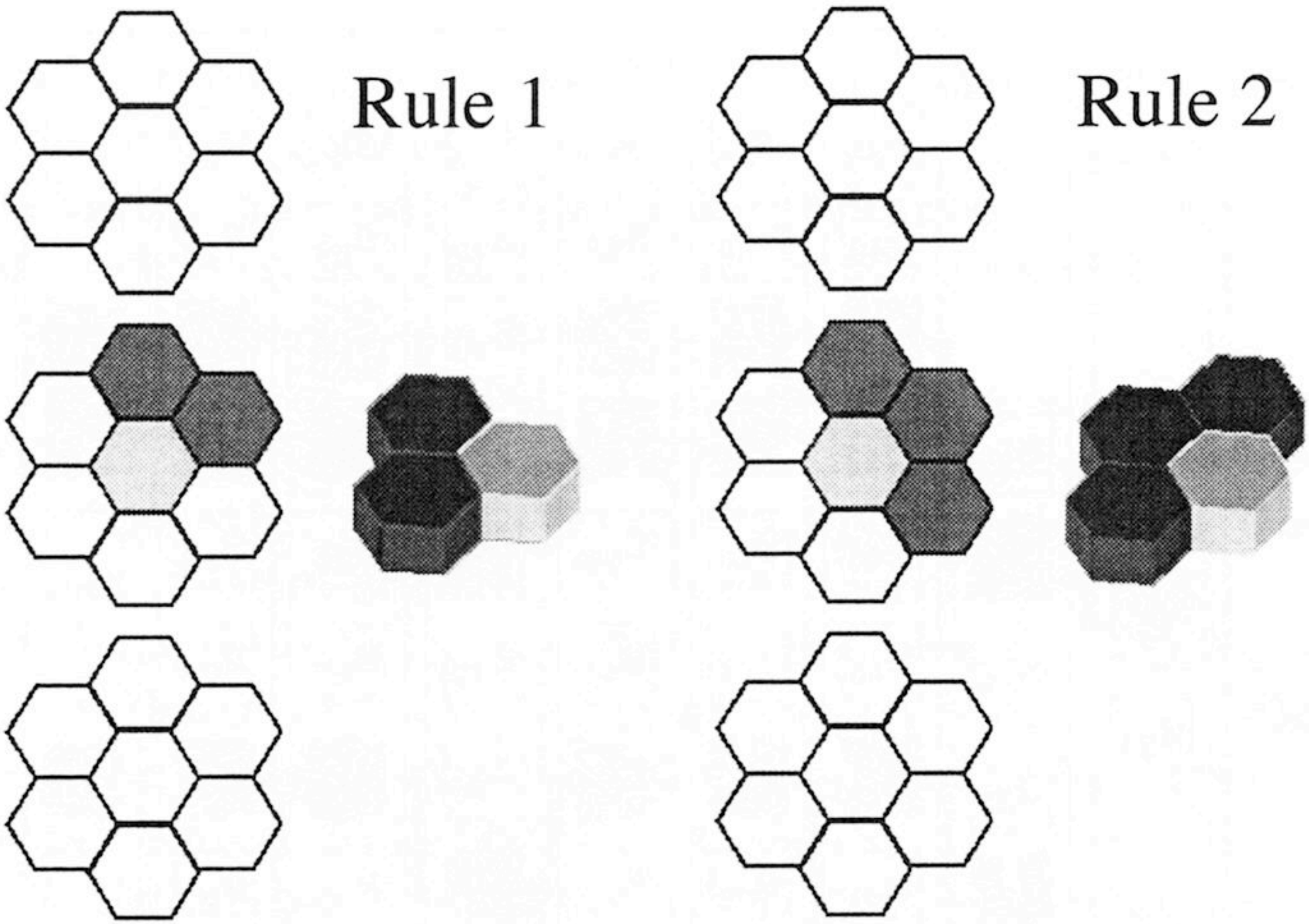


Fig. from *Self-Org. in Biol. Sys.*

Result from Deterministic Rules

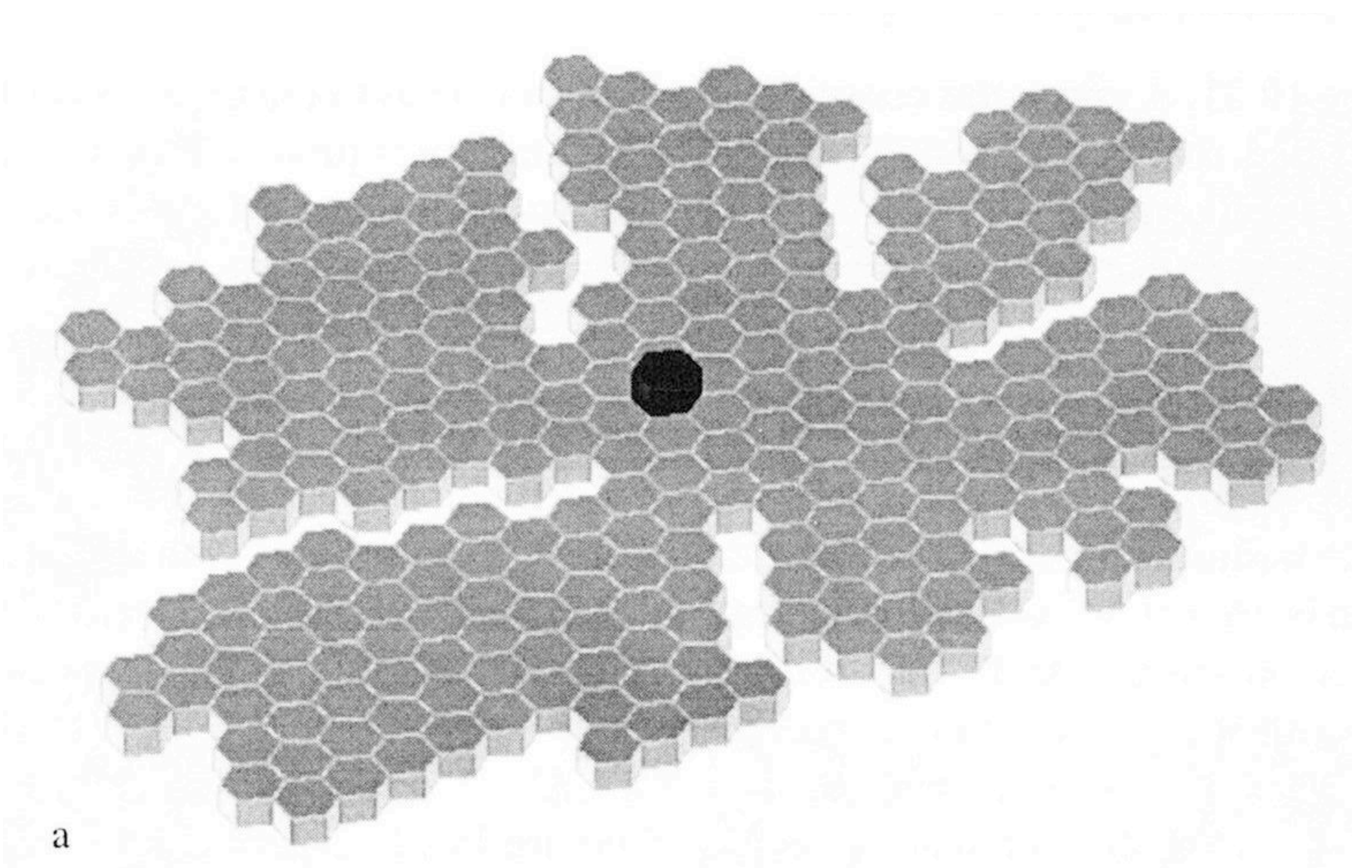


Fig. from *Self-Org. in Biol. Sys.*

Result from Probabilistic Rules

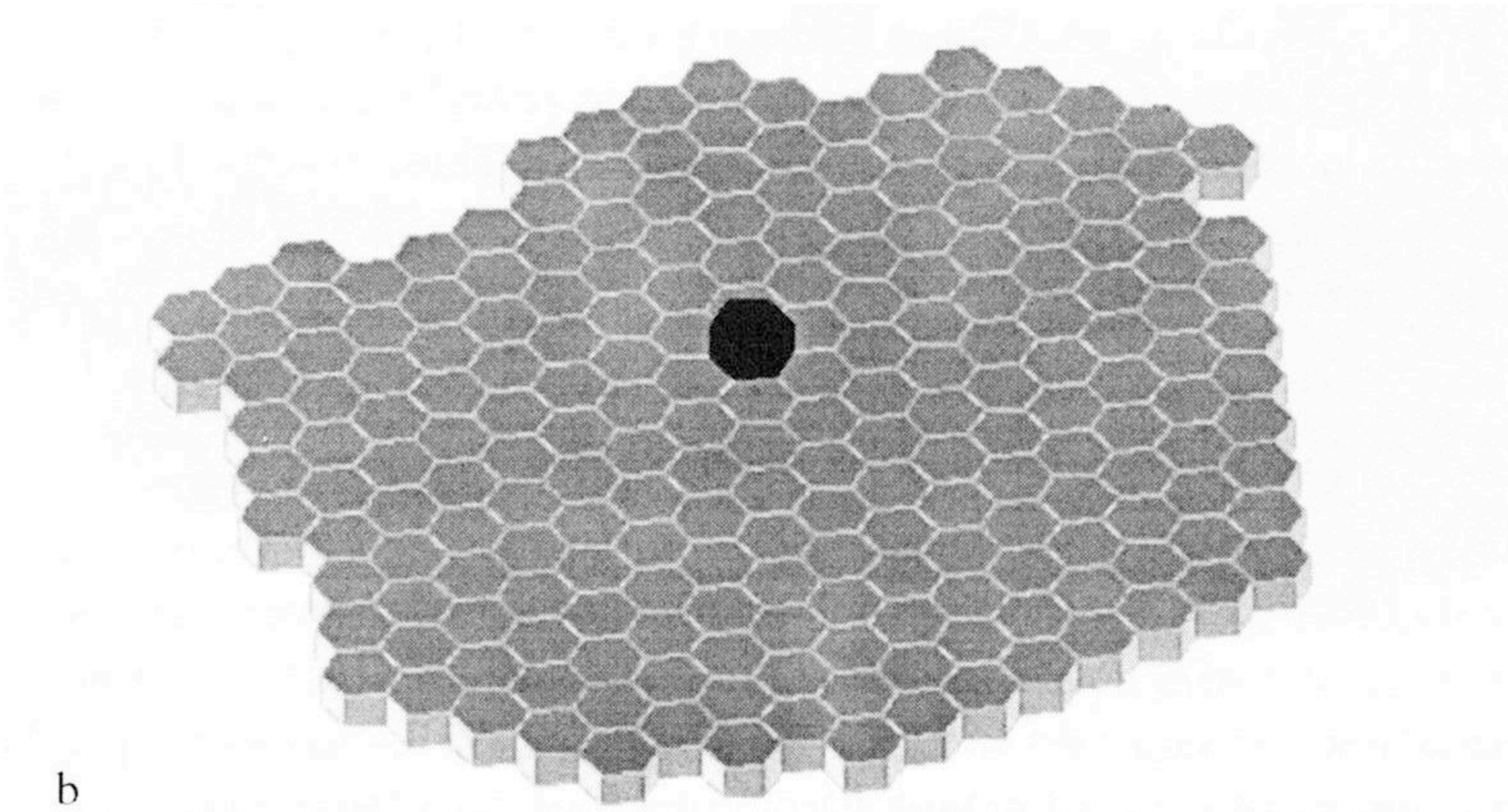


Fig. from *Self-Org. in Biol. Sys.*

Example Rules for a More Complex Architecture

The following stimulus configurations cause the agent to deposit a type-1 brick:

$$(1.1)\quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(1.2)\quad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 1 & \bullet & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

B

$$(2.1)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\1&\bullet&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.2)\quad \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}1&2&0\\0&\bullet&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.3)\quad \begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&\bullet&0\\1&2&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix}$$

$$(2.4)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}2&0&0\\2&\bullet&0\\2&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.5)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\2&\bullet&0\\2&2&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.6)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}2&2&0\\2&\bullet&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.7)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}2&2&2\\0&\bullet&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.8)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&\bullet&0\\2&2&2\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.9)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}1&2&0\\0&\bullet&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.10)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&\bullet&0\\1&2&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.11)^{*}\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}2&2&0\\2&\bullet&0\\2&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.12)^{*}\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&2&2\\0&\bullet&2\\0&0&2\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.13)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}2&2&2\\2&\bullet&2\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.14)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}2&2&0\\2&\bullet&0\\2&2&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.15)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\2&\bullet&2\\2&2&2\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.16)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&2&2\\0&\bullet&2\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.17)\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&\bullet&2\\0&2&2\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$(2.18)^{*}\quad \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\times\begin{bmatrix}2&0&0\\2&\bullet&0\\2&2&2\end{bmatrix}\times\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

Second Group of Rules

- For these configurations, deposit a type-2 brick

Result

- 20□20□20 lattice
- 10 wasps
- After 20 000 simulation steps
- Axis and plateaus
- Resembles nest of *Parachartergus*

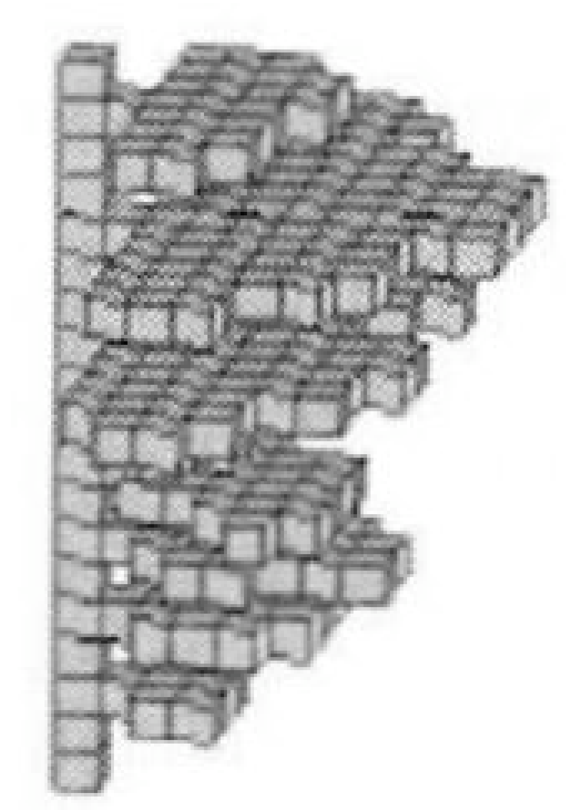


Fig. from Bonabeau & al., *Swarm Intell*.

Architectures Generated from Other Rule Sets

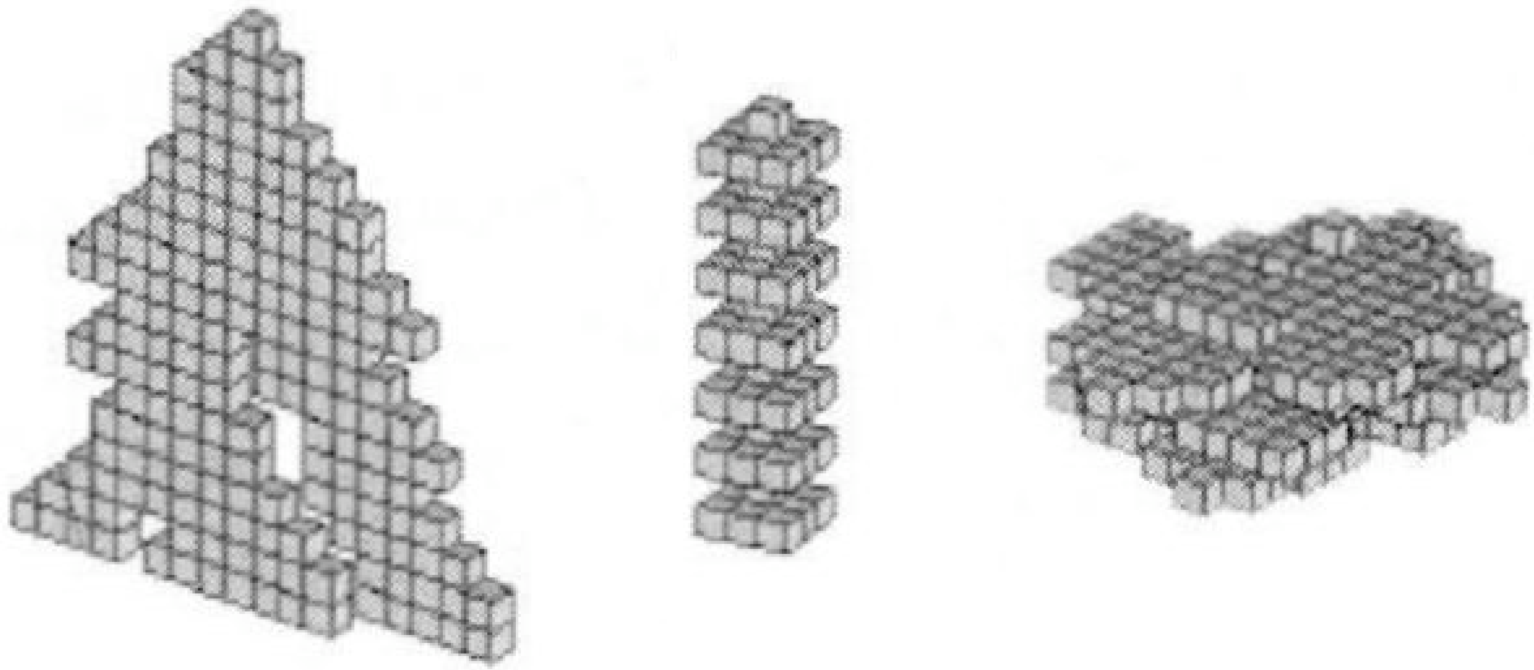


Fig. from Bonabeau & al., *Swarm Intell*.

More Examples

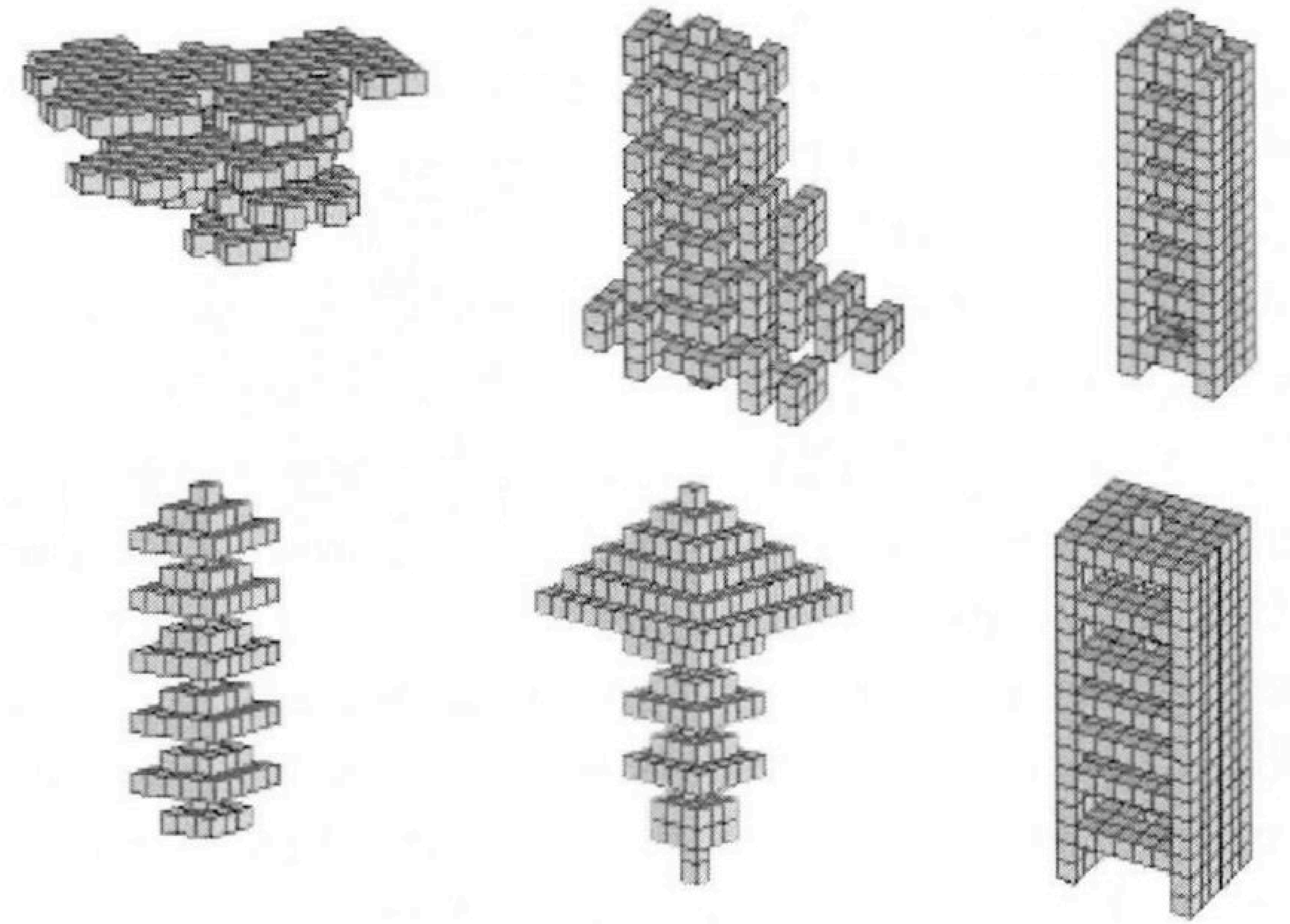
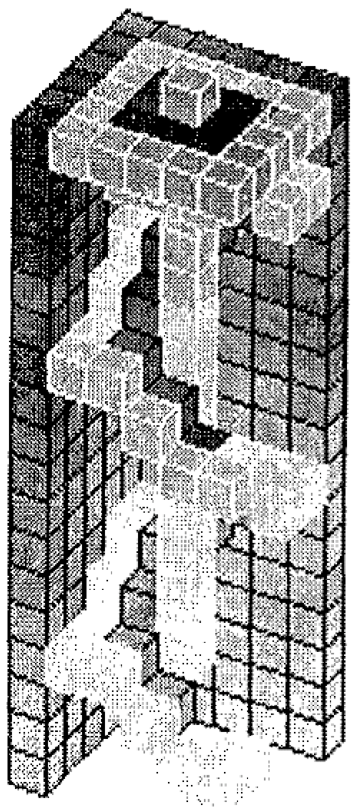


Fig. from Bonabeau & al., *Swarm Intell*.

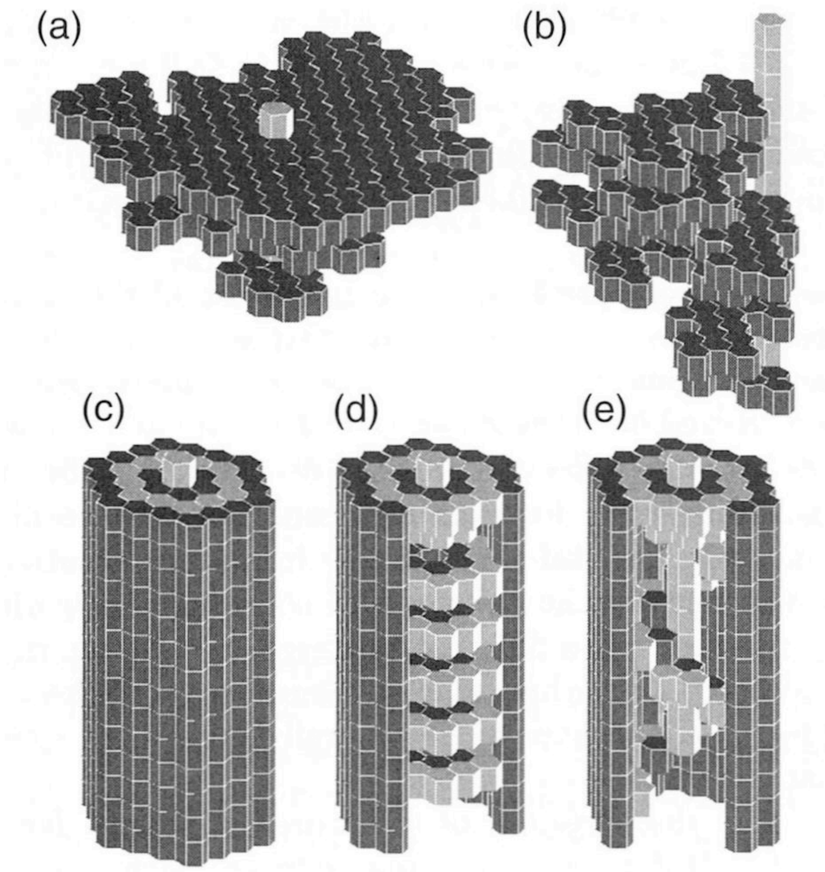
An Interesting Example



- Includes
  - central axis
  - external envelope
  - long-range helical ramp
- Similar to *Apicotermes* termite nest

Fig. from Theraulaz & Bonabeau (1995)

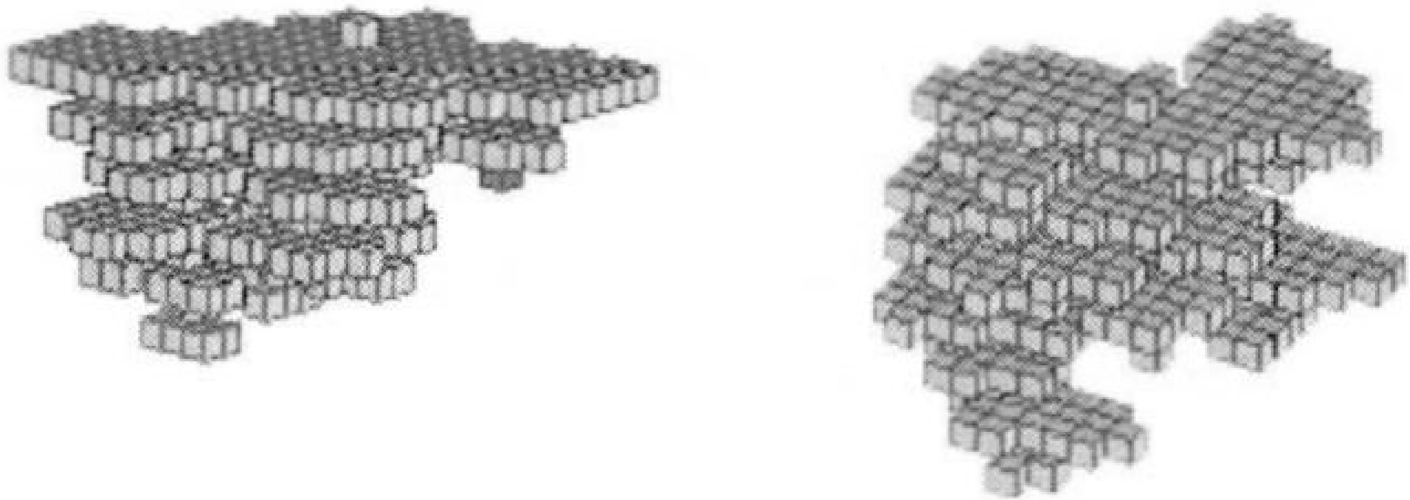
Similar Results with Hexagonal Lattice



- 20×20×20 lattice
- 10 wasps
- All resemble nests of wasp species
- (d) is (c) with envelope cut away
- (e) has envelope cut away

Fig. from Bonabeau & al., *Swarm Intell*.

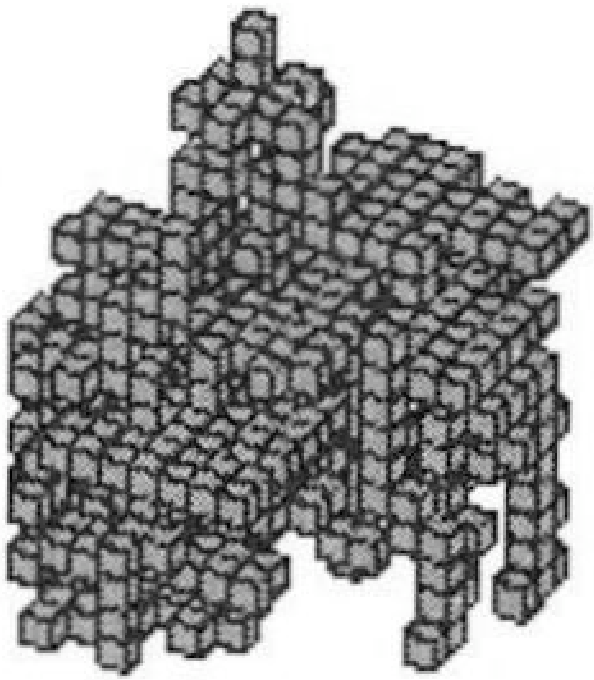
Effects of Randomness (Coordinated Algorithm)



- Specifically different (i.e., different in details)
- Generically the same (qualitatively identical)
- Sometimes results are fully constrained

Fig. from Bonabeau & al., *Swarm Intell*.

Effects of Randomness (Non-coordinated Algorithm)



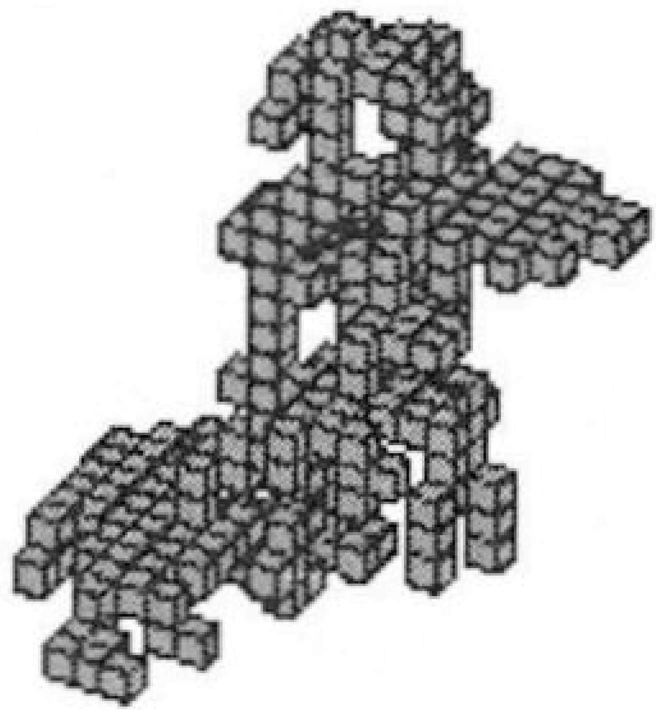


Fig. from Bonabeau & al., *Swarm Intell*.

Non-coordinated Algorithms

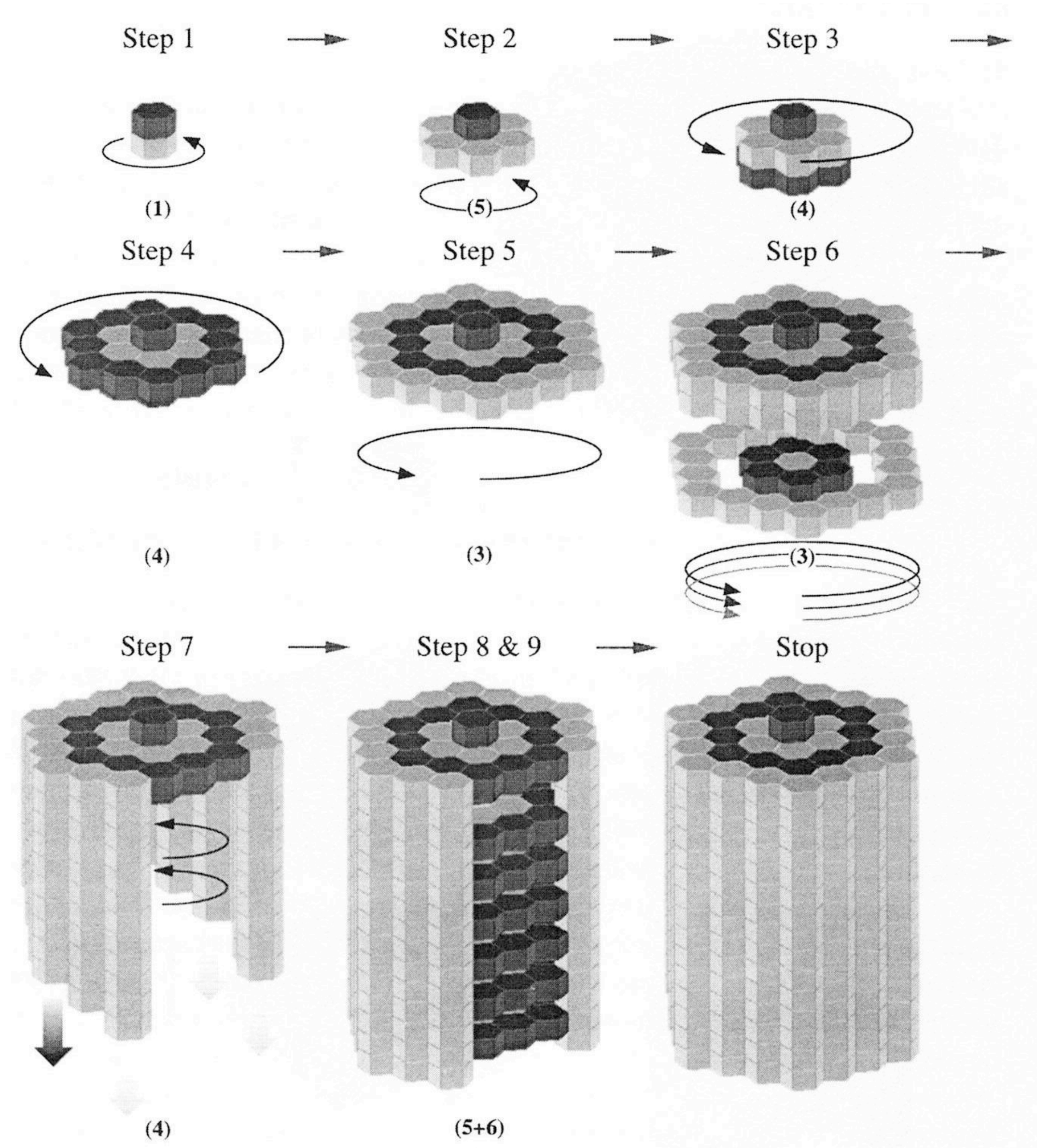
- Stimulating configurations are not ordered in time and space
- Many of them overlap
- Architecture grows without any coherence
- May be convergent, but are still unstructured

Coordinated Algorithm

- Non-conflicting rules
  - can’t prescribe two different actions for the same configuration
- Stimulating configurations for different building stages cannot overlap
- At each stage, “handshakes” and “interlocks” are required to prevent conflicts in parallel assembly

More Formally…

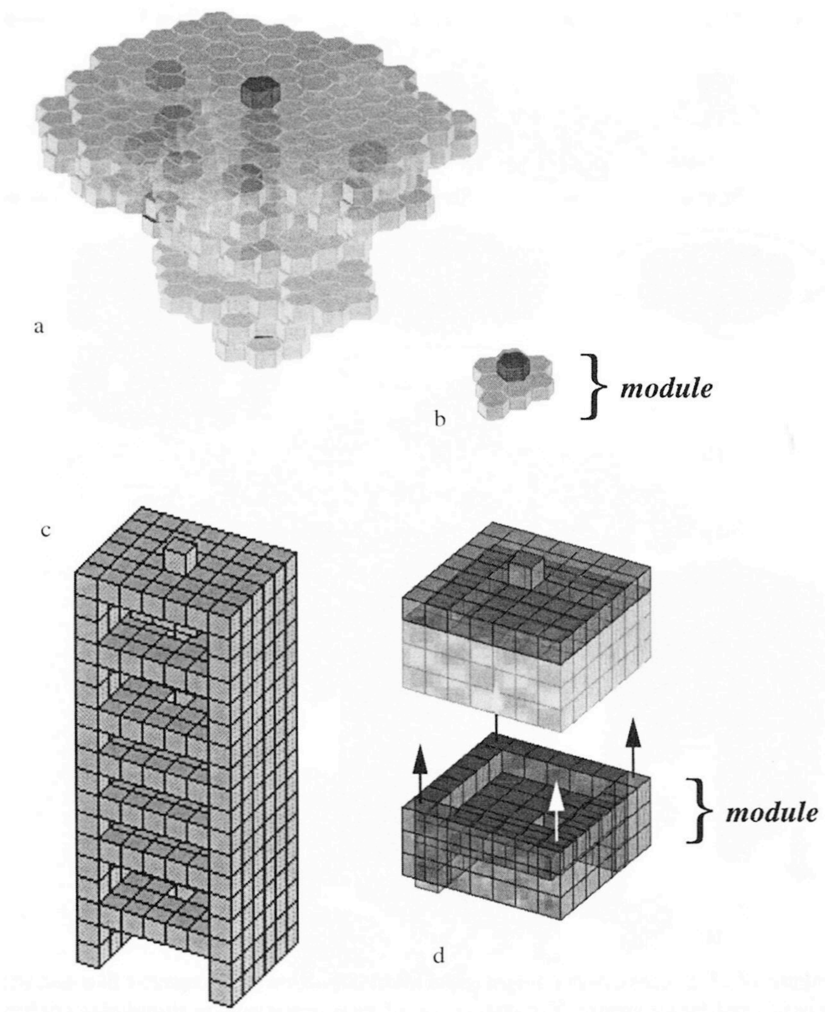
- Let $C = \{c_1, c_2, \ldots, c_n\}$ be the set of local stimulating configurations
- Let $(S_1, S_2, \ldots, S_m)$ be a sequence of assembly stages
- These stages partition C into mutually disjoint subsets $C(S_p)$
- Completion of S_p signaled by appearance of a configuration in $C(S_{p+1})$



Example

Fig. from Camazine &al., *Self-Org. Biol. Sys.*

Modular Structure



- Recurrent states induce cycles in group behavior
- These cycles induce modular structure
- Each module is built during a cycle
- Modules are qualitatively similar

Fig. from Camazine &al., *Self-Org. Biol. Sys.*

Possible Termination Mechanisms

- Qualitative
  - the assembly process leads to a configuration that is not stimulating
- Quantitative
  - a separate rule inhibiting building when nest a certain size relative to population
  - “empty cells rule”: make new cells only when no empties available
  - growing nest may inhibit positive feedback mechanisms

Observations

- Random algorithms tend to lead to uninteresting structures
  - random or space-filling shapes
- Similar structured architectures tend to be generated by similar coordinated algorithms
- Algorithms that generate structured architectures seem to be confined to a small region of rule-space

Analysis

- Define matrix M:
  - 12 columns for 12 sample structured architectures
  - 211 rows for stimulating configurations
  - $M_{ij} = 1$ if architecture j requires configuration i

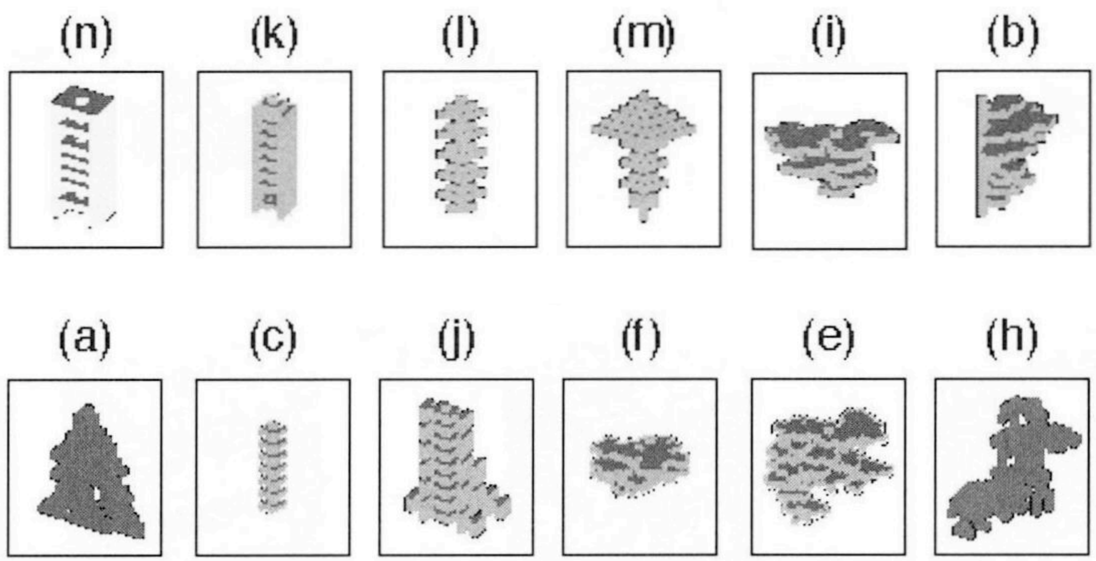


Fig. from Bonabeau & al., *Swarm Intell*.

Factorial Correspondence Analysis

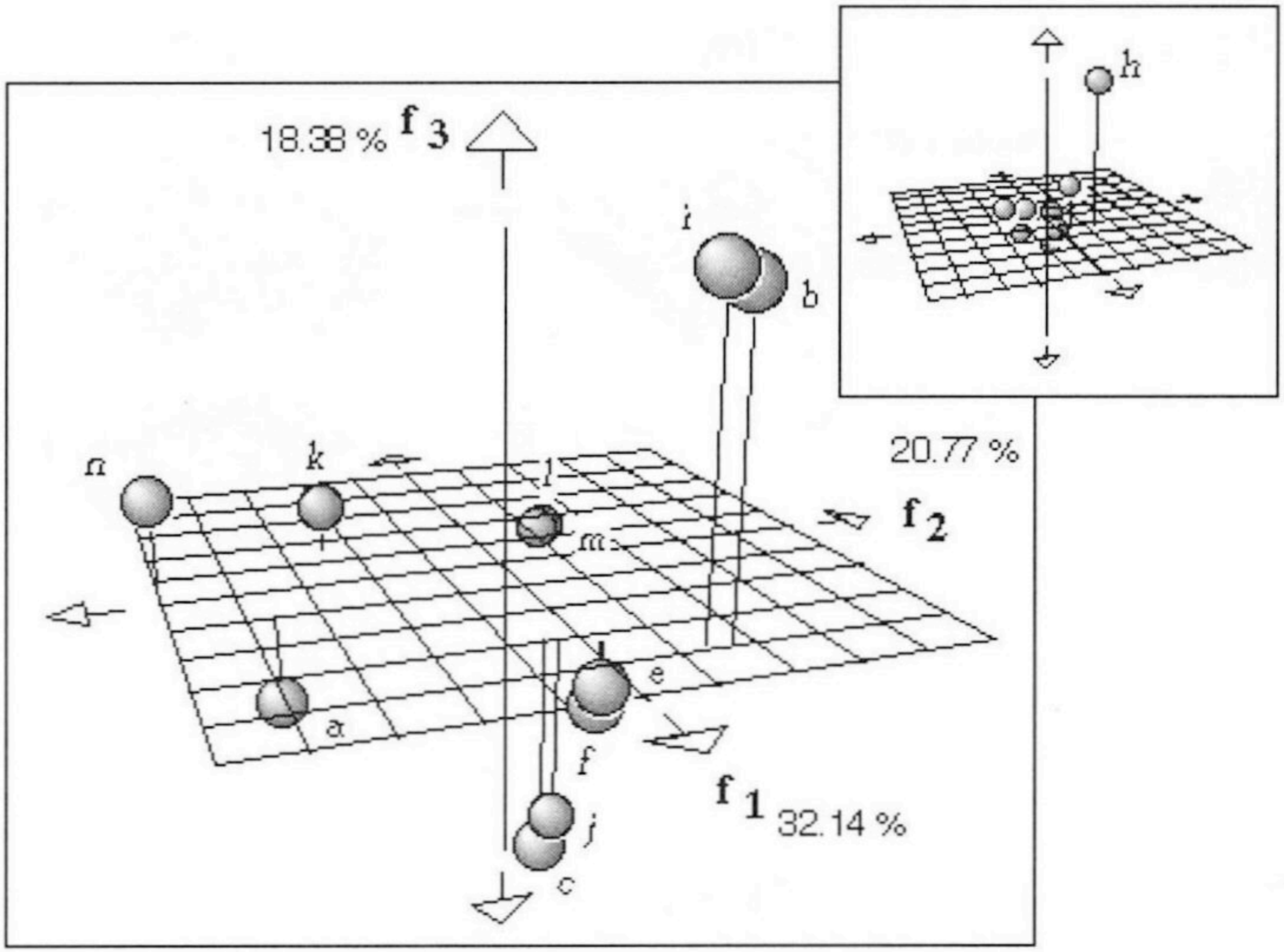


Fig. from Bonabeau & al., *Swarm Intell.*

Conclusions

- Simple rules that exploit discrete (qualitative) stigmergy can be used by autonomous agents to assemble complex, 3D structures
- The rules must be non-conflicting and coordinated according to stage of assembly
- The rules corresponding to interesting structures occupy a comparatively small region in rule-space

Langton's Vants
(Virtual Ants)

Vants

- Square grid
- Squares can be black or white
- Vants can face N, S, E, W
- Behavioral rule:
  - take a step forward,
  - **if** on a white square **then**
    paint it black & turn 90° right
  - **if** on a black square **then**
    paint it white & turn 90° left

Example

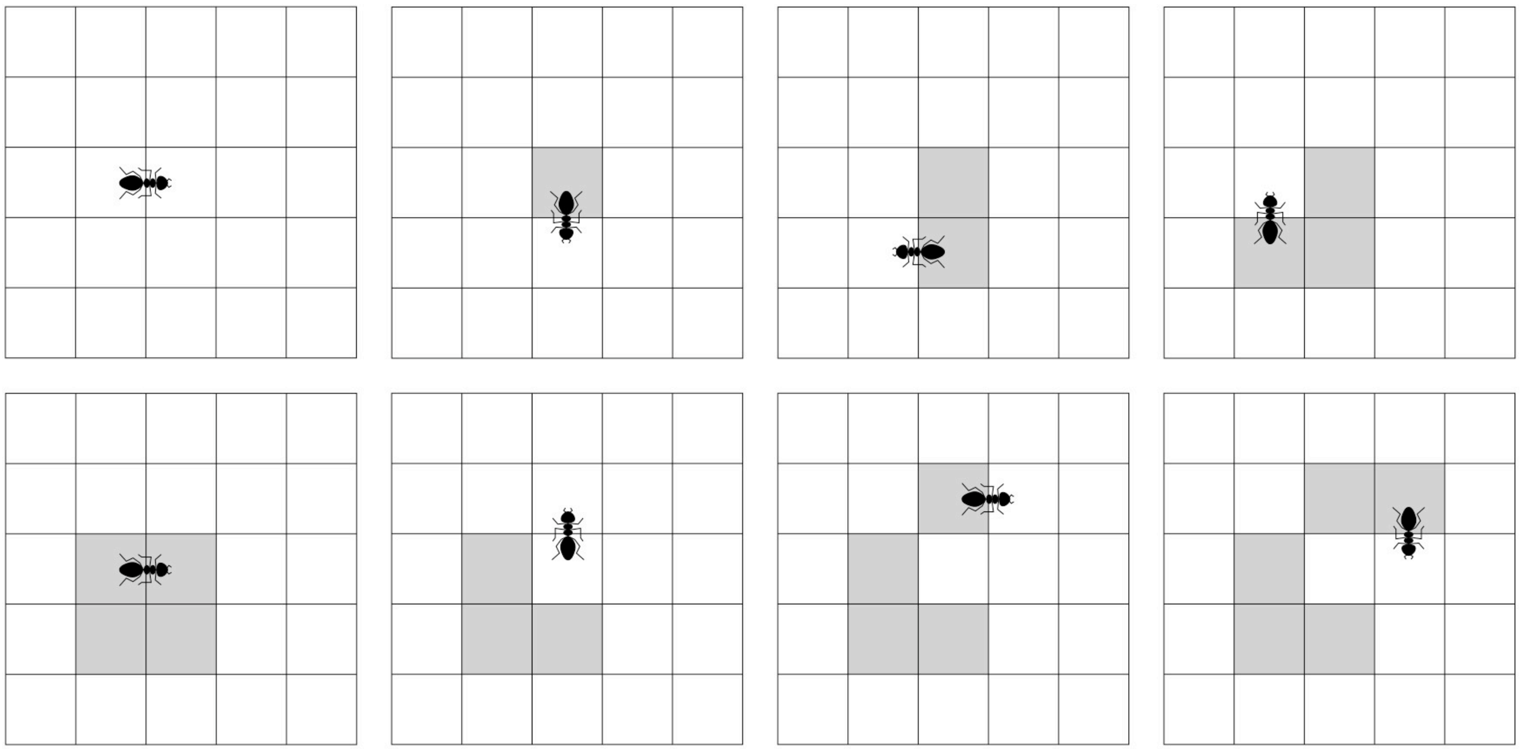


Figure 16.2 Eight steps of Langton's virtual ant, starting from an initially blank grid

Time Reversibility

- Vants are time-reversible
- But time reversibility does not imply global simplicity
- Even a single vant interacts with its own prior history
- But complexity does not always imply random-appearing behavior

Digression: Time-Reversibility and the Physical Limits of Computation

- Irreversible logic gate loses one bit of information
- This equals entropy decrease of $kT \ln 2$
- Therefore a conventional gate must dissipate at least $kT \ln 2$ joules
  - typical transistors dissipate about $10^8 kT$
- Reversible gates can dissipate arbitrarily little energy
- Charles H. Bennett (1973). See also *Feynman Lectures on Computation*, ch. 5

Demonstration of Vants

Run vants from CBN website

Conclusions

- Even simple, reversible local behavior can lead to complex global behavior
- Nevertheless, such complex behavior may create structures as well as apparently random behavior
- Perhaps another example of “edge of chaos” phenomena