594 Homework 1

- For asychronous updating, let *k* be the cell that is updated
- Then:

$$s_k(t+1) = \operatorname{sign}\left(h + J_1 \sum_{0<r_{kj} \le R_1} s_j(t) + J_2 \sum_{R_1<r_{kj}<R_2} s_j(t) \right)$$

- Note: for convenience cell *k* is not included in the R_1 neighborhood
- For all other cells *i*, $s_i(t+1) = s_i(t)$

Energy Function

- The energy function is defined by a summation over all the cells, including the one that changed:

$$E\{\mathbf{s}(t)\} = -\tfrac{1}{2}\sum_i s_i(t)\,\mathrm{sign}\left[h + J_1 \sum_{0<r_{ij}\le R_1} s_j(t) + J_2 \sum_{R_1<r_{ij}<R_2} s_j(t)\right]$$

- You need to show that

$$\Delta E = E\{\mathbf{s}(t+1)\} - E\{\mathbf{s}(t)\} \le 0$$

Ant Colony Optimization (ACO)

Developed in 1991 by Dorigo (PhD dissertation) in collaboration with Colorni & Maniezzo

Basis of all Ant-Based Algorithms

- Positive feedback
- Negative feedback
- Cooperation

Positive Feedback

- To reinforce portions of good solutions that contribute to their goodness
- To reinforce good solutions directly
- Accomplished by *pheromone accumulation*

Negative Feedback

- To avoid premature convergence (*stagnation*)
- Accomplished by *pheromone evaporation*

Cooperation

- For simultaneous exploration of different solutions
- Accomplished by:
  - *multiple ants* exploring solution space
  - *pheromone trail* reflecting multiple perspectives on solution space

Ant System for Traveling Salesman Problem (AS-TSP)

- During each iteration, each ant completes a tour
- During each tour, each ant maintains *tabu list* of cities already visited
- Each ant has access to
  - distance of current city to other cities
  - intensity of local pheromone trail
- Probability of next city depends on both

Transition Rule

- Let $\eta_{ij} = 1/d_{ij}$ = "nearness" of city j to current city i
- Let τ_{ij} = strength of trail from i to j
- Let J_i^k = list of cities ant k still has to visit after city i in current tour
- Then transition probability for ant k going from i to $j \in J_i^k$ in tour t is:

$$p_{ij}^k = \frac{\left[\tau_{ij}(t)\right]^{\alpha}\left[\eta_{ij}\right]^{\beta}}{\sum_{l \in J_i^k}\left[\tau_{il}(t)\right]^{\alpha}\left[\eta_{il}\right]^{\beta}}$$

Pheromone Deposition

- Let $T^k(t)$ be tour t of ant k
- Let $L^k(t)$ be the length of this tour
- After completion of a tour, each ant k contributes:

$$\Delta\tau_{ij}^{k} = \begin{cases} Q / L^k(t) & \text{if } (i,j) \in T^k(t) \\ 0 & \text{if } (i,j) \notin T^k(t) \end{cases}$$

Pheromone Decay

- Define total pheromone deposition for tour *t*:

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}(t)$$

- Let ρ be decay coefficient
- Define trail intensity for next round of tours:

$$\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) + \Delta\tau_{ij}(t)$$

Number of Ants is Critical

- Too many:
  - suboptimal trails quickly reinforced
  - □ early convergence to suboptimal solution
- Too few:
  - don't get cooperation before pheromone decays
- Good tradeoff:
  number of ants = number of cities
  ($m = n$)

Improvement: "Elitist" Ants

- Add a few ($e \approx 5$) "elitist" ants to population
- Let T^+ be best tour so far
- Let L^+ be its length
- Each "elitist" ant reinforces edges in T^+ by Q/L^+
- Add e more "elitist" ants
- This applies accelerating positive feedback to best tour

Time Complexity

- Let t be number of tours
- Time is $\mathcal{O}(tn^2m)$
- If $m = n$ then $\mathcal{O}(tn^3)$
  - that is, cubic in number of cities

Evaluation

- Both "very interesting and disappointing"
- For 30-cities:
  - beat genetic algorithm
  - matched or beat tabu search & simulated annealing
- For 50 & 75 cities and 3000 iterations
  - did not achieve optimum
  - but quickly found good solutions
- I.e., does not scale up well
- Like all general-purpose algorithms, it is out-performed by special purpose algorithms