Variables

$\mathbf{x}_k$ = current position of particle k

$\mathbf{v}_k$ = current velocity of particle k

$\mathbf{p}_k$ = best position found by particle k

$Q(\mathbf{x})$ = quality of position $\mathbf{x}$

g = index of best position found so far

  i.e., $g = \text{argmax}_k\, Q(\mathbf{p}_k)$

$\square_1$, $\square_2$ = random variables uniformly distributed over [0, 2]

w = inertia

Velocity & Position Updating

$\mathbf{v}_k' = w\,\mathbf{v}_k + \phi_1\,(\mathbf{p}_k - \mathbf{x}_k) + \phi_2\,(\mathbf{p}_g - \mathbf{x}_k)$

- $w\,\mathbf{v}_k$ maintains direction (*inertial* part)
- $\phi_1\,(\mathbf{p}_k - \mathbf{x}_k)$ turns toward private best (*cognition* part)
- $\phi_2\,(\mathbf{p}_g - \mathbf{x}_k)$ turns towards public best (*social* part)

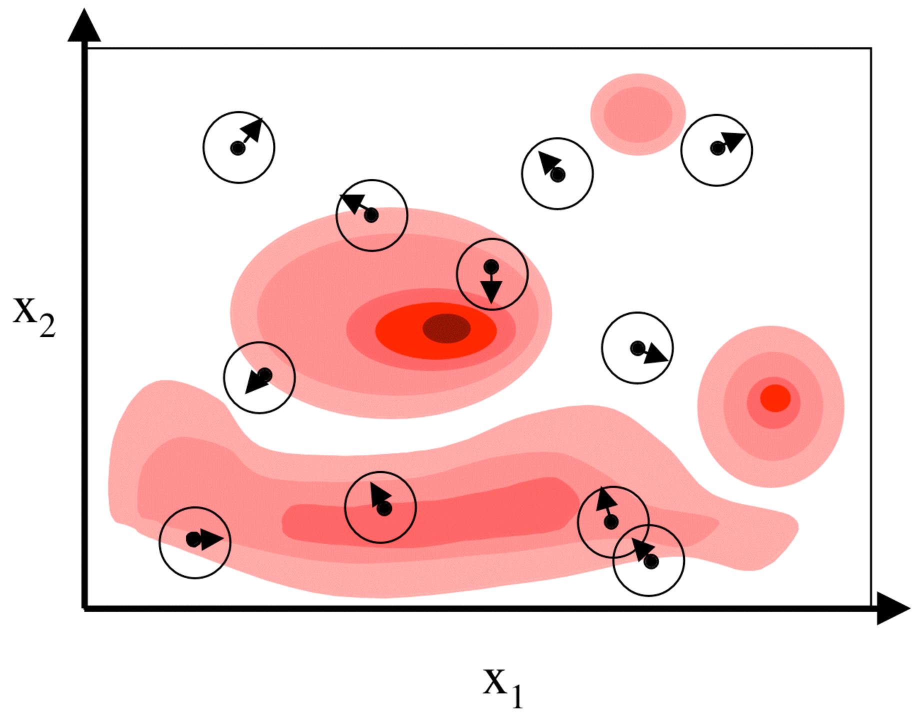
$\mathbf{x}_k' = \mathbf{x}_k + \mathbf{v}_k$

- Allowing $\phi_1, \phi_2 > 1$ permits overshooting and better exploration (*important!*)
- Good balance of *exploration* & *exploitation*
- Limiting $\mathbf{v}_k < \mathbf{v}_{\max}$ controls resolution of search

Improvements

- Alternative velocity update equation:
  $\mathbf{v}_k' = \chi\,[w\,\mathbf{v}_k + \varphi_1\,(\mathbf{p}_k - \mathbf{x}_k) + \varphi_2\,(\mathbf{p}_g - \mathbf{x}_k)]$
  χ = constriction coefficient (controls magnitude of $\mathbf{v}_k$)
- Alternative neighbor relations:
  - **star**: fully connected (each responds to best of all others; fast information flow)
  - **circle**: connected to *K* immediate neighbors (slows information flow)
  - **wheel**: connected to one axis particle (moderate information flow)

Spatial Extension



- Spatial extension avoids premature convergence
- Preserves diversity in population
- More like flocking/schooling models

Fig. from EVALife site

Some Applications of PSO

- integer programming
- minimax problems
  - in optimal control
  - engineering design
  - discrete optimization
  - Chebyshev approximation
  - game theory
- multiobjective optimization
- hydrologic problems
- musical improvisation!

Millonas' Five Basic Principles of Swarm Intelligence

1. *Proximity principle:*
   pop. should perform simple space & time computations
2. *Quality principle:*
   pop. should respond to quality factors in environment
3. *Principle of diverse response:*
   pop. should not commit to overly narrow channels
4. *Principle of stability:*
   pop. should not change behavior every time env. changes
5. *Principle of adaptability:*
   pop. should change behavior when it's worth comp. price

(Millonas 1994)

Kennedy & Eberhart on PSO

"This algorithm belongs ideologically to that philosophical school

that allows wisdom to emerge rather than trying to impose it,

that emulates nature rather than trying to control it,

and that seeks to make things simpler rather than more complex.

Once again nature has provided us with a technique for processing information that is at once elegant and versatile."

Additional Bibliography

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2. Bonabeau, E., Dorigo, M., & Theraulaz, G. *Swarm Intelligence: From Natural to Artificial Systems*. Oxford, 1999, chs. 2, 6.
3. Solé, R., & Goodwin, B. *Signs of Life: How Complexity Pervades Biology*. Basic Books, 2000, ch. 6.
4. Resnick, M. *Turtles, Termites, and Traffic Jams: Explorations in Massively Parallel Microworlds*. MIT Press, 1994, pp. 59-68, 75-81.
5. Kennedy, J., & Eberhart, R. "Particle Swarm Optimization," *Proc. IEEE Int'l. Conf. Neural Networks* (Perth, Australia), 1995. http://www.engr.iupui.edu/~shi/pso.html.

IV. Cooperation & Competition

Game Theory and the Iterated Prisoner's Dilemma

The Rudiments of Game Theory

Leibniz on Game Theory

- "Games combining chance and skill give the best representation of human life, particularly of military affairs and of the practice of medicine which necessarily depend partly on skill and partly on chance." — Leibniz (1710)
- "… it would be desirable to have a complete study made of games, treated mathematically."
 — Leibniz (1715)

Origins of Modern Theory

- 1928: John von Neumann: optimal strategy for two-person zero-sum games
  - von Neumann: mathematician & pioneer computer scientist (CAs, "von Neumann machine")
- 1944: von Neumann & Oskar Morgenstern:*Theory of Games and Economic Behavior*
  - Morgenstern: famous mathematical economist
- 1950: John Nash: *Non-cooperative Games*
  - his PhD dissertation (27 pages)
  - "genius," Nobel laureate (1994), schizophrenic