II. Cellular Automata

Cellular Automata (CAs)

- Invented by von Neumann in 1940s to study reproduction
- He succeeded in constructing a self-reproducing CA
- Have been used as:
  - massively parallel computer architecture
  - model of physical phenomena (Fredkin, Wolfram)
- Currently being investigated as model of quantum computation (QCAs)

Structure

- Discrete space (lattice) of regular *cells*
  - 1D, 2D, 3D, …
  - rectangular, hexagonal, …
- At each unit of time a cell changes state in response to:
  - its own previous state
  - states of neighbors (within some "radius")
- All cells obey same state update rule
  - an FSA
- Synchronous updating

Example: Conway's Game of Life

- Invented by Conway in late 1960s
- A simple CA capable of universal computation
- Structure:
  - 2D space
  - rectangular lattice of cells
  - binary states (alive/dead)
  - neighborhood of 8 surrounding cells (& self)
  - simple population-oriented rule

State Transition Rule

- Live cell has 2 or 3 live neighbors
  ⇒ stays as is (stasis)
- Live cell has < 2 live neighbors
  ⇒ dies (loneliness)
- Live cell has > 3 live neighbors
  ⇒ dies (overcrowding)
- Empty cell has 3 live neighbors
  ⇒ comes to life (reproduction)

Demonstration of Life

Go to CBN
Online Experimentation Center
(mitpress.mit.edu/books/FLAOH/cbnhtml/java.html)

Some Observations About Life

1. Long, chaotic-looking initial transient
   – unless initial density too low or high
2. Intermediate phase
   – isolated islands of complex behavior
   – matrix of static structures & “blinkers”
   – gliders creating long-range interactions
3. Cyclic attractor
   – typically short period

From Life to CAs in General

- What gives Life this very rich behavior?
- Is there some simple, general way of characterizing CAs with rich behavior?
- It belongs to Wolfram’s Class IV

The four classes of feedback behaviour

(a) Fixed points

(b) Simple periodic orbits

(c) Period-n orbit

(d) Chaos

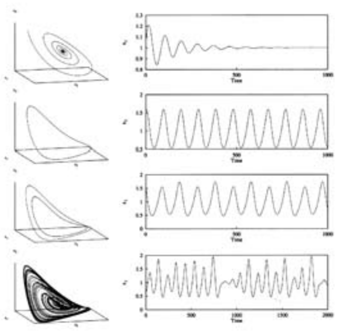

 fig. from Flake via EVALife

Wolfram's Classification

- Class I: evolve to fixed, homogeneous state
  - ~ limit point
- Class II: evolve to simple separated periodic structures
  - ~ limit cycle
- Class III: yield chaotic aperiodic patterns
  - ~ strange attractor (chaotic behavior)
- Class IV: complex patterns of localized structure
  - ~ long transients, no analog in dynamical systems

Langton's Investigation

Under what conditions can we expect a complex dynamics of information to emerge spontaneously and come to dominate the behavior of a CA?

Approach

- Investigate 1D CAs with:
  - random transition rules
  - starting in random initial states
- Systematically vary a simple parameter characterizing the rule
- Evaluate qualitative behavior (Wolfram class)

Assumptions

- Periodic boundary conditions
  - no special place
- Strong quiescence:
  - if all the states in the neighborhood are the same, then the new state will be the same
  - persistence of uniformity
- Spatial isotropy:
  - all rotations of neighborhood state result in same new state
  - no special direction
- Totalistic [not used by Langton]:
  - depend only on sum of states in neighborhood
  - implies spatial isotropy

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Langton's Lambda

- Designate one state to be quiescent state
- Let K = number of states
- Let $N = 2r + 1$ = area of neighborhood
- Let $T = K^N$ = number of entries in table
- Let n_q = number mapping to quiescent state
- Then

$$\lambda = \frac{T - n_q}{T}$$

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Range of Lambda Parameter

- If *all* configurations map to quiescent state: $\lambda = 0$
- If *no* configurations map to quiescent state: $\lambda = 1$
- If every state is represented *equally*: $\lambda = 1 - 1/K$
- A sort of measure of "excitability"

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