Lecture 4
Langton's Investigation

Under what conditions can we expect a complex dynamics of information to emerge spontaneously and come to dominate the behavior of a CA?

Approach

- Investigate 1D CAs with:
  - random transition rules
  - starting in random initial states
- Systematically vary a simple parameter characterizing the rule
- Evaluate qualitative behavior (Wolfram class)

Assumptions

- Periodic boundary conditions
  - no special place
- Strong quiescence:
  - if all the states in the neighborhood are the same, then the new state will be the same
  - persistence of uniformity
- Spatial isotropy:
  - all rotations of neighborhood state result in same new state
  - no special direction
- Totalistic [not used by Langton]:
  - depend only on sum of states in neighborhood
  - implies spatial isotropy

Langton's Lambda

- Designate one state to be quiescent state
- Let K = number of states
- Let $N = 2r + 1$ = area of neighborhood
- Let $T = K^N$ = number of entries in table
- Let n_q = number mapping to quiescent state
- Then

$$\lambda = \frac{T - n_q}{T}$$

Range of Lambda Parameter

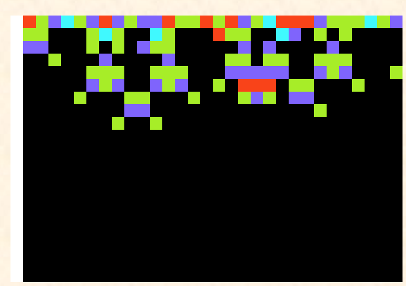
- If *all* configurations map to quiescent state: $\lambda = 0$
- If *no* configurations map to quiescent state: $\lambda = 1$
  - If every state is represented *equally*: $\lambda = 1 - 1/K$
  - A sort of measure of "excitability"

Example

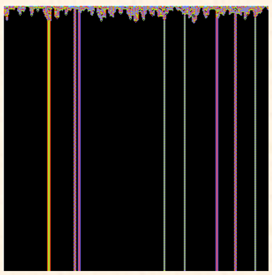
- States: $K = 5$
- Radius: $r = 1$
- Initial state: random
- Transition function: random (given λ)

Class I ($\lambda = 0.2$)

Class I ($\lambda = 0.2$) Closeup



Class II (λ = 0.4)



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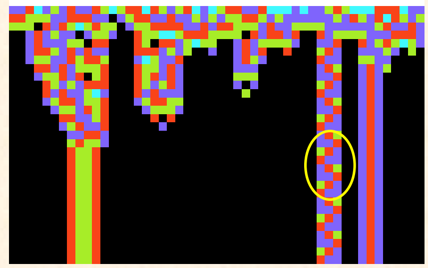
Class II (λ = 0.4) Closeup



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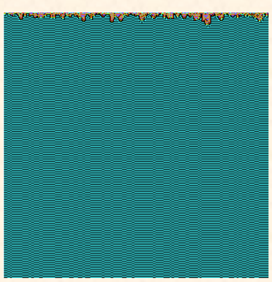
Class II (λ = 0.31)



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Class II (λ = 0.31) Closeup

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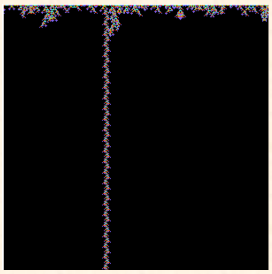
Class II (λ = 0.37)



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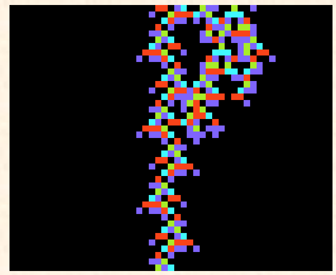
Class II (λ = 0.37) Closeup



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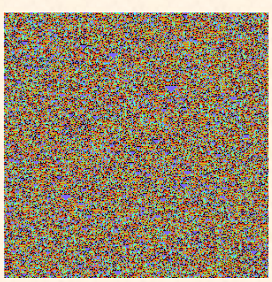
Class III (λ = 0.5)



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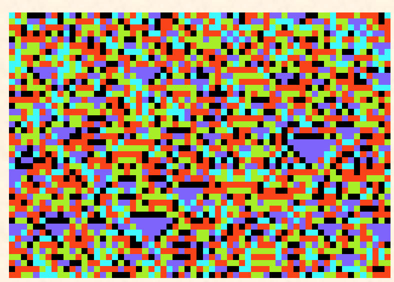
Class III (λ = 0.5) Closeup



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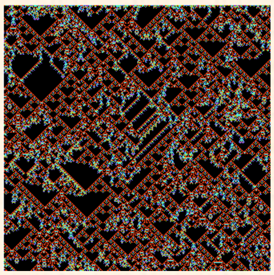
Class IV (λ = 0.35)



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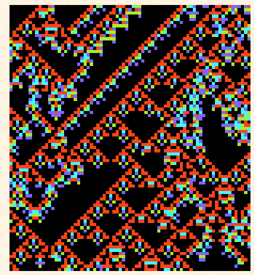
Class IV (λ = 0.35) Closeup



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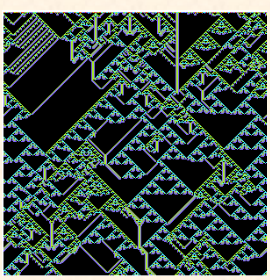
Class IV (λ = 0.34)



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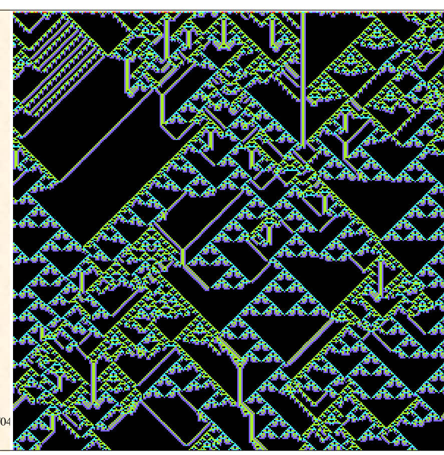

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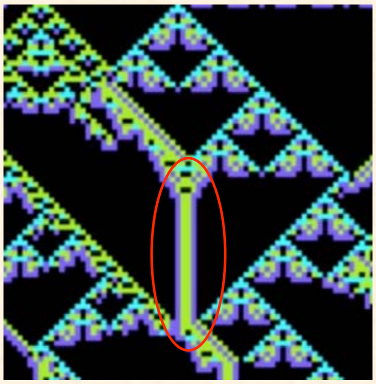

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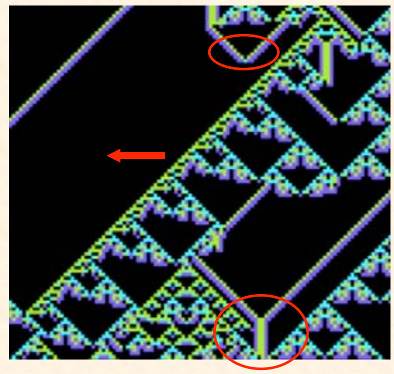

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Class IV Shows Some of the Characteristics of Computation

- Persistent, but not perpetual storage
- Terminating cyclic activity
- Global transfer of control/information

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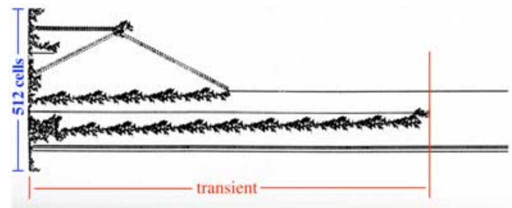
λ of Life

- For Life, $\lambda \approx 0.273$
- which is near the critical region for CAs with:

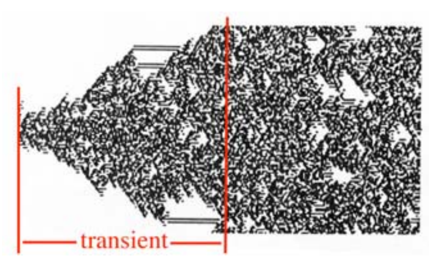
  $K = 2$

  $N = 9$

Transient Length (I, II)



Transient Length (III)



Shannon Information

(very briefly!)

- Information varies directly with surprise
- Information varies inversely with probability
- Information is additive
- ∴The information content of a message is proportional to the negative log of its probability

$$I\{s\} = -\lg \Pr\{s\}$$

Entropy

- Suppose have source S of symbols from ensemble $\{s_1, s_2, \ldots, s_N\}$
- Average information per symbol:

$$\sum_{k=1}^{N} \Pr\{s_k\} I\{s_k\} = \sum_{k=1}^{N} \Pr\{s_k\}\left(-\lg \Pr\{s_k\}\right)$$

- This is the *entropy* of the source:

$$H\{S\} = -\sum_{k=1}^{N} \Pr\{s_k\} \lg \Pr\{s_k\}$$

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Maximum and Minimum Entropy

- Maximum entropy is achieved when all signals are equally likely
  - No ability to guess; maximum surprise
  - $H_{\max} = \lg N$
- Minimum entropy occurs when one symbol is certain and the others are impossible
  - No uncertainty; no surprise
  - $H_{\min} = 0$

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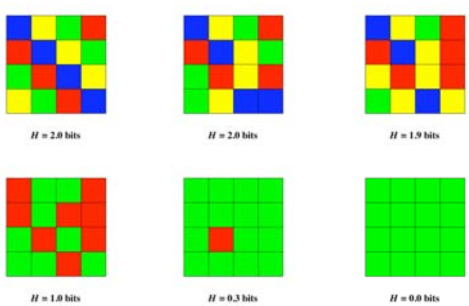
Entropy Examples



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Entropy of Transition Rules

- Among other things, a way to measure the uniformity of a distribution

$$H = -\sum_i p_i \lg p_i$$

- Distinction of quiescent state is arbitrary
- Let n_k = number mapping into state k
- Then $p_k = n_k / T$

$$H = \lg T - \frac{1}{T}\sum_{k=1}^{K} n_k \lg n_k$$

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Entropy Range

- Maximum entropy ($\lambda = 1 - 1/K$):
  uniform as possible
  all $n_k = T/K$
  $H_{max} = \lg K$
- Minimum entropy ($\lambda = 0$ or $\lambda = 1$):
  non-uniform as possible
  one $n_s = T$
  all other $n_r = 0$ ($r \neq s$)
  $H_{min} = 0$

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Further Investigations by Langton

- 2-D CAs
- $K = 8$
- $N = 5$
- 64×64 lattice
- periodic boundary conditions

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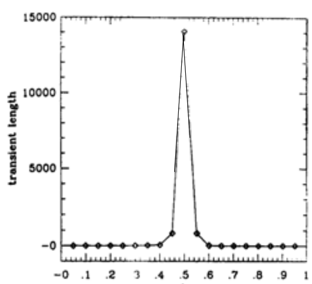
Avg. Transient Length vs. λ (*K*=4, *N*=5)



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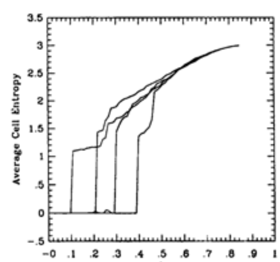
Avg. Cell Entropy vs. λ (*K*=4, *N*=5)



$$H(A) = \sum_{k=1}^{K} p_k \lg p_k$$

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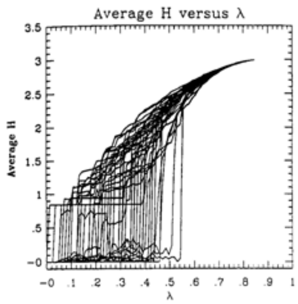
Avg. Cell Entropy vs. λ (*K*=4, *N*=5)



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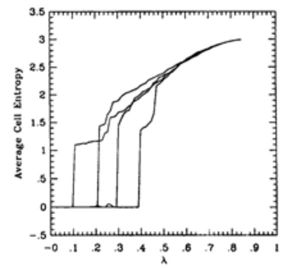
Avg. Cell Entropy vs. λ (*K*=4, *N*=5)



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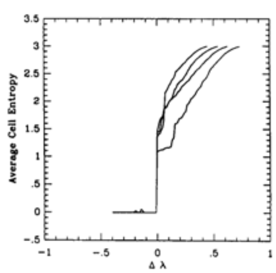
Avg. Cell Entropy vs. Δλ (*K*=4, *N*=5)



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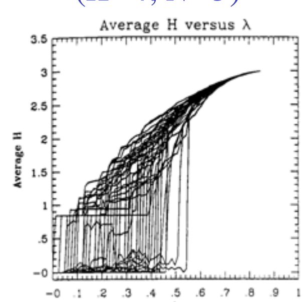
Avg. Cell Entropy vs. λ (*K*=4, *N*=5)



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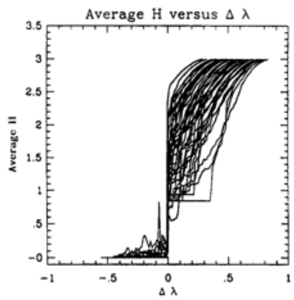
Avg. Cell Entropy vs. $\Delta\lambda$ (K=4, N=5)



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Entropy of Independent Systems

- Suppose sources A and B are independent
- Let $p_j = \Pr\{a_j\}$ and $q_k = \Pr\{b_k\}$
- Then $\Pr\{a_j, b_k\} = \Pr\{a_j\} \Pr\{b_k\} = p_j q_k$

$$H(A,B) = \sum_{j,k} \Pr(a_j,b_k) \lg \Pr(a_j,b_k)$$

$$= \sum_{j,k} p_j q_k \lg(p_j q_k) = \sum_{j,k} p_j q_k (\lg p_j + \lg q_k)$$

$$= \sum_j p_j \lg p_j + \sum_k q_k \lg q_k = H(A) + H(B)$$

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Mutual Information

- *Mutual information* measures the degree to which two sources are not independent
- A measure of their correlation

$$I(A,B) = H(A) + H(B) - H(A,B)$$

- $I(A,B) = 0$ for completely independent sources
- $I(A,B) = H(A) = H(B)$ for completely correlated sources

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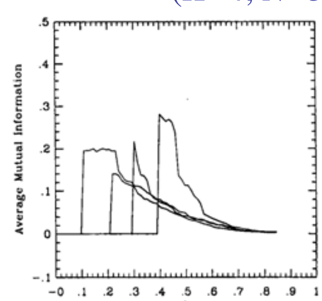
Avg. Mutual Info vs. λ (K=4, N=5)



$I(A,B) = H(A) + H(B) - H(A,B)$

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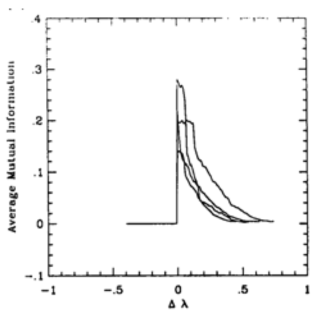
Avg. Mutual Info vs. Δλ (*K*=4, *N*=5)



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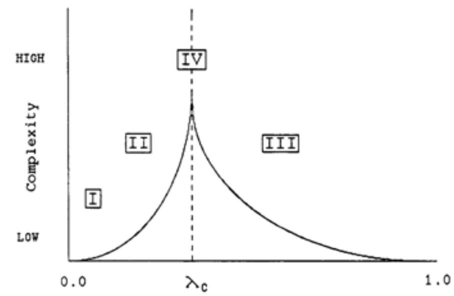
Complexity vs. λ



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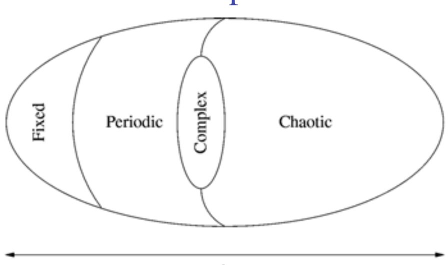
Schematic of CA Rule Space vs. λ



8/27/04 Fig. from Langton, "Life at Edge of Chaos" 47

Additional Bibliography

1. Langton, Christopher G. "Computation at the Edge of Chaos: Phase Transitions and Emergent Computation," in *Emergent Computation*, ed. Stephanie Forrest. North-Holland, 1990.
2. Langton, Christopher G. "Life at the Edge of Chaos," in *Artificial Life II*, ed. Langton et al. Addison-Wesley, 1992.
3. Emmeche, Claus. *The Garden in the Machine: The Emerging Science of Artificial Life*. Princeton, 1994.

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