Langton's Vants (Virtual Ants)

Vants

- Square grid
- Squares can be black or white
- Vants can face N, S, E, W
- Behavioral rule:
  - take a step forward,
  - **if** on a white square **then** paint it black & turn 90° right
  - **if** on a black square **then** paint it white & turn 90° left

Example

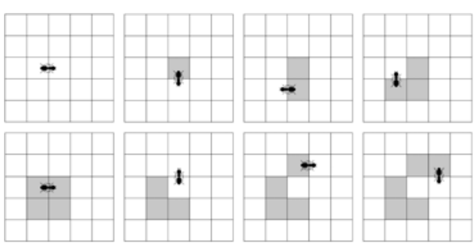

Figure 16.2 Eight steps of Langton's virtual ant, starting from an initially blank grid

Time Reversibility

- Vants are time-reversible
- But time reversibility does not imply global simplicity
- Even a single vant interacts with its own prior history
- But complexity does not always imply random-appearing behavior

Demonstration of Vants

Run vants from CBN website

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Conclusions

- Even simple, reversible local behavior can lead to complex global behavior
- Nevertheless, such complex behavior may create structures as well as apparently random behavior
- Perhaps another example of "edge of chaos" phenomena

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Digression: Time-Reversibility and the Physical Limits of Computation

Work done by:

- Rolf Landauer (1961)
- Charles Bennett (1973)
- Richard Feynman (1981–3)

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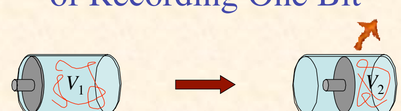
Thermodynamics of Recording One Bit



$$\Delta S = k \ln \frac{V_1}{V_2}$$

if $V_2 = V_1/2$, then $\Delta S = -k \ln 2$

also, $\Delta F = kT \ln 2$

- ΔS derived by gas laws & classical thermodynamics
- Boltzmann constant: $k = 1.381 \times 10^{-23}$ J K^{-1}

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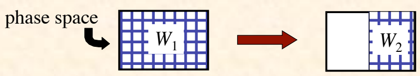
Entropy Change in Terms of Phase Space



- Let W = number of microstates corresponding to a macrostate
- Entropy $S = k \ln W$
- Then $\Delta S = k \ln W_2 - k \ln W_1 = k \ln (W_2 / W_1)$
- If $W_2 = W_1 / 2$, then $\Delta S = -k \ln 2$

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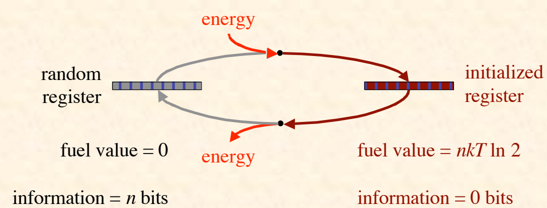
Information and Energy



- initialization equivalent to storing energy
- information and energy are complementary

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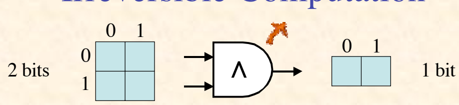
Minimum Energy for Irreversible Computation



- Loss of one bit of information
  - an irreversible operation (many to one)
- $\Delta S = -k \ln 2$
  - entropy decrease must be compensated by heat dissipation
- Minimum energy required: $\Delta F = kT \ln 2$
  - transistors: $\sim 10^8\, kT$; RNA polymerase: $\sim 100\, kT$

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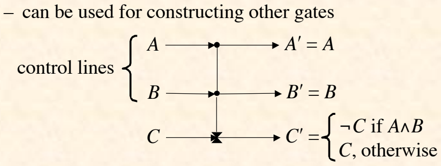
Reversible Gates

- Can make dissipation arbitrarily small by using reversible gates
- All outputs must go somewhere
- Cannot *ever* throw information away
- The Fredkin CCN gate ("Controlled Controlled Not") is reversible
  - can be used for constructing other gates



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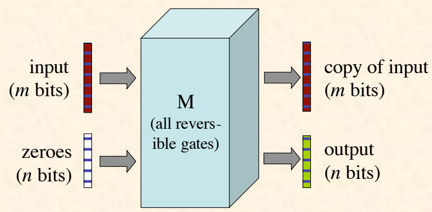
Reversible Computer



- Reversible because get input back
- Only loss is resetting machine for next job
  - energy is proportional to n, number of output bits

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Summary: Energy Required for Reversible Computing

- There is no lower limit on the energy required for basic operations (gates, bit copying, etc.) provided:
  - it is done reversibly
  - it is done sufficiently slowly
- What is the fundamental relation between speed and energy dissipation?

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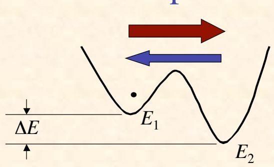
Energy and the Speed of Computation



- Let r be ratio of forward to backward rate
- Statistical mechanics shows: $kT \ln r = \Delta E$
- Greater "driving energy" ⇒ greater rate

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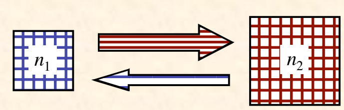
Entropy and the Speed of Computation



- Consider number of accessible microstates, n_1 and n_2
- Can show: $r = n_2 / n_1$
- Hence, $kT \ln r = kT (\ln n_2 - \ln n_1)$ $= (S_2 - S_1)T = T\Delta S$

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Conclusions

- Entropy increase and energy dissipation can be made arbitrarily small by doing reversible computation
- However, the speed of computation is an exponential function of the driving energy or entropy increase

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Additional Information

1. Feynman, R. P. *Feynman Lectures on Computation*, ed. by A.J.G Hey & R.W. Allen. Perseus, 1996.
2. Hey, A.J.G. (ed.) *Feynman and Computation: Exploring the Limits of Computation*. Perseus, 1999.

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