

### Extensions of the Concept of a Rational Solution

- Every maximin solution is a dominant strategy equilibrium
- Every dominant strategy equilibrium is a Nash equilibrium

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### Cooperation Better for Both: A Dilemma

Price Competition		Beta		
		$p = 1$	$p = 2$	$p = 3$
Alpha	$p = 1$	0, 0	50, -10	40, -20
	$p = 2$	-10, 50	20, 20	90, 10
	$p = 3$	-20, 40	10, 90	50, 50

Cooperation

10/20/04 Example from McCain's *Game Theory: An Introductory Sketch* 2

### Dilemmas

- Dilemma: “A situation that requires choice between options that are or seem equally unfavorable or mutually exclusive”  
*– Am. Her. Dict.*
- In game theory: each player acts rationally, but the result is undesirable (less reward)

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### The Prisoners' Dilemma

- Devised by Melvin Dresher & Merrill Flood in 1950 at RAND Corporation
- Further developed by mathematician Albert W. Tucker in 1950 presentation to psychologists
- It “has given rise to a vast body of literature in subjects as diverse as philosophy, ethics, biology, sociology, political science, economics, and, of course, game theory.” — S.J. Hagenmayer
- “This example, which can be set out in one page, could be the most influential one page in the social sciences in the latter half of the twentieth century.” — R.A. McCain

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### Prisoners' Dilemma: The Story

- Two criminals have been caught
- They cannot communicate with each other
- If both confess, they will each get 10 years
- If one confesses and accuses other:
  - confessor goes free
  - accused gets 20 years
- If neither confesses, they will both get 1 year on a lesser charge

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### Prisoners' Dilemma Payoff Matrix

		Bob	
		cooperate	defect
Ann	cooperate	-1, -1	-20, 0
	defect	0, -20	-10, -10

- defect = confess, cooperate = don't
- payoffs < 0 because punishments (losses)

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### Ann's "Rational" Analysis (Dominant Strategy)

		Bob	
		cooperate	defect
Ann	cooperate	-1, -1	-20, 0
	defect	0, -20	-10, -10

- if cooperates, may get 20 years
- if defects, may get 10 years
- ∴, best to defect

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### Bob's "Rational" Analysis (Dominant Strategy)

		Bob	
		cooperate	defect
Ann	cooperate	-1, -1	-20, 0
	defect	0, -20	-10, -10

- if he cooperates, may get 20 years
- if he defects, may get 10 years
- ∴, best to defect

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### Suboptimal Result of “Rational” Analysis

		Bob	
		cooperate	defect
Ann	cooperate	-1, -1	-20, 0
	defect	0, -20	-10, -10

- each acts individually rationally  $\Rightarrow$  get 10 years (dominant strategy equilibrium)
- “irrationally” decide to cooperate  $\Rightarrow$  only 1 year

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### Summary

- Individually rational actions lead to a result that all agree is less desirable
- In such a situation you cannot act unilaterally in your own best interest
- Just one example of a (game-theoretic) *dilemma*
- Can there be a situation in which it would make sense to cooperate unilaterally?
  - Yes, if the players can expect to interact again in the future

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### Classification of Dilemmas

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### General Payoff Matrix

		Bob	
		cooperate	defect
Ann	cooperate	CC ( <i>R</i> ) Reward	CD ( <i>S</i> ) Sucker
	defect	DC ( <i>T</i> ) Temptation	DD ( <i>P</i> ) Punishment

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### General Conditions for a Dilemma

- You always benefit if the other cooperates:
  - $CC > CD$  and  $DC > DD$
- You sometimes benefit from defecting:
  - $DC > CC$  or  $DD > CD$
- Mutual coop. is preferable to mut. def.
  - $CC > DD$
- Consider relative size of CC, CD, DC, DD
  - think of as permutations of  $R, S, T, P$
  - only three result in dilemmas

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### Three Possible Orders

The three dilemmas: *TRSP*, *RTPS*, *TRPS*

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### The Three Dilemmas

- Chicken (*TRSP*)
  - $DC > CC > CD > DD$
  - characterized by mutual defection being worst
  - two Nash equilibria (DC, CD)
- Stag Hunt (*RTPS*)
  - $CC > DC > DD > CD$
  - better to cooperate with cooperator
  - Nash equilibrium is CC
- Prisoners' Dilemma (*TRPS*)
  - $DC > CC > DD > CD$
  - better to defect on cooperator
  - Nash equilibrium is DD

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## The Iterated Prisoners' Dilemma

and Robert Axelrod's Experiments

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### Assumptions

- No mechanism for enforceable threats or commitments
- No way to foresee a player's move
- No way to eliminate other player or avoid interaction
- No way to change other player's payoffs
- Communication only through direct interaction

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### Axelrod's Experiments

- Intuitively, expectation of future encounters may affect rationality of defection
- Various programs compete for 200 rounds
  - encounters each other and self
- Each program can remember:
  - its own past actions
  - its competitors' past actions
- 14 programs submitted for first experiment

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### IPD Payoff Matrix

		B	
		cooperate	defect
A	cooperate	3, 3	0, 5
	defect	5, 0	1, 1

N.B. Unless  $DC + CD < 2 CC$  (i.e.  $T + S < 2 R$ ), can win by alternating defection/cooperation

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### Indefinite Number of Future Encounters

- Cooperation depends on expectation of **indefinite** number of future encounters
- Suppose a known finite number of encounters:
  - No reason to C on last encounter
  - Since expect D on last, no reason to C on next to last
  - And so forth: there is no reason to C at all

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### Analysis of Some Simple Strategies

- Three simple strategies:
  - **ALL-D**: always defect
  - **ALL-C**: always cooperate
  - **RAND**: randomly cooperate/defect
- Effectiveness depends on environment
  - **ALL-D** optimizes local (individual) fitness
  - **ALL-C** optimizes global (population) fitness
  - **RAND** compromises

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### Expected Scores

↓ playing ⇒	ALL-C	RAND	ALL-D	Average
ALL-C	3.0	1.5	0.0	1.5
RAND	4.0	2.0	0.5	2.166...
ALL-D	5.0	3.0	1.0	3.0

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### Result of Axelrod's Experiments

- Winner is Rapoport's **TFT** (Tit-for-Tat)
  - cooperate on first encounter
  - reply in kind on succeeding encounters
- Second experiment:
  - 62 programs
  - all know **TFT** was previous winner
  - **TFT** wins again

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### Characteristics of Successful Strategies

- *Don't be envious*
  - at best **TFT** ties other strategies
- *Be nice*
  - i.e. don't be first to defect
- *Reciprocate*
  - reward cooperation, punish defection
- *Don't be too clever*
  - sophisticated strategies may be unpredictable & look random; be clear

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### Tit-for-Two-Tats

- More forgiving than **TFT**
- Wait for two successive defections before punishing
- Beats **TFT** in a noisy environment
- E.g., an unintentional defection will lead **TFTs** into endless cycle of retaliation
- May be exploited by feigning accidental defection

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### Effects of Many Kinds of Noise Have Been Studied

- Misimplementation noise
- Misperception noise
  - noisy channels
- Stochastic effects on payoffs
- General conclusions:
  - sufficiently little noise  $\Rightarrow$  generosity is best
  - greater noise  $\Rightarrow$  generosity avoids unnecessary conflict but invites exploitation

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### More Characteristics of Successful Strategies

- Should be a generalist (robust)
  - i.e. do sufficiently well in wide variety of environments
- Should do well with its own kind
  - since successful strategies will propagate
- Should be cognitively simple
- Should be evolutionary stable strategy
  - i.e. resistant to invasion by other strategies

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### Kant's Categorical Imperative

“Act on maxims that can at the same time have for their object themselves as universal laws of nature.”

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