Reading

- CS 420/594: Flake, ch. 18 (Natural & Analog Computation)
- CS 594: Bar-Yam, ch. 2 (Neural Networks I), sections 2.1-2.2 (pp. 295-322)

Ecological & Spatial Models

Ecological Model

- What if more successful strategies spread in population at expense of less successful?
- Models success of programs as fraction of total population
- Fraction of strategy = probability random program obeys this strategy

Variables

- $P_i(t)$ = probability = proportional population of strategy i at time t
- $S_i(t)$ = score achieved by strategy i
- $R_{ij}(t)$ = relative score achieved by strategy i playing against strategy j over many rounds
  - fixed (not time-varying) for now

Computing Score of a Strategy

- Let n = number of strategies in ecosystem
- Compute score achieved by strategy i:

$$S_i(t) = \sum_{k=1}^{n} R_{ik}(t) P_k(t)$$

$$\mathbf{S}(t) = \mathbf{R}(t)\mathbf{P}(t)$$

Updating Proportional Population

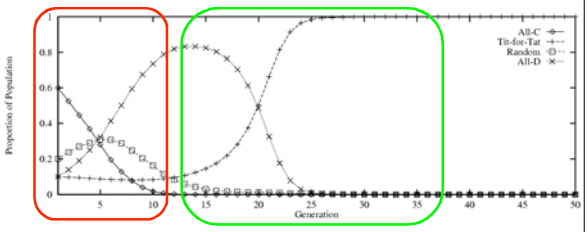
$$P_i(t+1) = \frac{P_i(t) S_i(t)}{\sum_{j=1}^{n} P_j(t) S_j(t)}$$

Some Simulations

- Usual Axelrod payoff matrix
- 200 rounds per step

Demonstration Simulation

- 60% ALL-C
- 20% RAND
- 10% ALL-D, TFT



Collectively Stable Strategy

- Let w = probability of future interactions
- Suppose cooperation based on reciprocity has been established
- Then no one can do better than **TFT** provided:

$$w \geq \max\left(\frac{T-R}{R-S}, \frac{T-R}{T-P}\right)$$

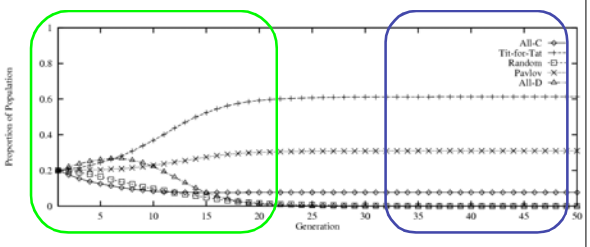
- The **TFT** users are in a Nash equilibrium

"Win-Stay, Lose-Shift" Strategy

- Win-stay, lose-shift strategy:
  - begin cooperating
  - if other cooperates, continue current behavior
  - if other defects, switch to opposite behavior
- Called **PAV** (because suggests Pavlovian learning)

Simulation without Noise

- 20% each
- no noise

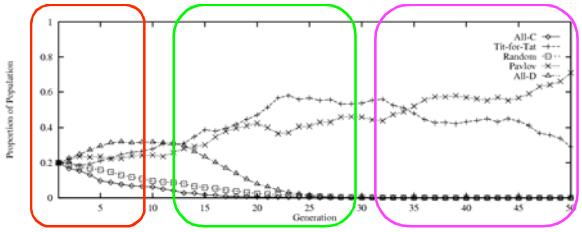


Effects of Noise

- Consider effects of noise or other sources of error in response
- **TFT**:
  - cycle of alternating defections (CD, DC)
  - broken only by another error
- **PAV**:
  - eventually self-corrects (CD, DC, DD, CC)
  - can exploit **ALL-C** in noisy environment
- Noise added into computation of $R_{ij}(t)$

Simulation with Noise

- 20% each
- 0.5% noise



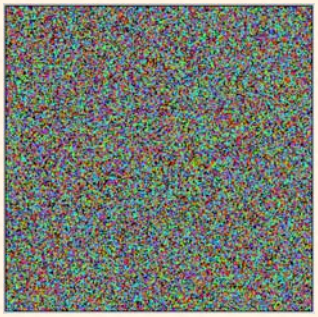
Spatial Effects

- Previous simulation assumes that each agent is equally likely to interact with each other
- So strategy interactions are proportional to fractions in population
- More realistically, interactions with "neighbors" are more likely
  - "Neighbor" can be defined in many ways
- Neighbors are more likely to use the same strategy

Spatial Simulation

- Toroidal grid
- Agent interacts only with eight neighbors
- Agent adopts strategy of most successful neighbor
- Ties favor current strategy

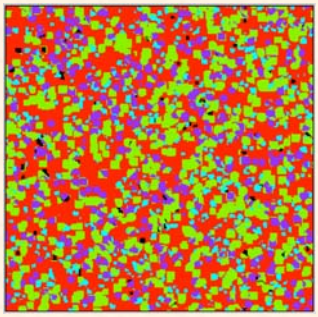
Typical Simulation (t = 1)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

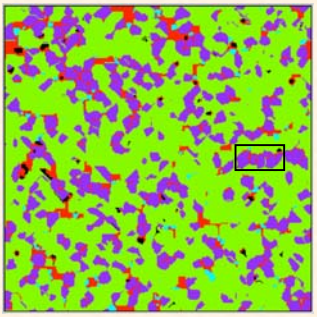
Typical Simulation (t = 5)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

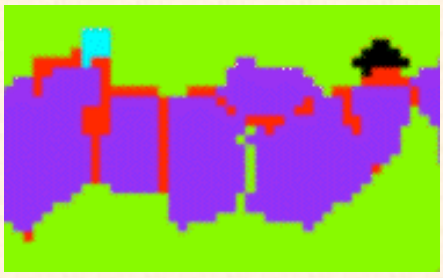
Typical Simulation (t = 10)



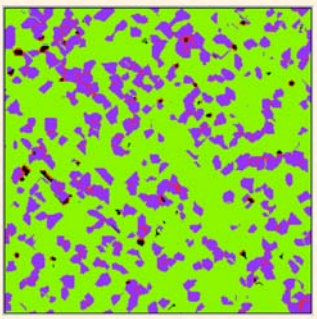
Colors:

ALL-C
TFT
RAND
PAV
ALL-D

Typical Simulation (t = 10) Zooming In



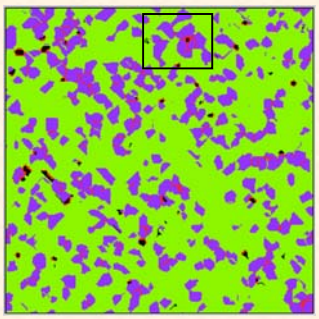
Typical Simulation (t = 20)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

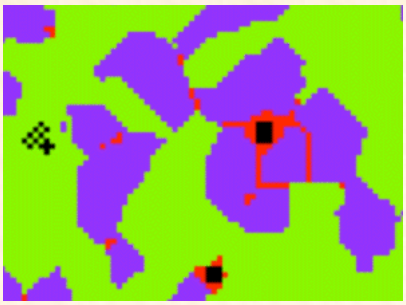
Typical Simulation (t = 50)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

Typical Simulation (t = 50) Zoom In



Simulation of Spatial Iterated Prisoners Dilemma

Run sipd simulator

SIPD Without Noise

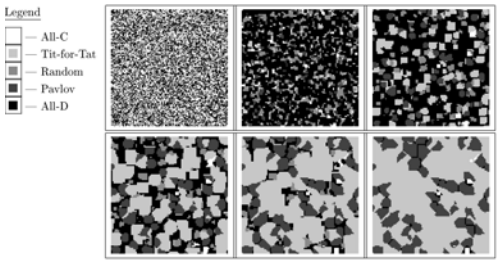


Figure 17.4 Competition in the spatial iterated Prisoner's Dilemma without noise

Conclusions: Spatial IPD

- Small clusters of cooperators can exist in hostile environment
- Parasitic agents can exist only in limited numbers
- Stability of cooperation depends on expectation of future interaction
- Adaptive cooperation/defection beats unilateral cooperation or defection

Additional Bibliography

1. von Neumann, J., & Morgenstern, O. *Theory of Games and Economic Behavior*, Princeton, 1944.
2. Morgenstern, O. "Game Theory," in *Dictionary of the History of Ideas*, Charles Scribners, 1973, vol. 2, pp. 263-75.
3. Axelrod, R. *The Evolution of Cooperation*. Basic Books, 1984.
4. Axelrod, R., & Dion, D. "The Further Evolution of Cooperation," *Science* **242** (1988): 1385-90.
5. Poundstone, W. *Prisoner's Dilemma*. Doubleday, 1992.