Part B
Ants (Natural and Artificial)

Langton's Vants
(Virtual Ants)

Vants

- Square grid
- Squares can be black or white
- Vants can face N, S, E, W
- Behavioral rule:
  - take a step forward,
  - **if** on a white square **then**
    paint it black & turn 90° right
  - **if** on a black square **then**
    paint it white & turn 90° left

Example

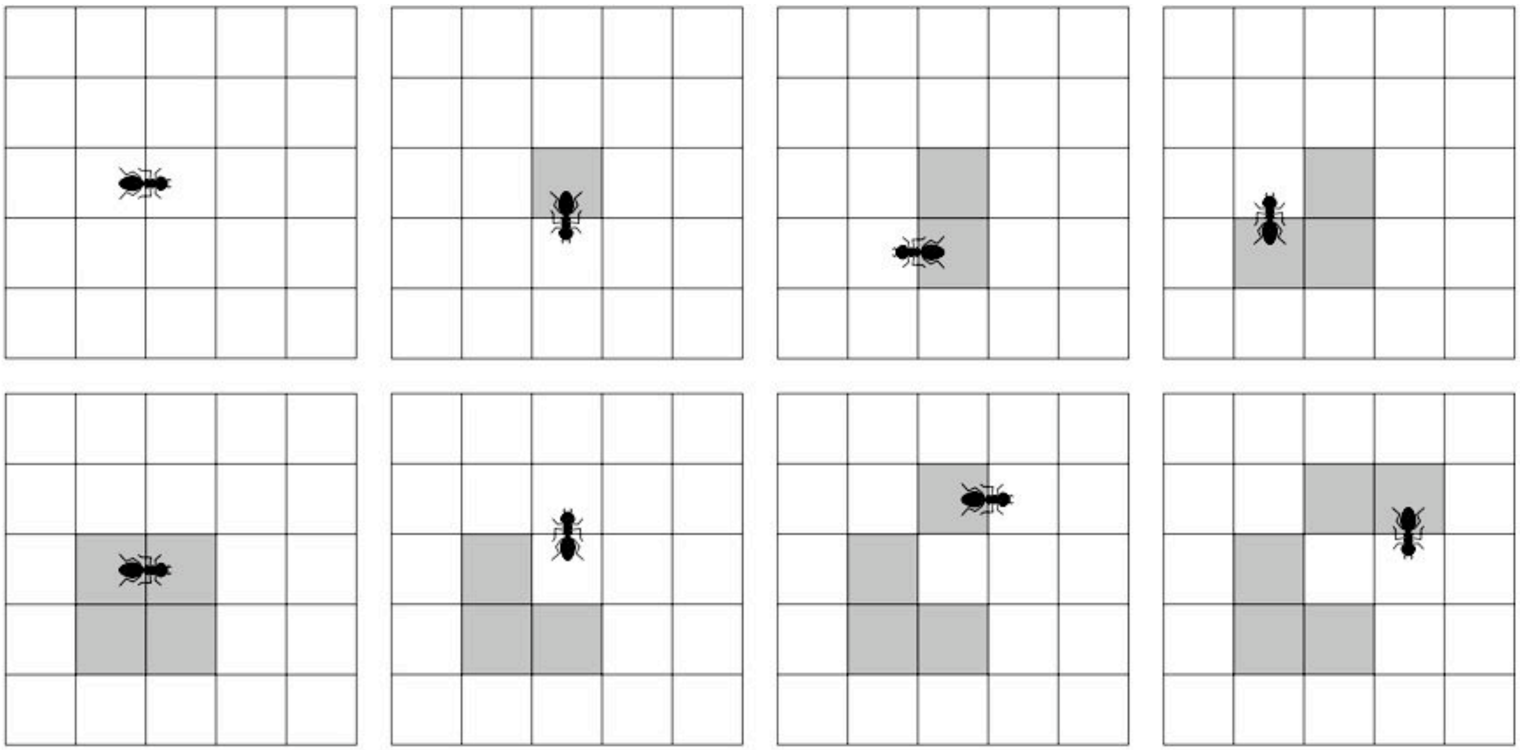


Figure 16.2 Eight steps of Langton's virtual ant, starting from an initially blank grid

Time Reversibility

- Vants are time-reversible
- But time reversibility does not imply global simplicity
- Even a single vant interacts with its own prior history
- But complexity does not always imply random-appearing behavior

Demonstration of Vants

(NetLogo Simulation)

Run Vants-Large-Field.nlogo

Demonstration of Generalized Vants
(NetLogo Simulation)

Run Generalized-Vants.nlogo

Conclusions

- Even simple, reversible local behavior can lead to complex global behavior
- Nevertheless, such complex behavior may create structures as well as apparently random behavior
- Perhaps another example of "edge of chaos" phenomena

Digression: Time-Reversibility and the Physical Limits of Computation

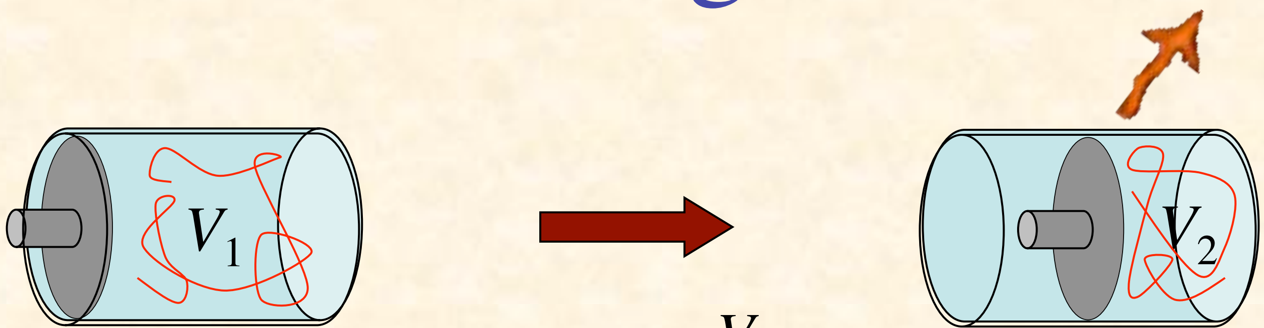
Work done by:

- Rolf Landauer (1961)
- Charles Bennett (1973)
- Richard Feynman (1981–3)

Entropy of Physical Systems

- Recall information content of N equally likely messages: $I_2 = \lg N$ bits.
- Can also use natural logs: $I_e = \ln N$ nats $= I_2 \ln 2$.
- To specify position & momentum of each particle of a physical system:
  $S = k \ln N = k\, I_2 \ln 2$.
  - k is Boltzmann's constant
- This is the entropy of the system
  - entropies of 10 bits/atom are typical

Thermodynamics of Recording One Bit



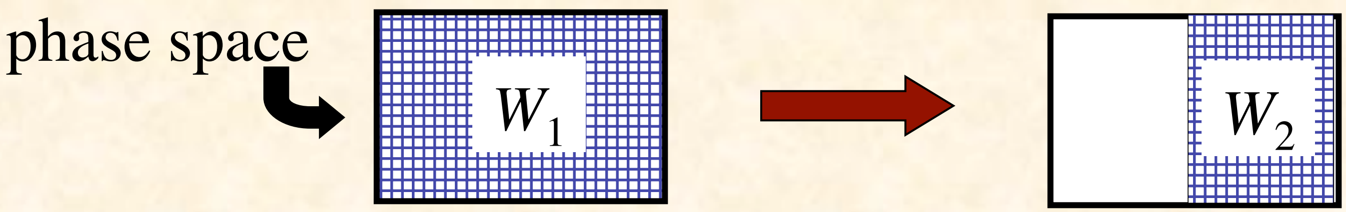
$$\Delta S = k \ln \frac{V_2}{V_1}$$

$$\text{if } V_2 = V_1/2, \text{ then } \Delta S = -k \ln 2$$

$$\text{also, } \Delta F = kT \ln 2$$

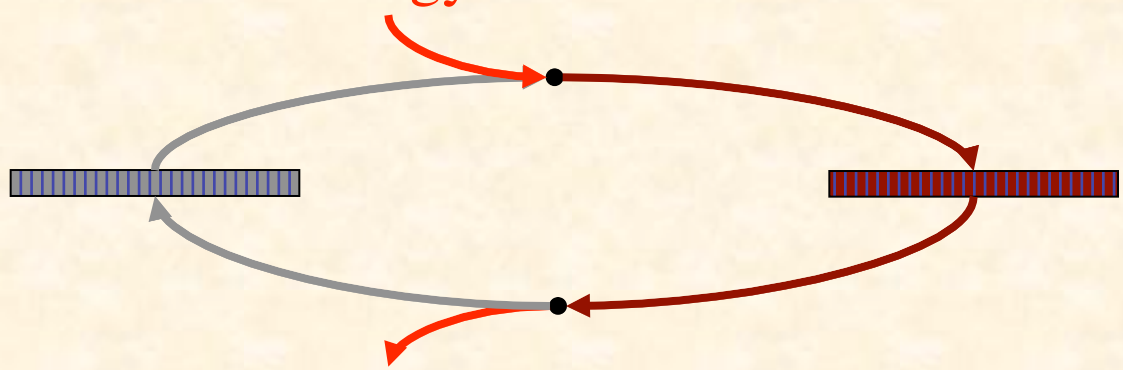
- ΔS derived by gas laws & classical thermodynamics
- Boltzmann constant: $k = 1.381 \times 10^{-23}$ J K^{-1}

Entropy Change in Terms of Phase Space



- Let W = number of microstates corresponding to a macrostate
- Entropy $S = k \ln W$
- Then $\Delta S = k \ln W_2 - k \ln W_1 = k \ln (W_2 / W_1)$
- If $W_2 = W_1 / 2$, then $\Delta S = -k \ln 2$

Information and Energy



fuel value = 0

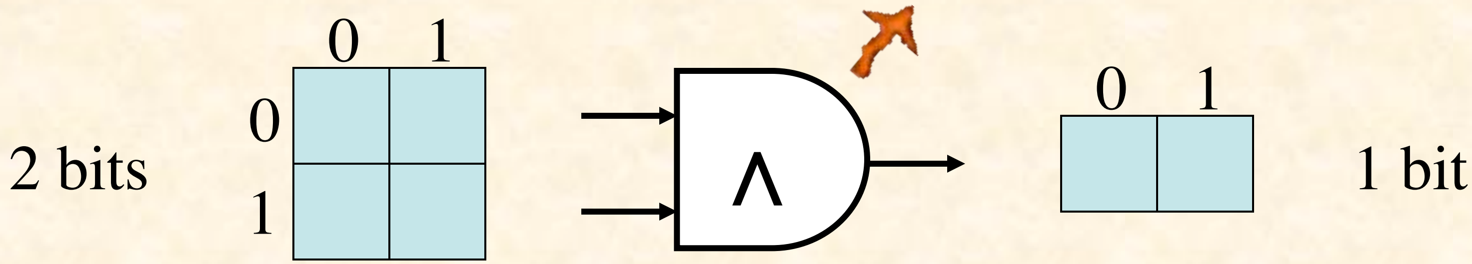
information = n bits

fuel value = $nkT \ln 2$

information = 0 bits

- initialization equivalent to storing energy
- information and energy are complementary

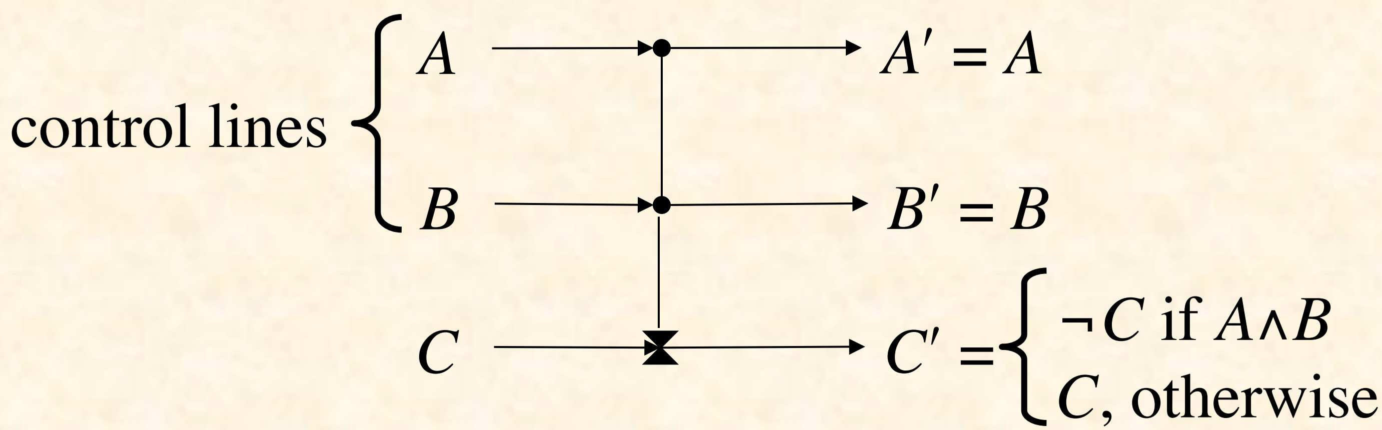
Minimum Energy for Irreversible Computation



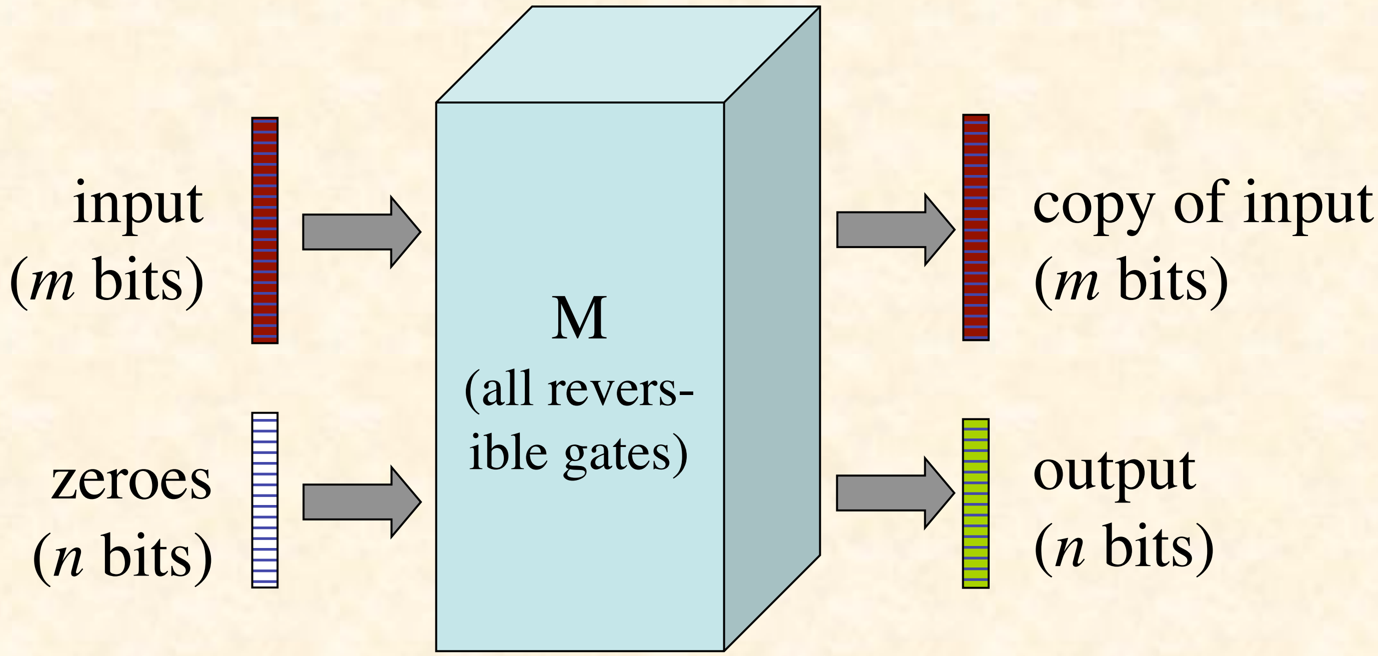
- Loss of one bit of information
  - an irreversible operation (many to one)
- $\Delta S = -\,k \ln 2$
  - entropy decrease must be compensated by heat dissipation
- Minimum energy required: $\Delta F = kT \ln 2$
  - transistors: $\sim 10^8\ kT$; RNA polymerase: $\sim 100\ kT$

Reversible Gates

- Can make dissipation arbitrarily small by using reversible gates
- All outputs must go somewhere
- Cannot *ever* throw information away
- The Fredkin CCN gate ("Controlled Controlled Not") is reversible
  - can be used for constructing other gates



Reversible Computer

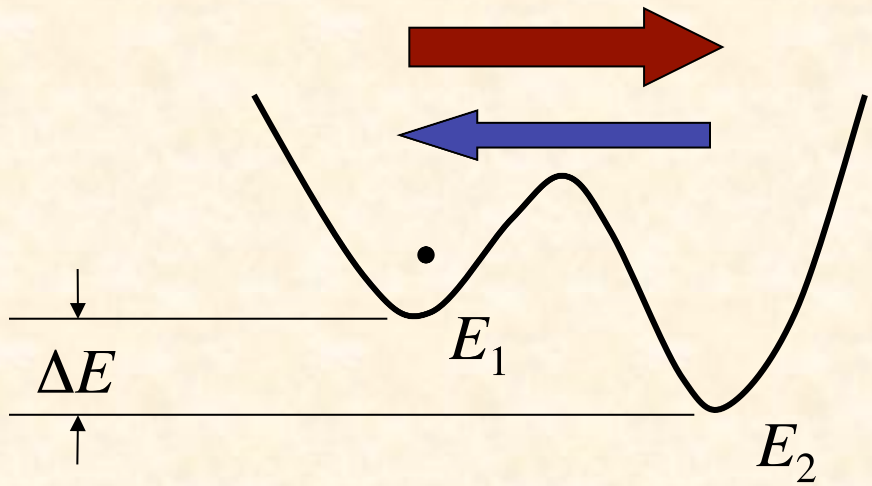


- Reversible because get input back
- Only loss is resetting machine for next job
  - energy is proportional to *n*, number of output bits

Summary: Energy Required for Reversible Computing

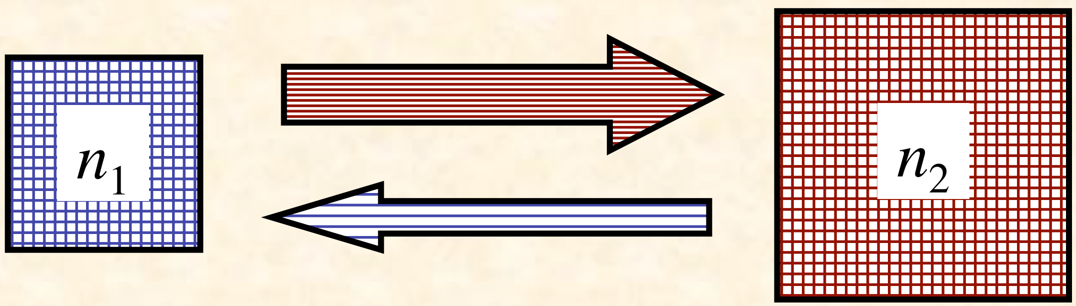
- There is no lower limit on the energy required for basic operations (gates, bit copying, etc.) provided:
  - it is done reversibly
  - it is done sufficiently slowly
- What is the fundamental relation between speed and energy dissipation?

Energy and the Speed of Computation



- Let r be ratio of forward to backward rate
- Statistical mechanics shows: $kT \ln r = \Delta E$
- Greater "driving energy" ⇒ greater rate

Entropy and the Speed of Computation



- Consider number of accessible microstates, n_1 and n_2
- Can show: $r = n_2 / n_1$
- Hence, $kT \ln r = kT (\ln n_2 - \ln n_1)$
  $= (S_2 - S_1)T = T\Delta S$

Conclusions

- Entropy increase and energy dissipation can be made arbitrarily small by doing reversible computation
- However, the speed of computation is an exponential function of the driving energy or entropy increase

Additional Information

1. Feynman, R. P. *Feynman Lectures on Computation*, ed. by A.J.G Hey & R.W. Allen. Perseus, 1996.
2. Hey, A.J.G. (ed.) *Feynman and Computation: Exploring the Limits of Computation*. Perseus, 1999.

Real Ants

(especially the black garden ant,
Lasius niger)

Adaptive Significance

- Selects most profitable from array of food sources
- Selects shortest route to it
  - longer paths abandoned within 1–2 hours
- Adjusts amount of exploration to quality of identified sources
- Collective decision making can be as accurate and effective as some vertebrate individuals

Observations on Trail Formation

- Two equal-length paths presented at same time: ants choose one at random
- Sometimes the longer path is initially chosen
- Ants may remain “trapped” on longer path, once established
- Or on path to lower quality source, if it’s discovered first
- But there may be advantages to sticking to paths
  - easier to follow
  - easier to protect trail & source
  - safer

Process of Trail Formation

1. Trail laying
2. Trail following

Trail Laying

- On discovering food, forager lays chemical trail while returning to nest
  - only ants who have found food deposit pheromone
- Others stimulated to leave nest by:
  - the trail
  - the recruitor exciting nestmates (sometimes)
- In addition to defining trail, pheromone:
  - serves as general orientation signal for ants outside nest
  - serves as arousal signal for ants inside

Additional Complexities

- Some ants begin marking on return from discovering food
- Others on their first return trip to food
- Others not at all, or variable behavior
- Probability of trail laying decreases with number of trips

Frequency of Trail Marking

- Ants modulate frequency of trail marking
- May reflect quality of source
  - hence more exploration if source is poor
- May reflect orientation to nest
  - ants keep track of general direction to nest
  - and of general direction to food source
  - trail laying is less intense if the angle to homeward direction is large

Trail Following

- Ants preferentially follow stronger of two trails
  - show no preference for path they used previously
- Ant may double back, because of:
  - decrease of pheromone concentration
  - unattractive orientation

Probability of Choosing One of Two Branches

- Let C_{L} and C_{R} be units of pheromone deposited on left & right branches
- Let P_{L} and P_{R} be probabilities of choosing them
- Then:

$$P_{\mathrm{L}} = \frac{\left(C_{\mathrm{L}} + 6\right)^2}{\left(C_{\mathrm{L}} + 6\right)^2 + \left(C_{\mathrm{R}} + 6\right)^2}$$

- Nonlinearity amplifies probability

Additional Adaptations

- If a source is crowded, ants may return to nest or explore for other sources
- New food sources are preferred if they are near to existing sources
- Foraging trails may rotate systematically around a nest

Pheromone Evaporation

- Trails can persist from several hours to several months
- Pheromone has mean lifetime of 30-60 min.
- But remains detectable for many times this
- Long persistence of pheromone prevents switching to shorter trail
- Artificial ant colony systems rely more heavily on evaporation

Resnick’s Ants

Environment

- Nest emits *nest-scent*, which
  - diffuses uniformly
  - decays slowly
  - provides general orientation signal
  - by diffusing around barriers, shows possible paths around barriers
- Trail pheromone
  - emitted by ants carrying food
  - diffuses uniformly
  - decays quickly
- Food detected only by contact

Resnick Ant Behavior

1. Looking for food:

   if trail pheromone weak **then** wander

   else move toward increasing concentration

2. Acquiring food:

   if at food **then**

   pick it up, turn around, & begin depositing pheromone

3. Returning to nest:

   deposit pheromone & decrease amount available

   move toward increasing nest-scent

4. Depositing food:

   if at nest **then**

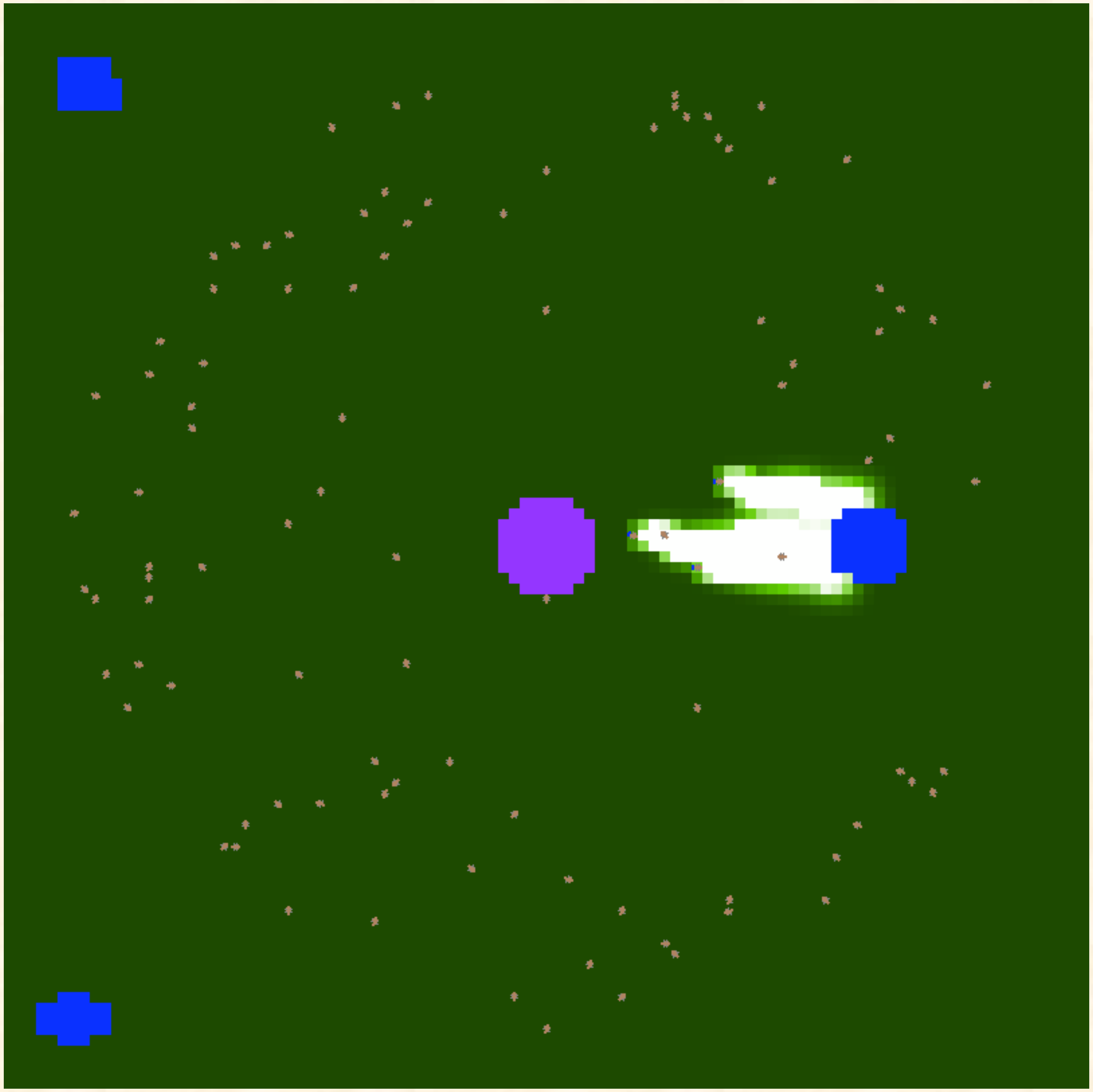
   deposit food, stop depositing pheromone, & turn around

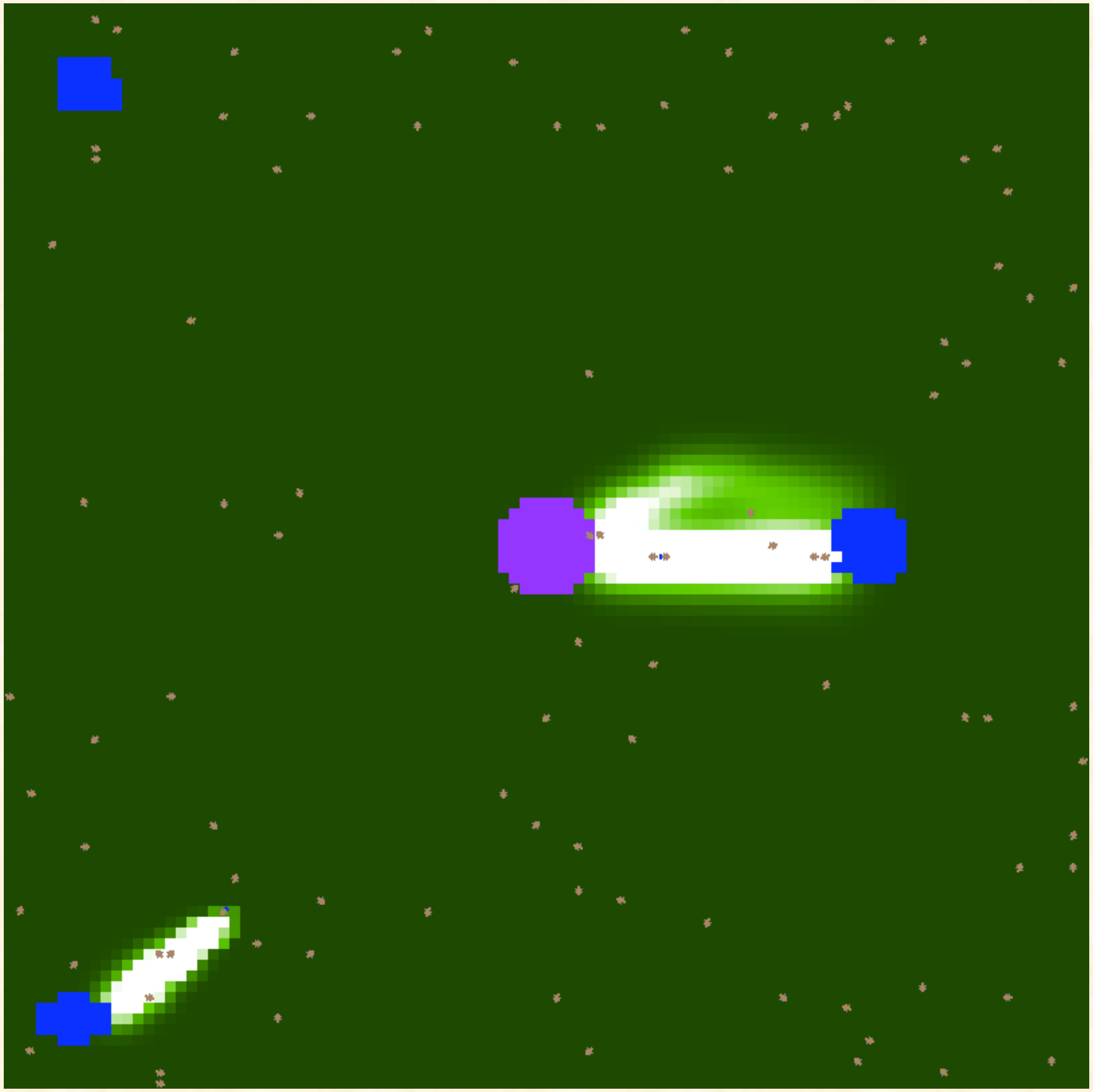
5. Repeat forever

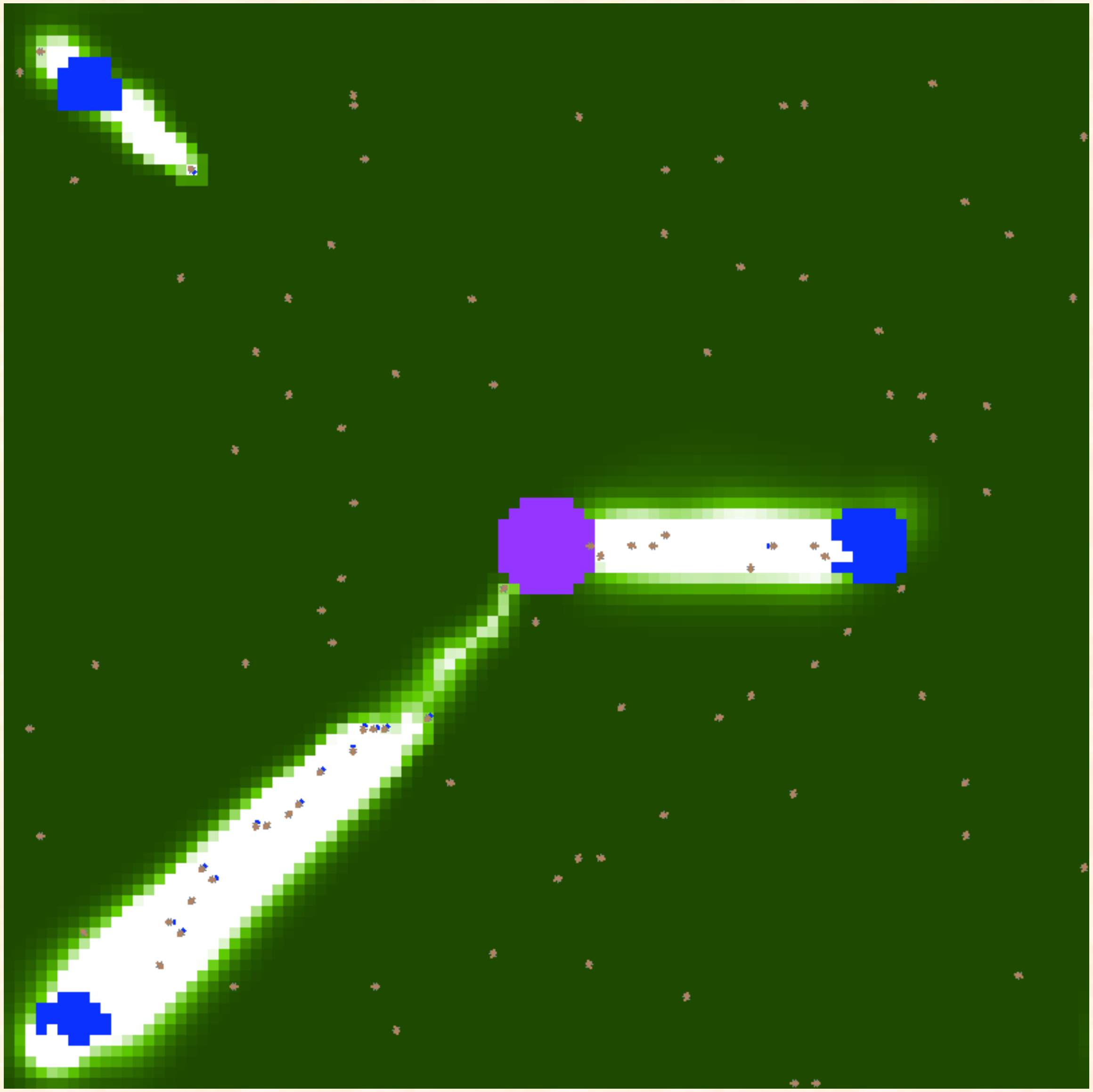
Demonstration of Resnick Ants

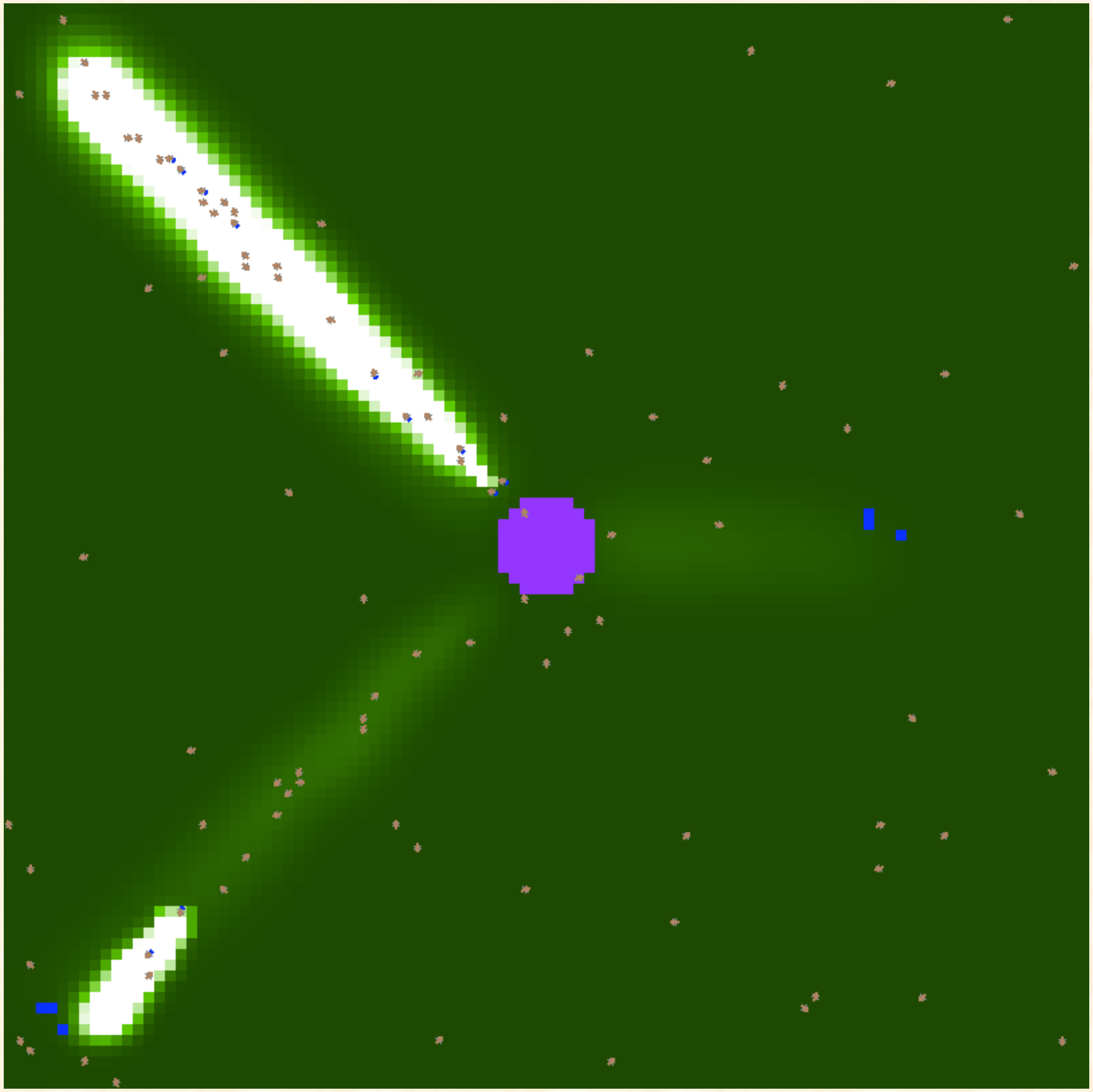
Run Ants.nlogo

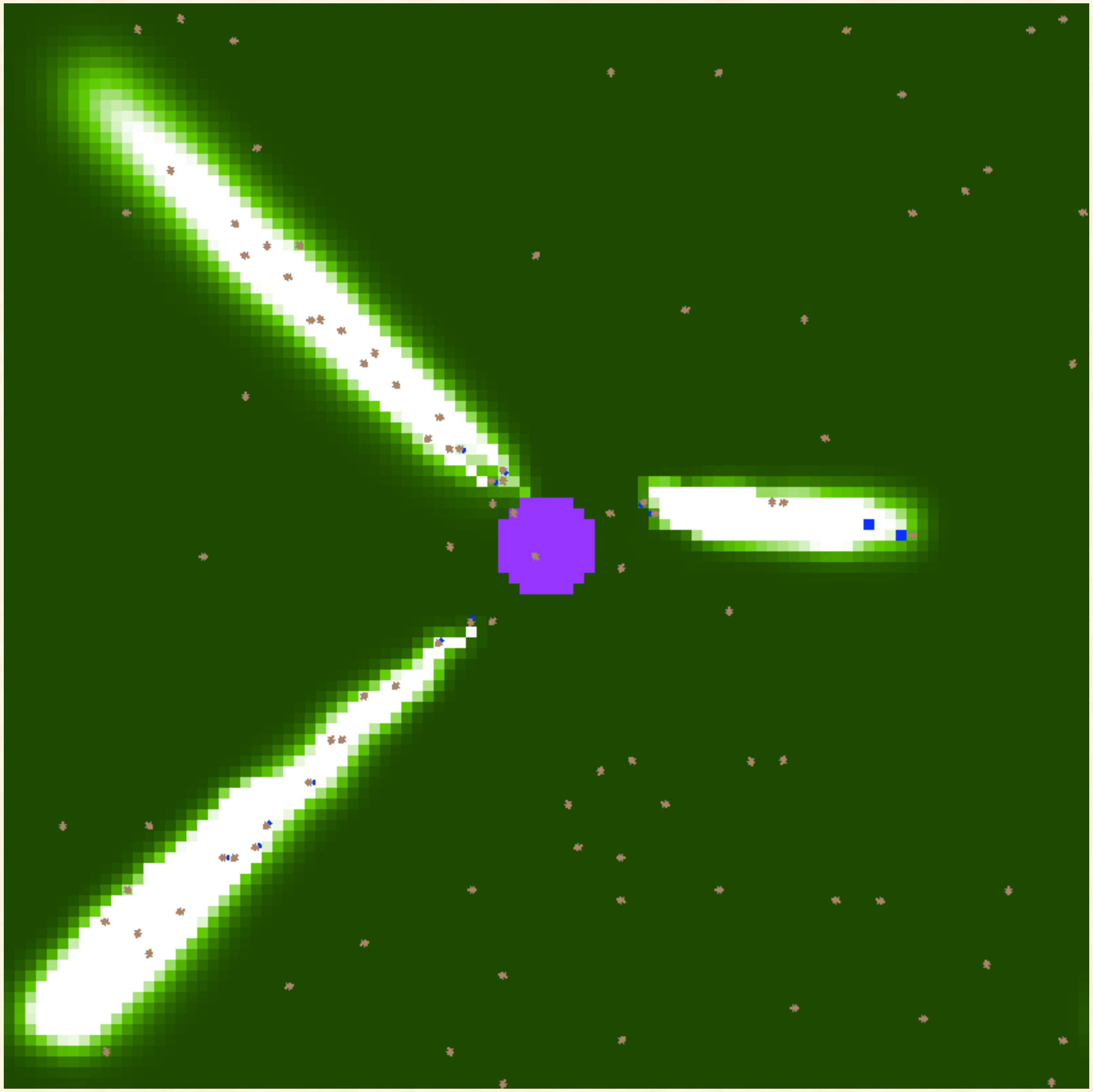
Exploitation of Multiple Food Sources

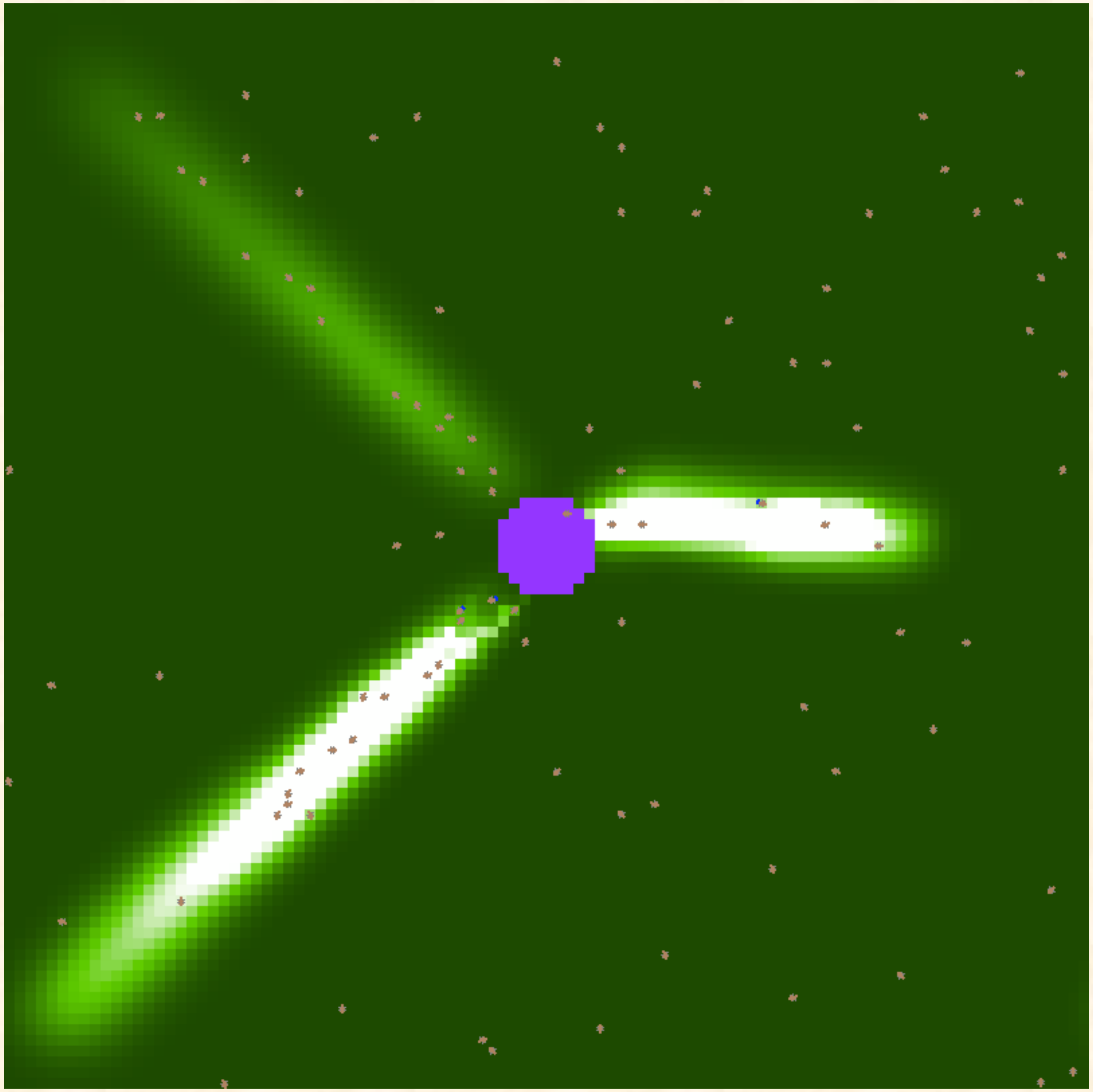




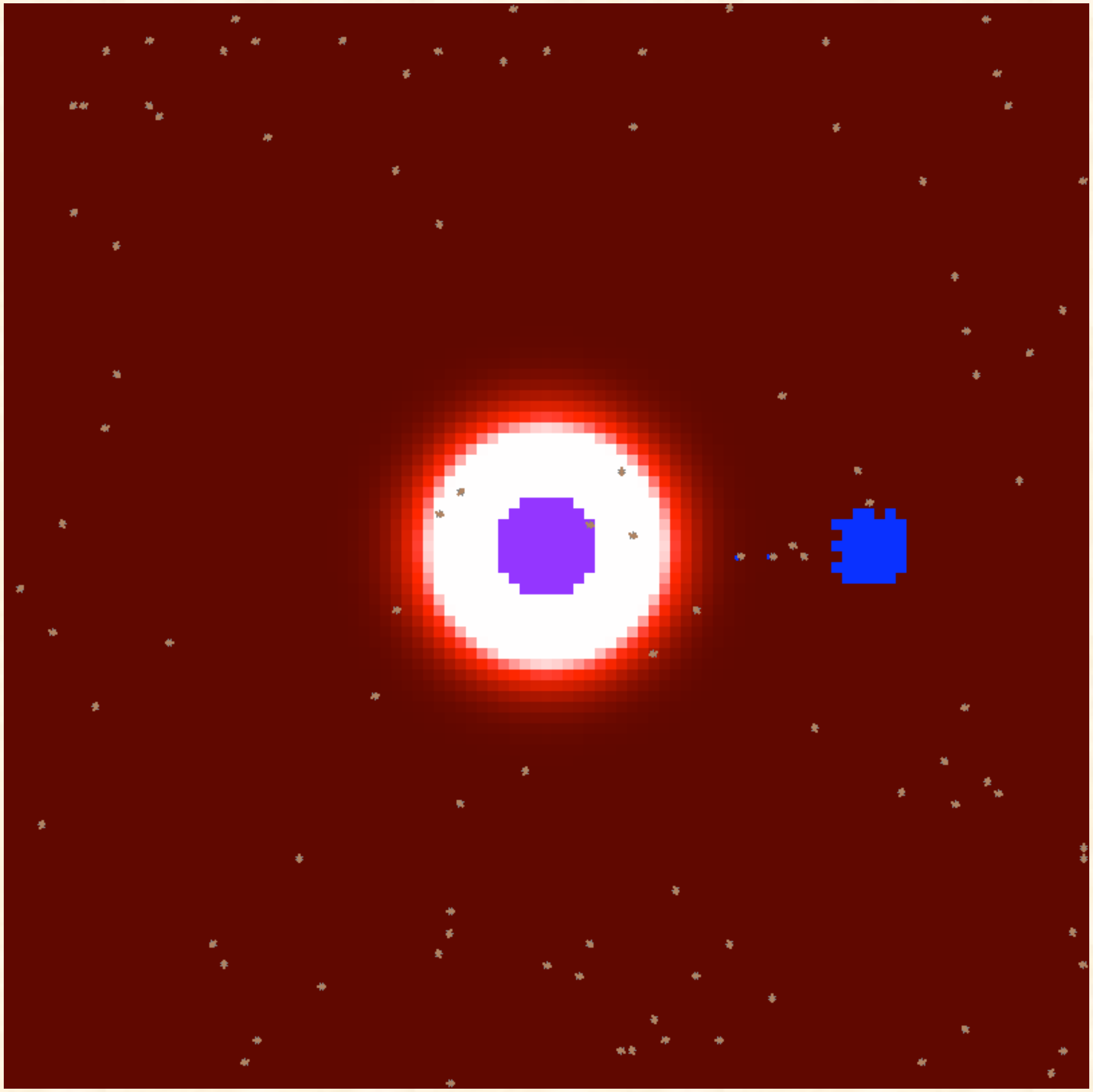


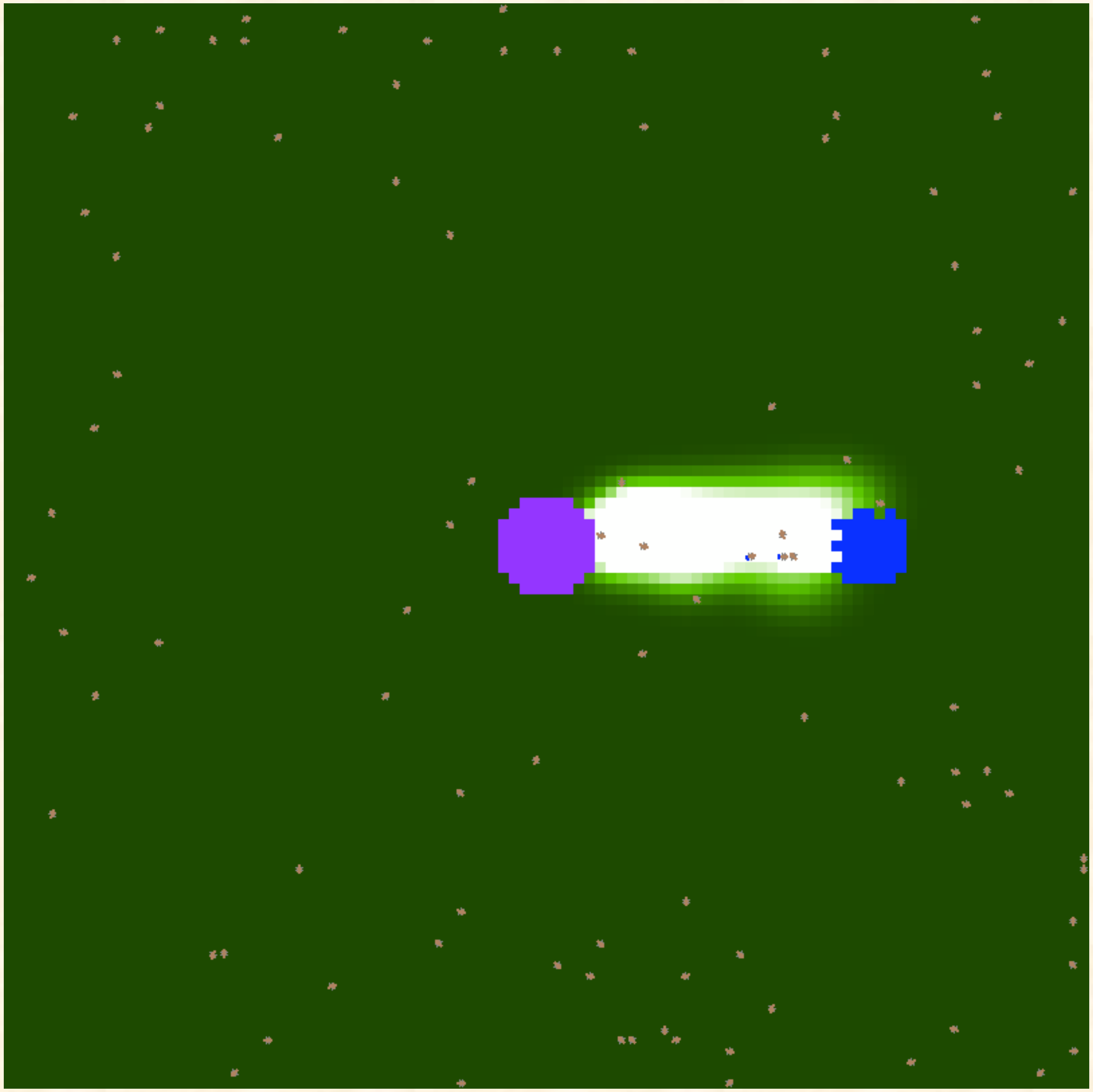


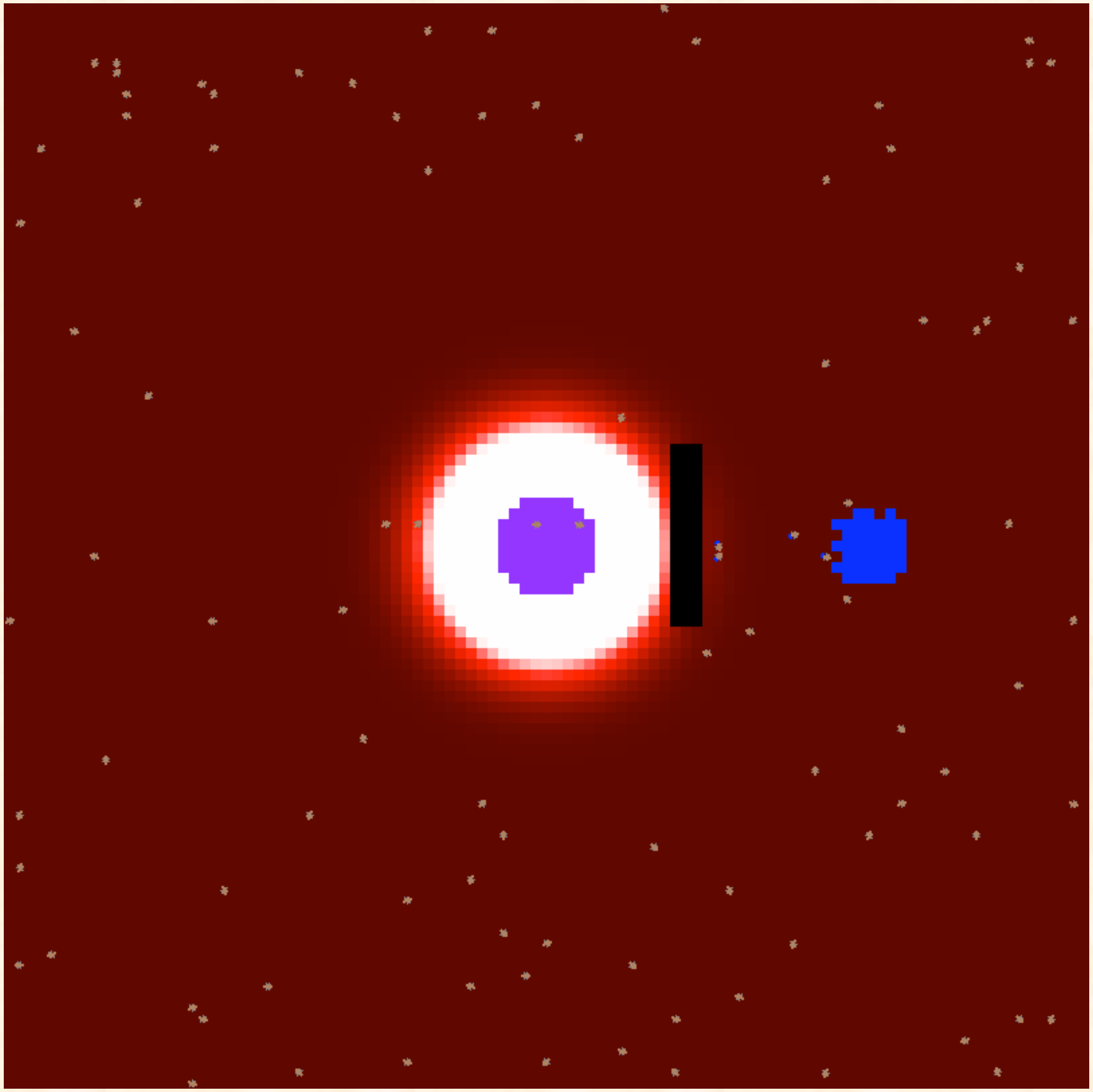


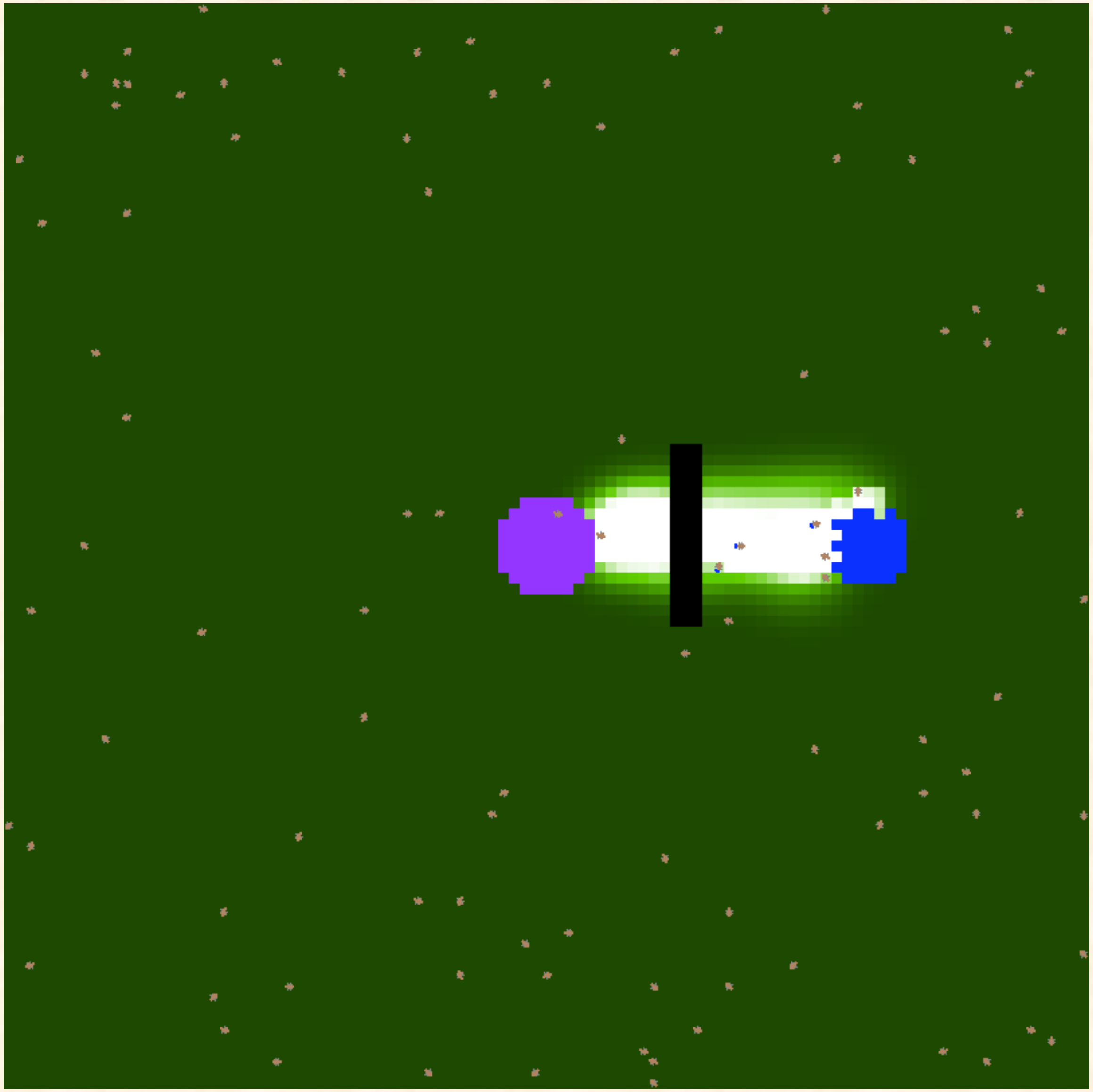


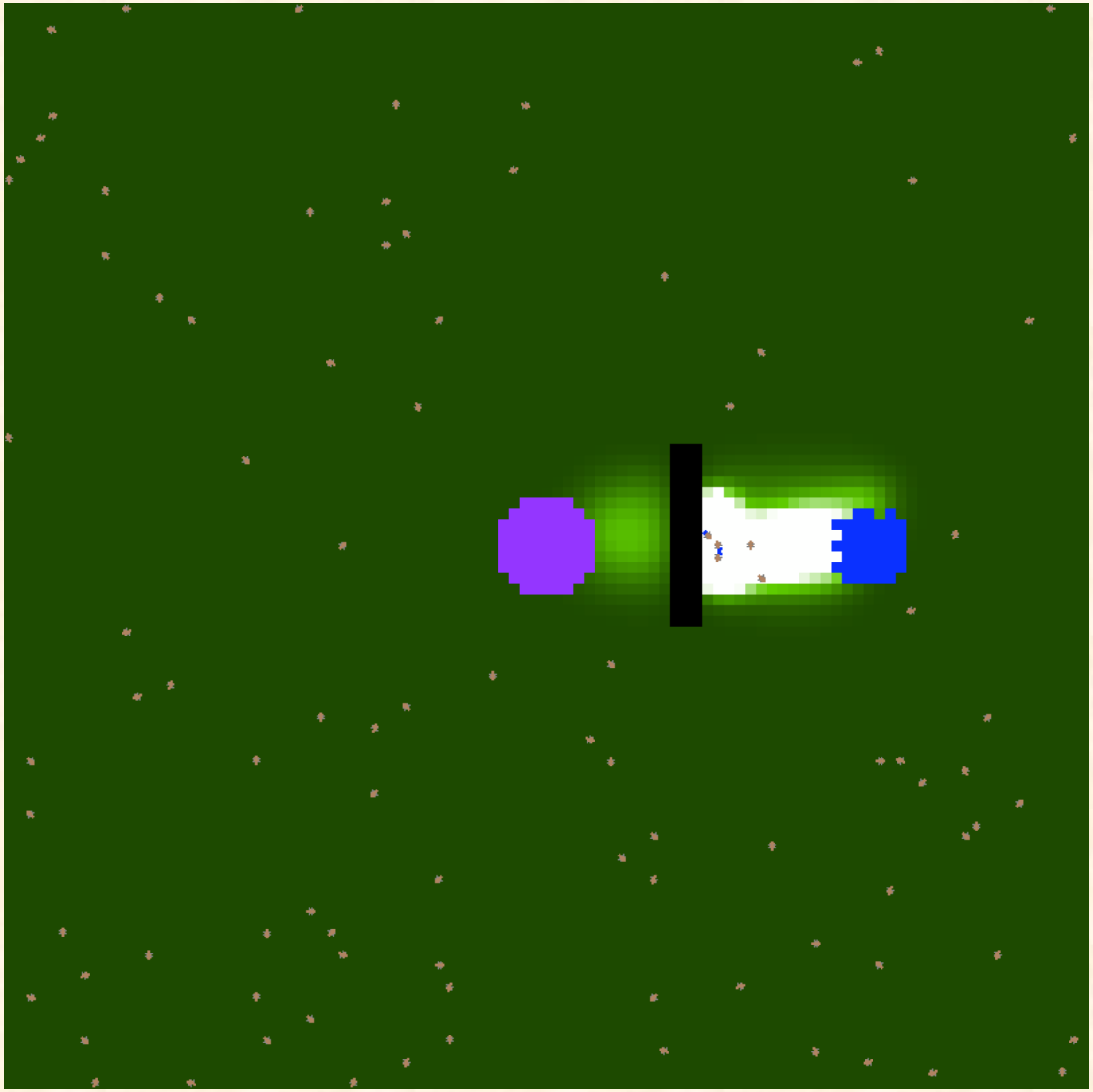
Insertion of Barrier

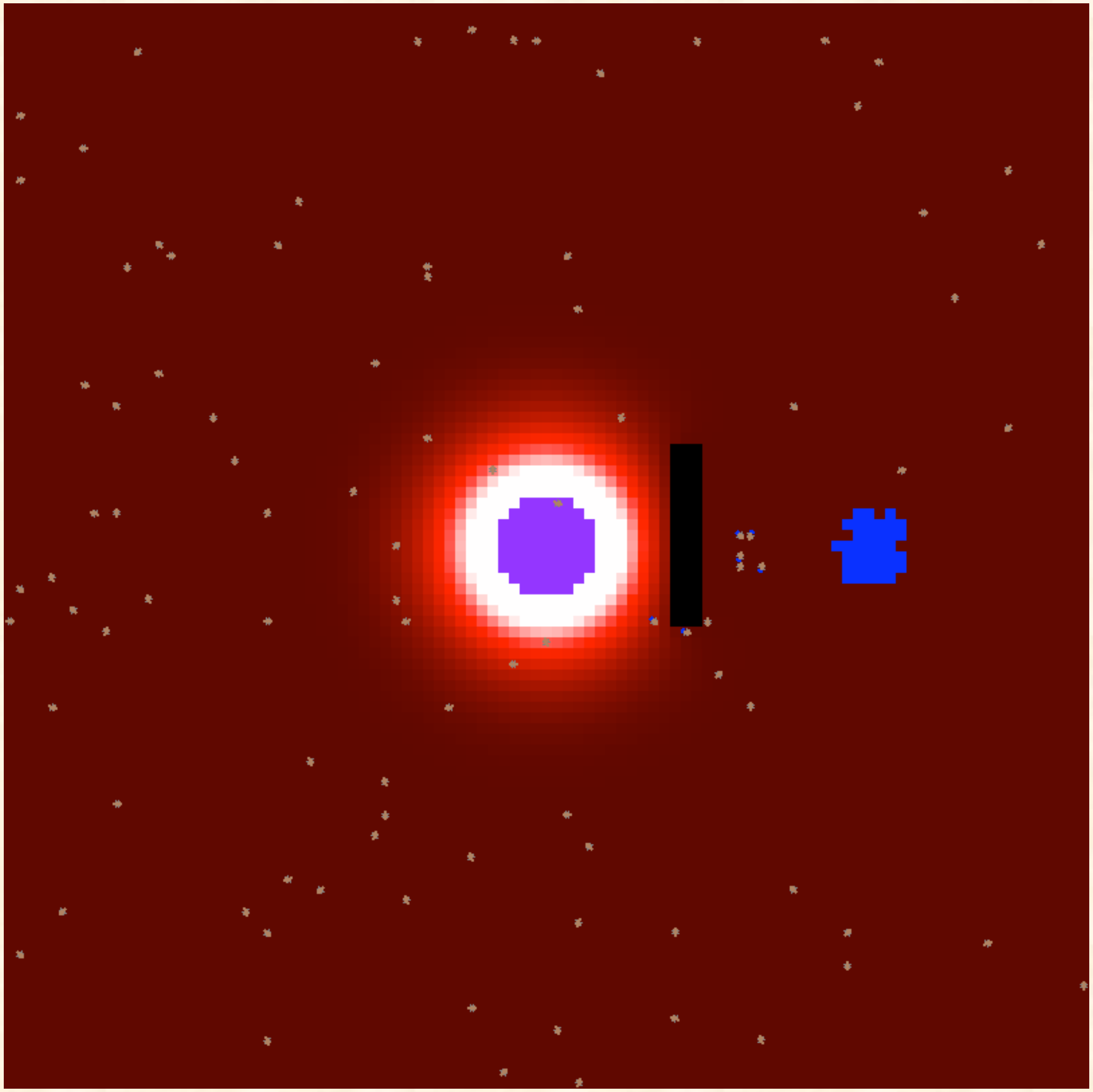


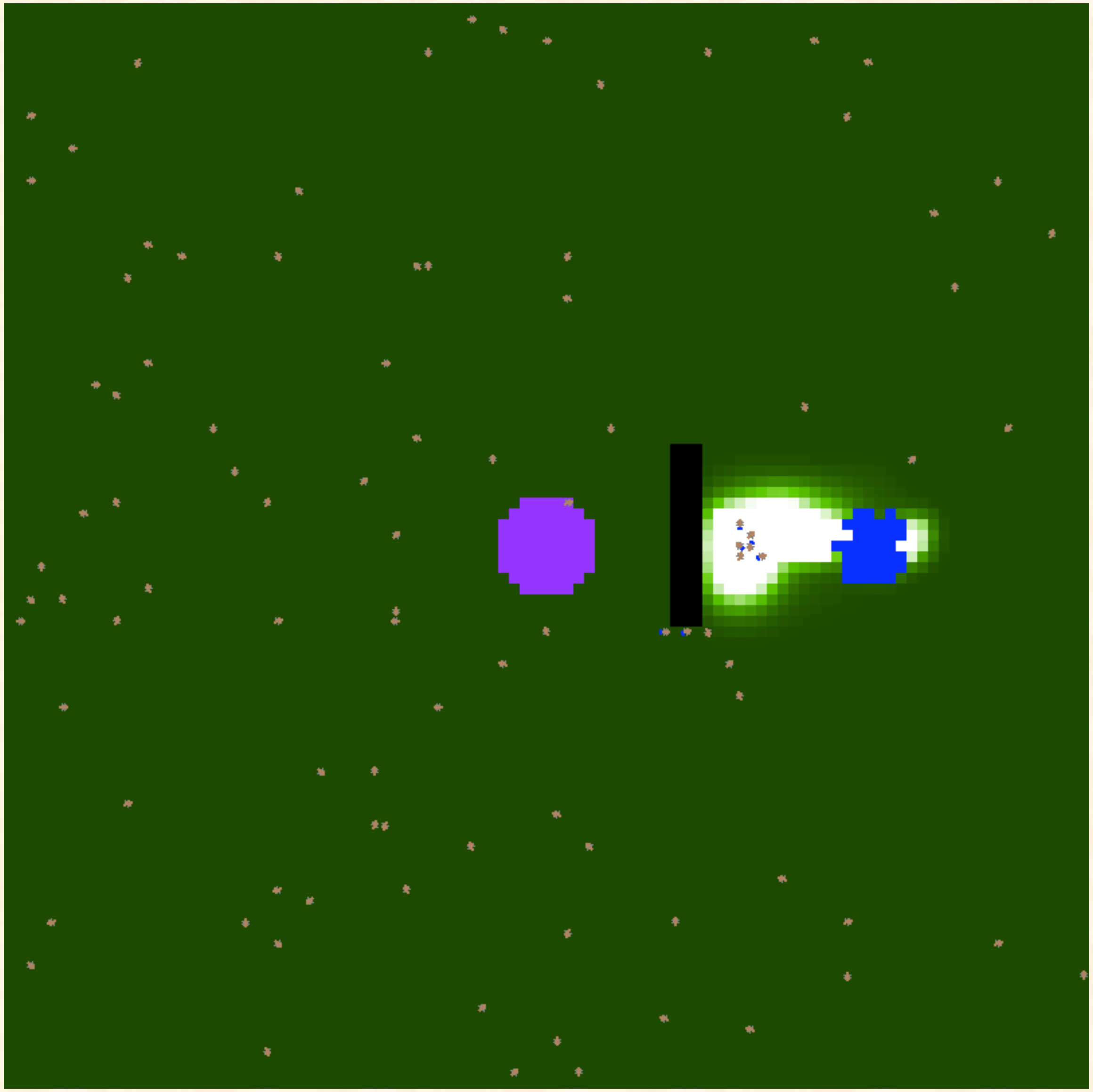


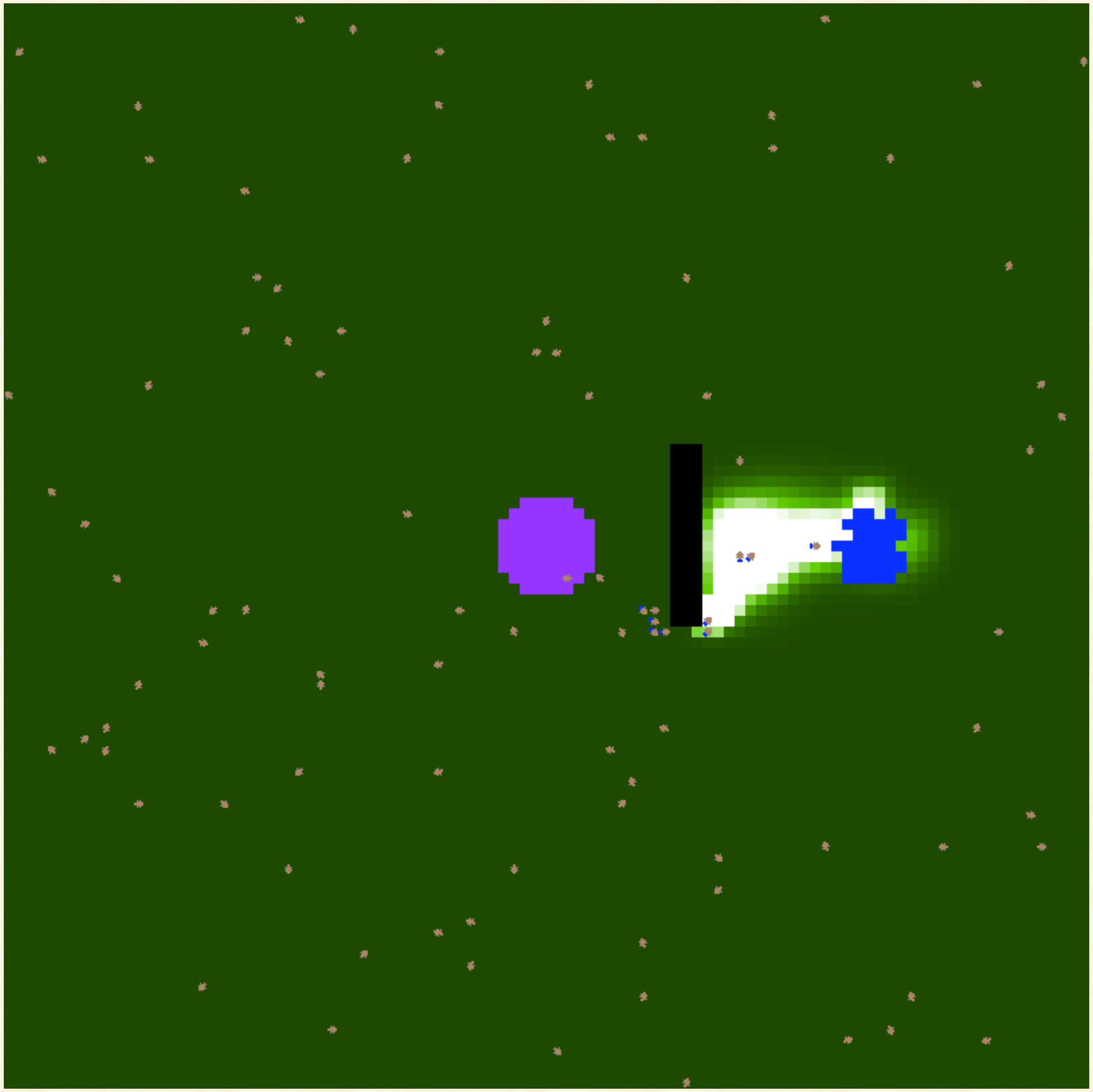


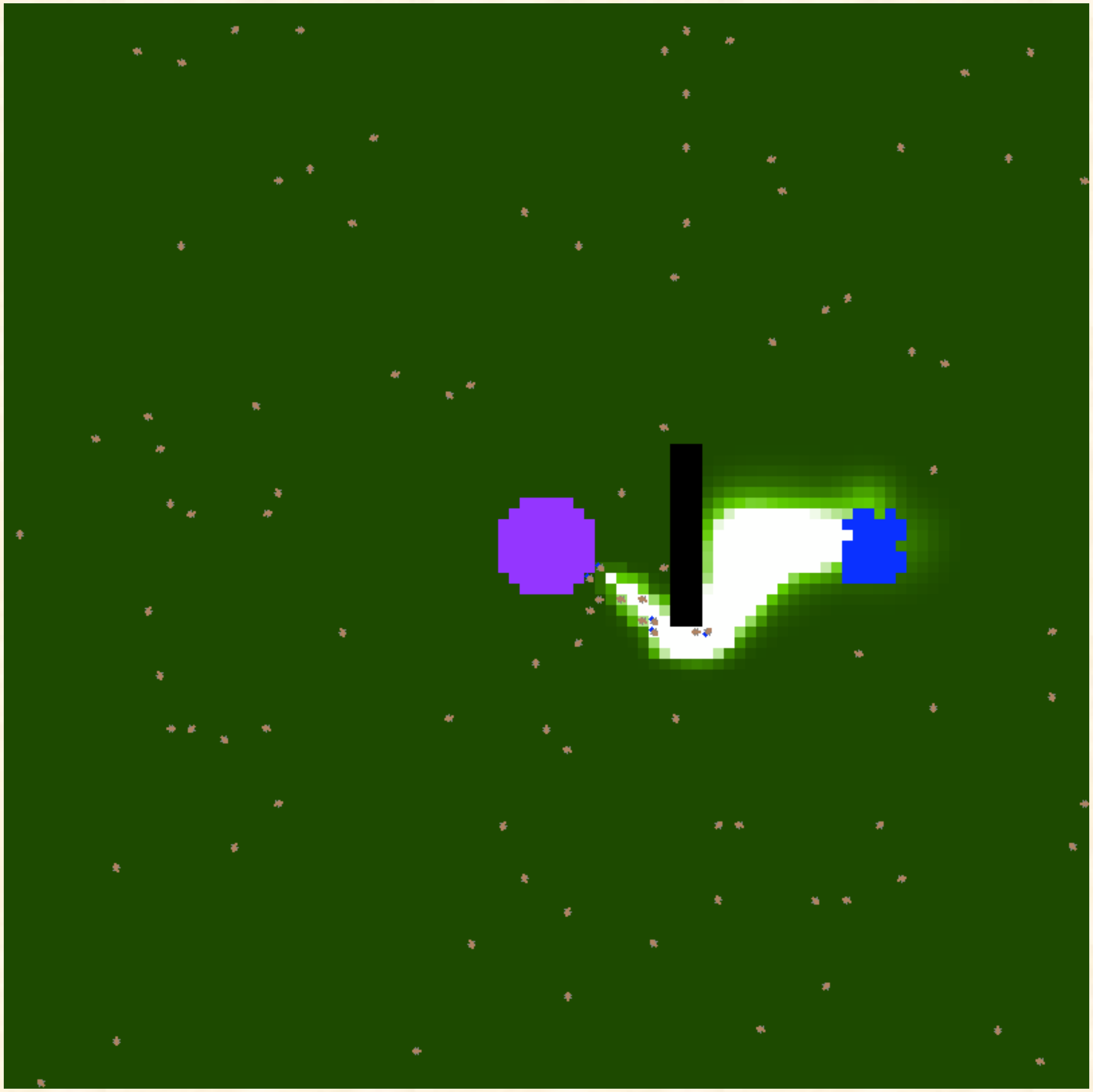


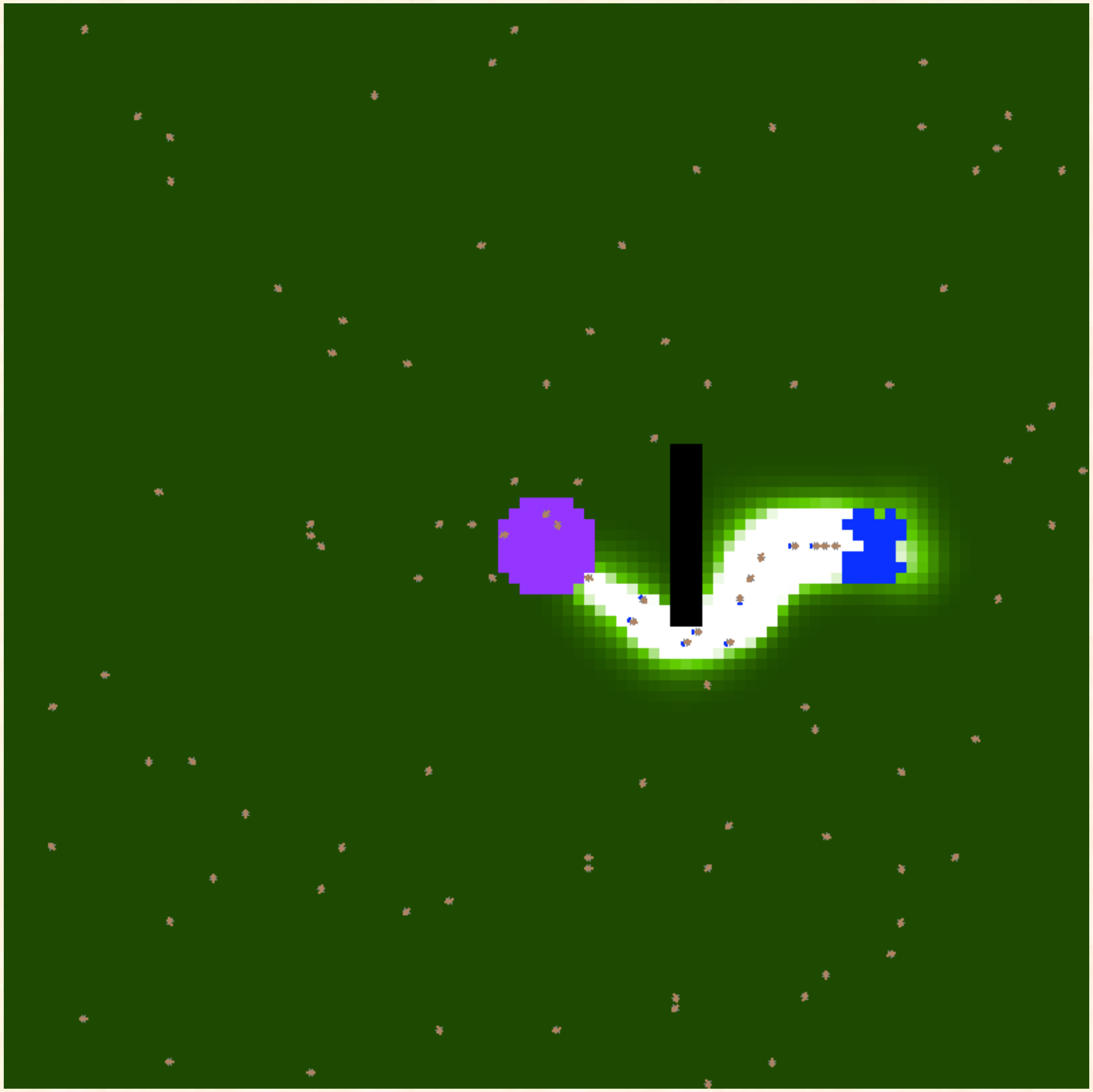




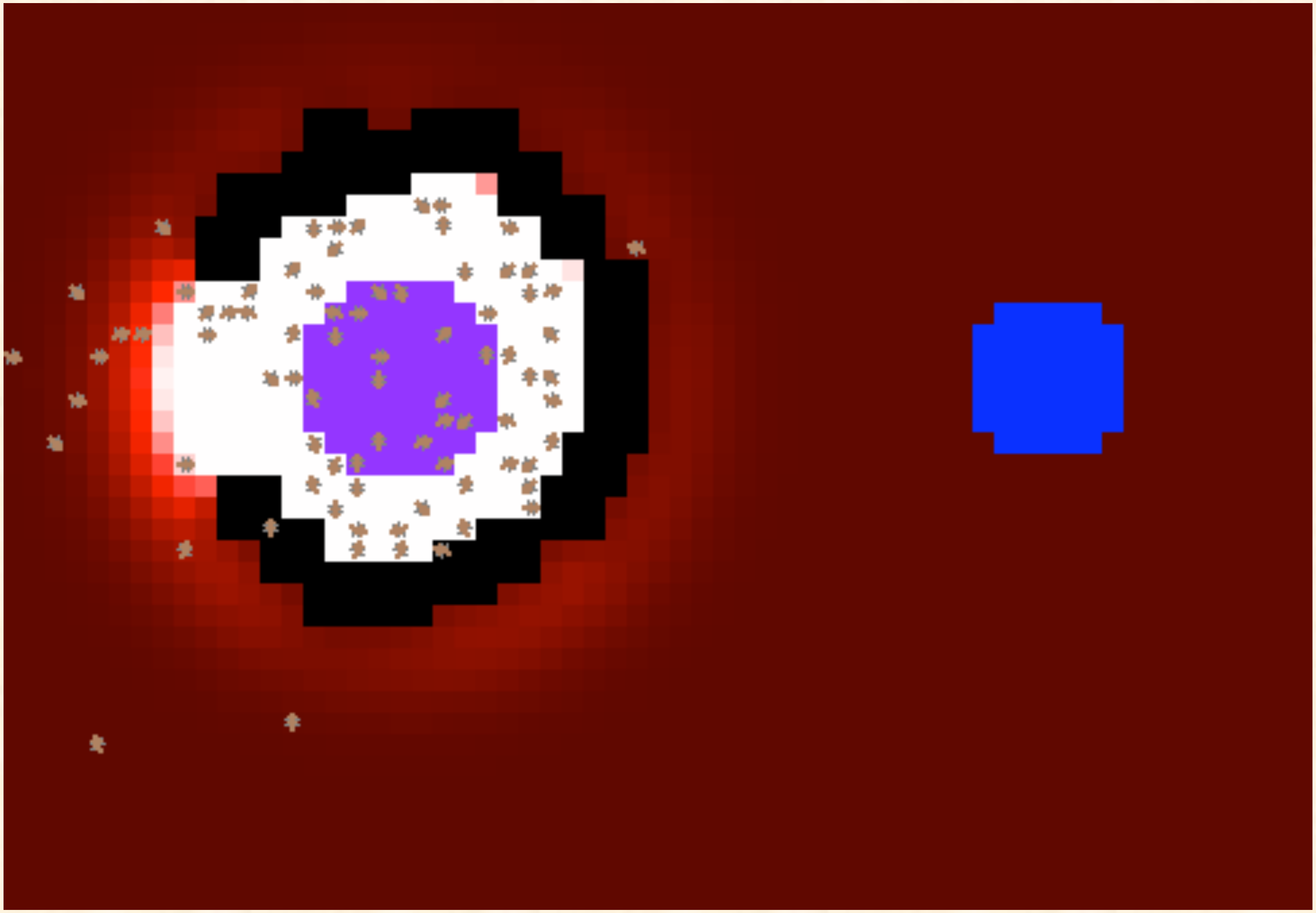


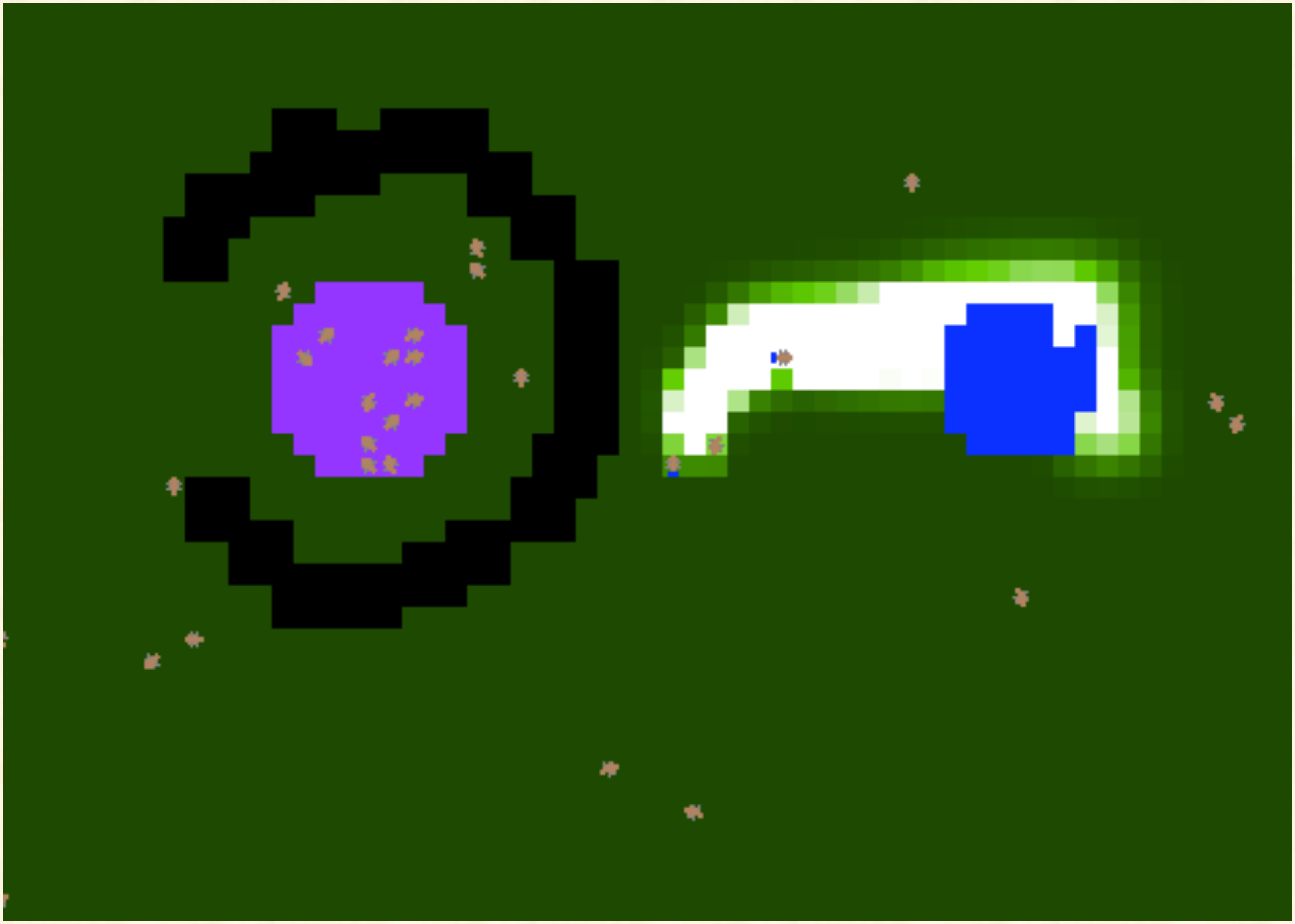


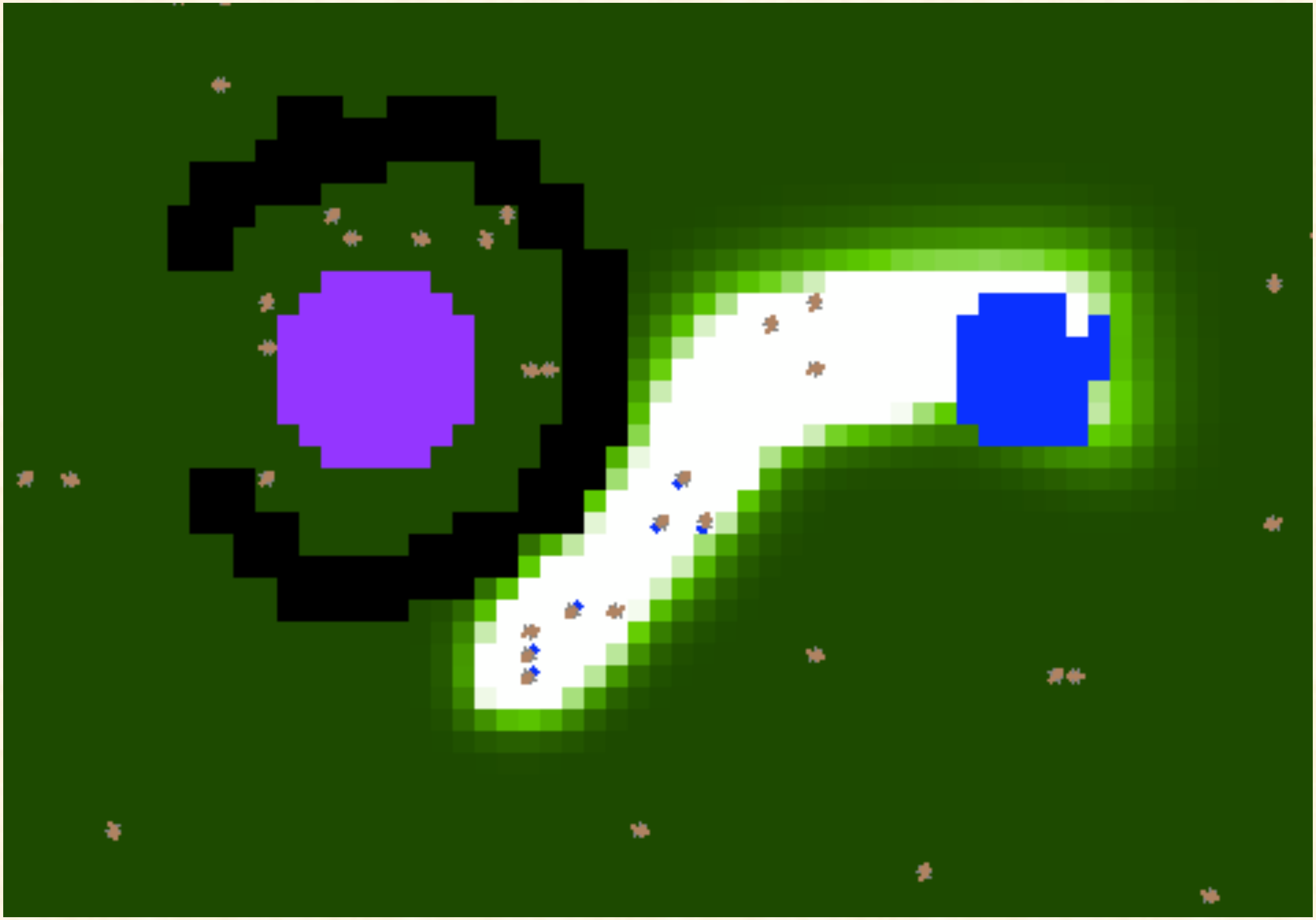


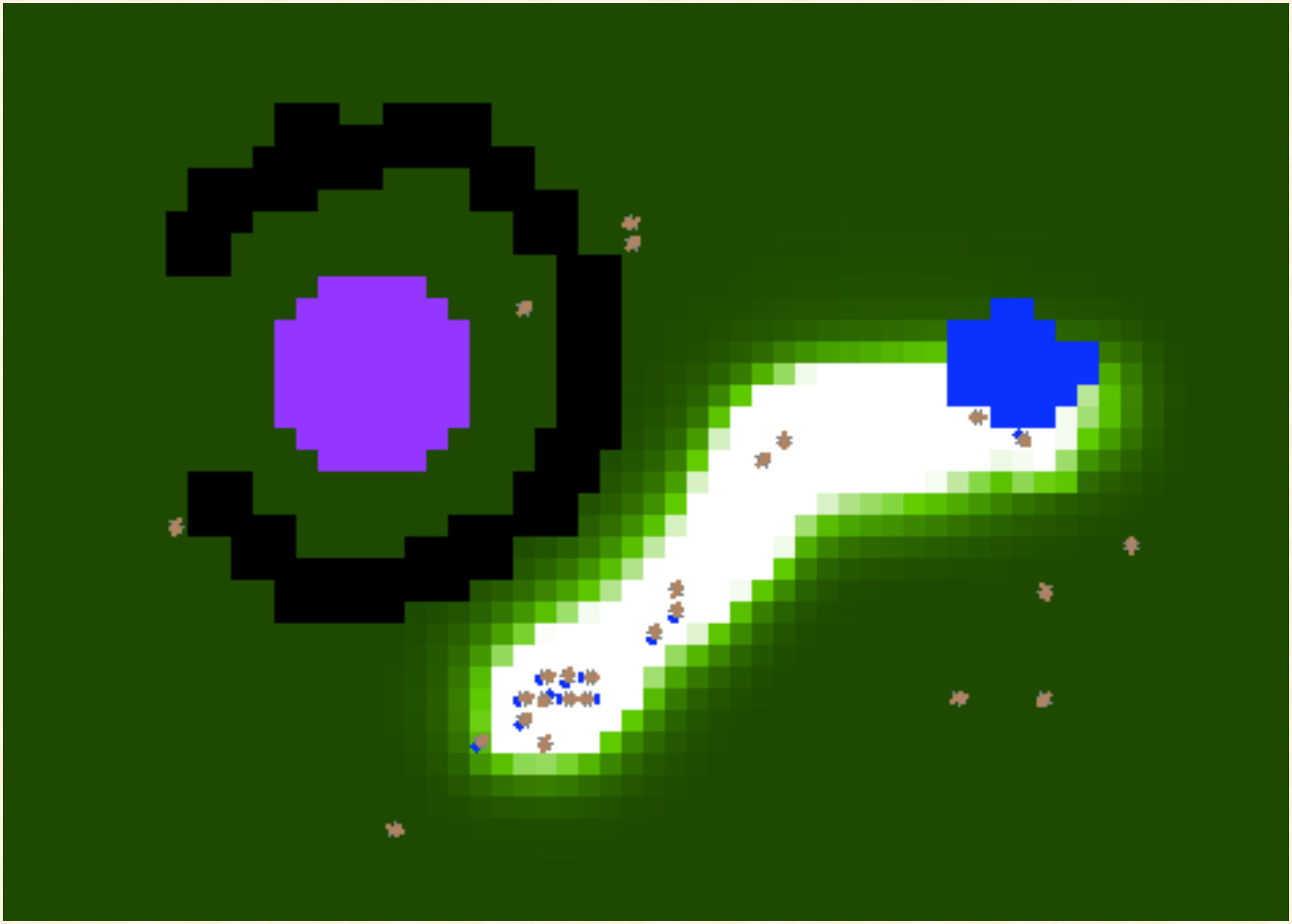


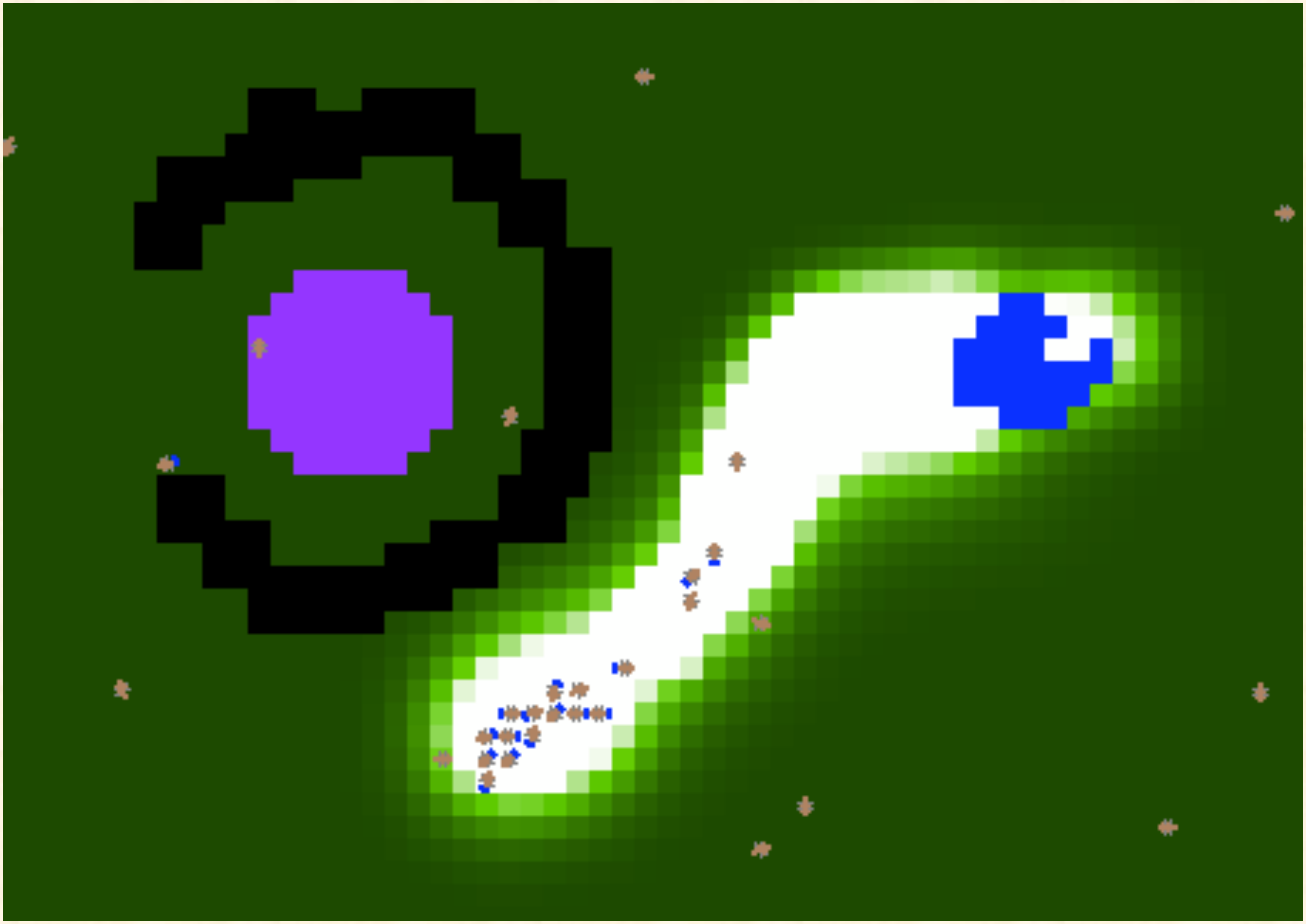
Avoiding a Barrier

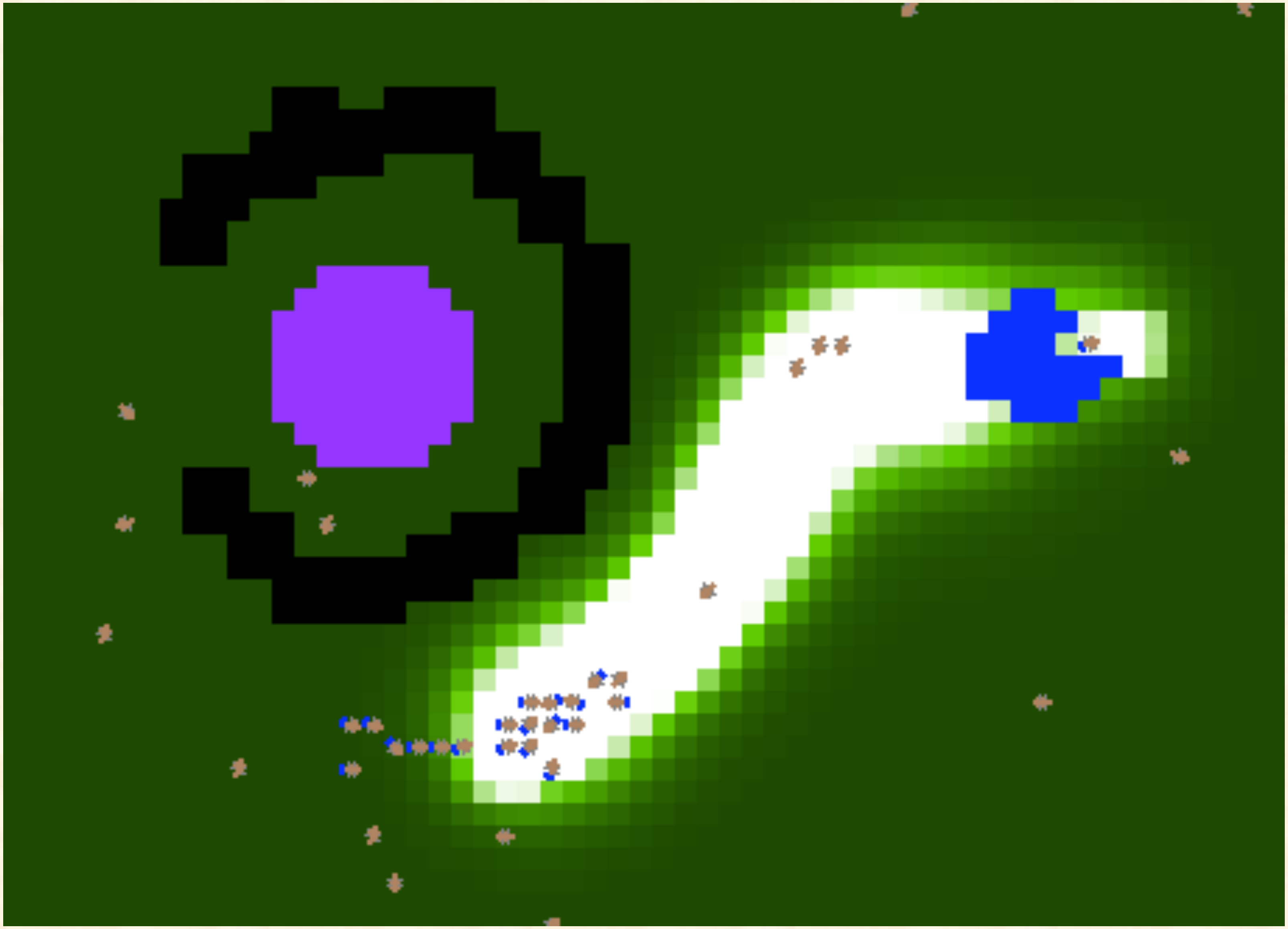


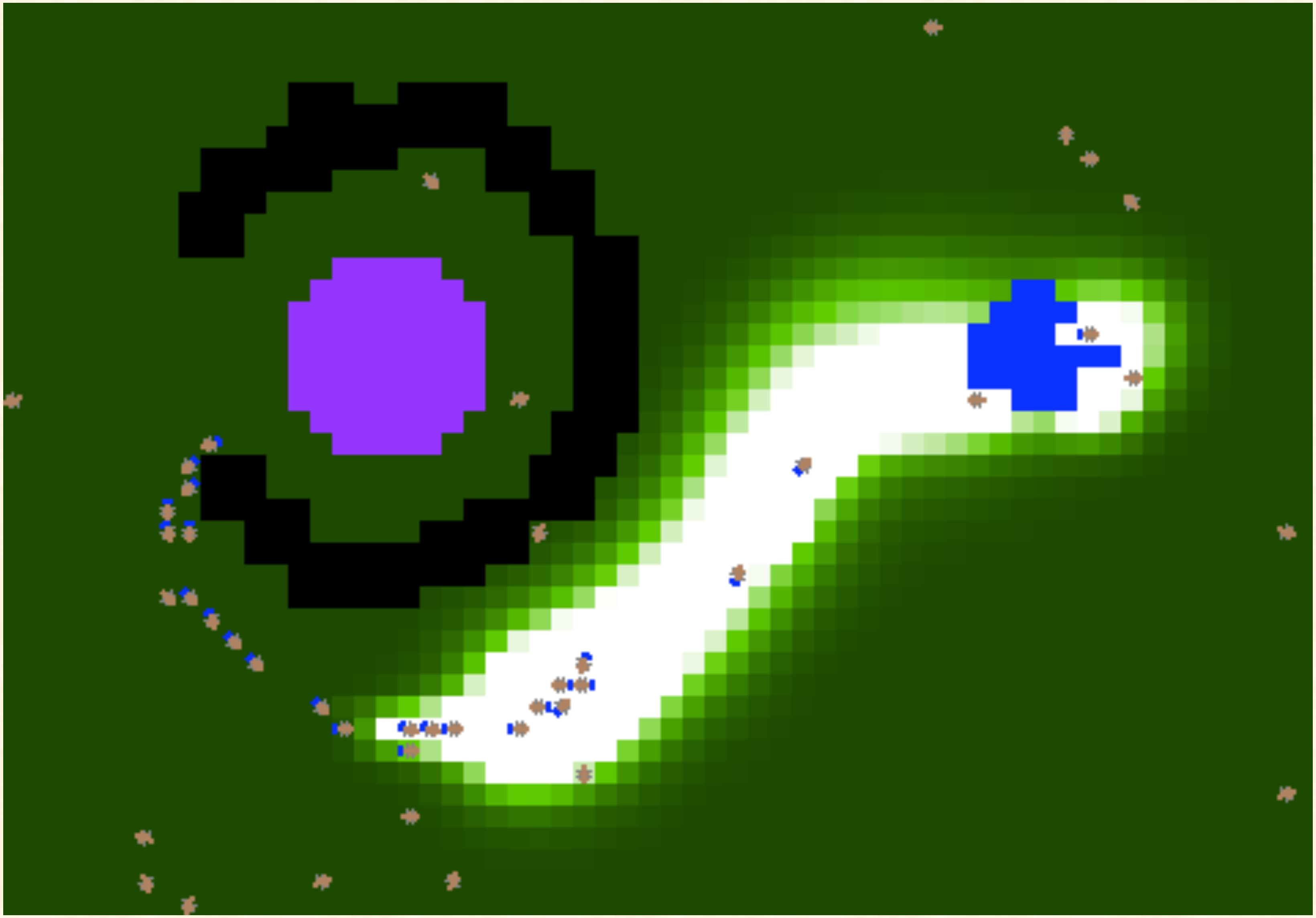


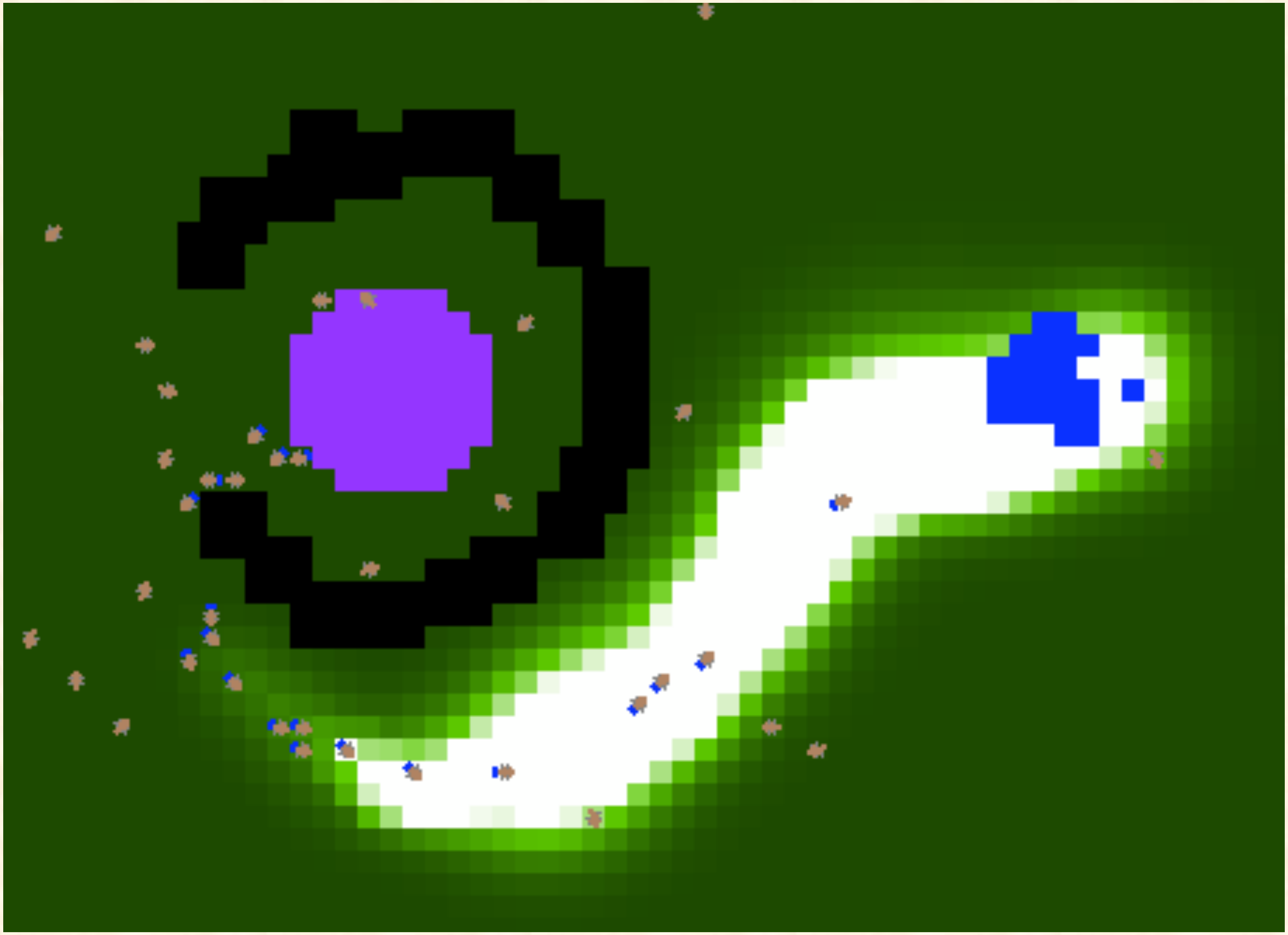


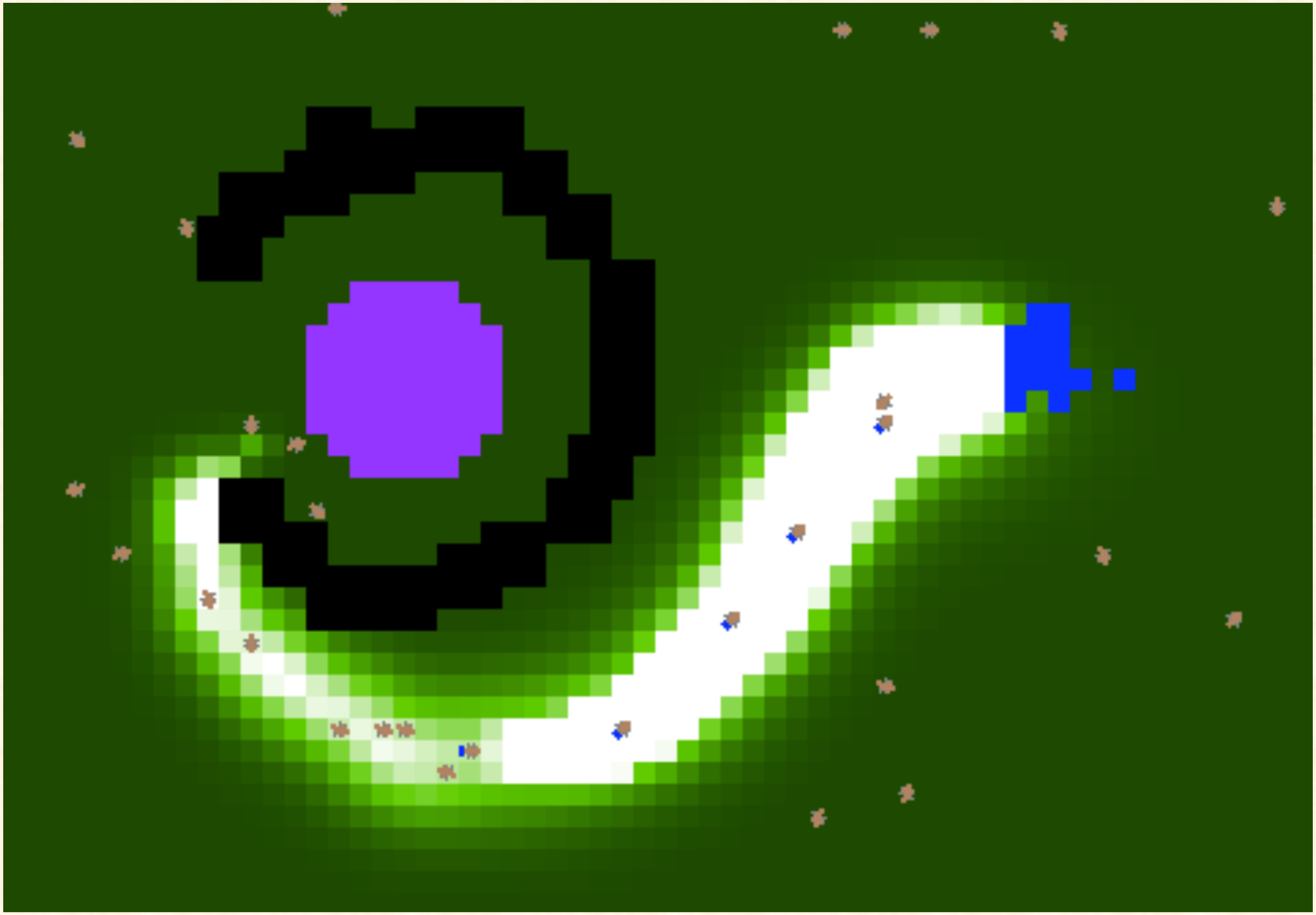


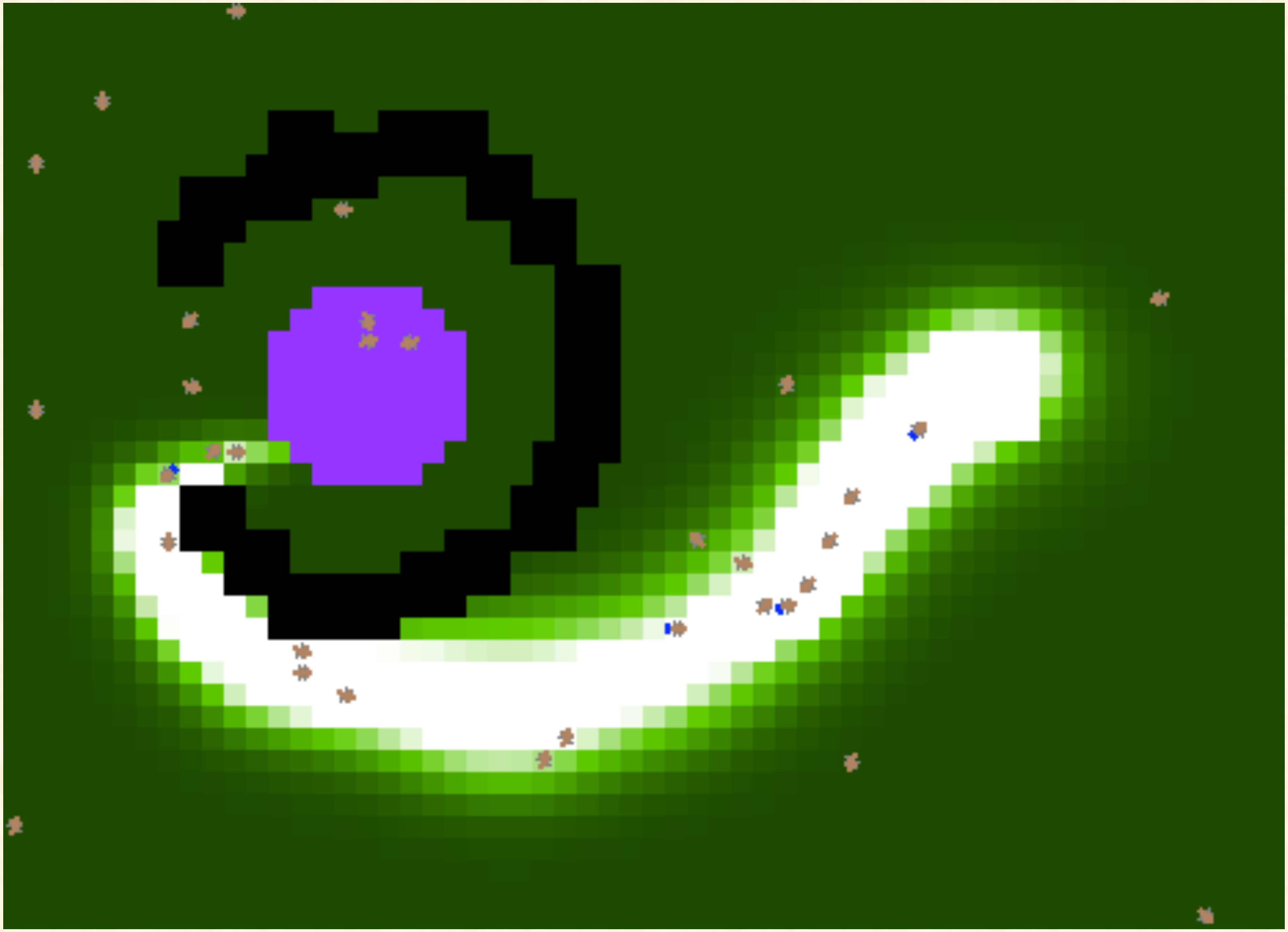


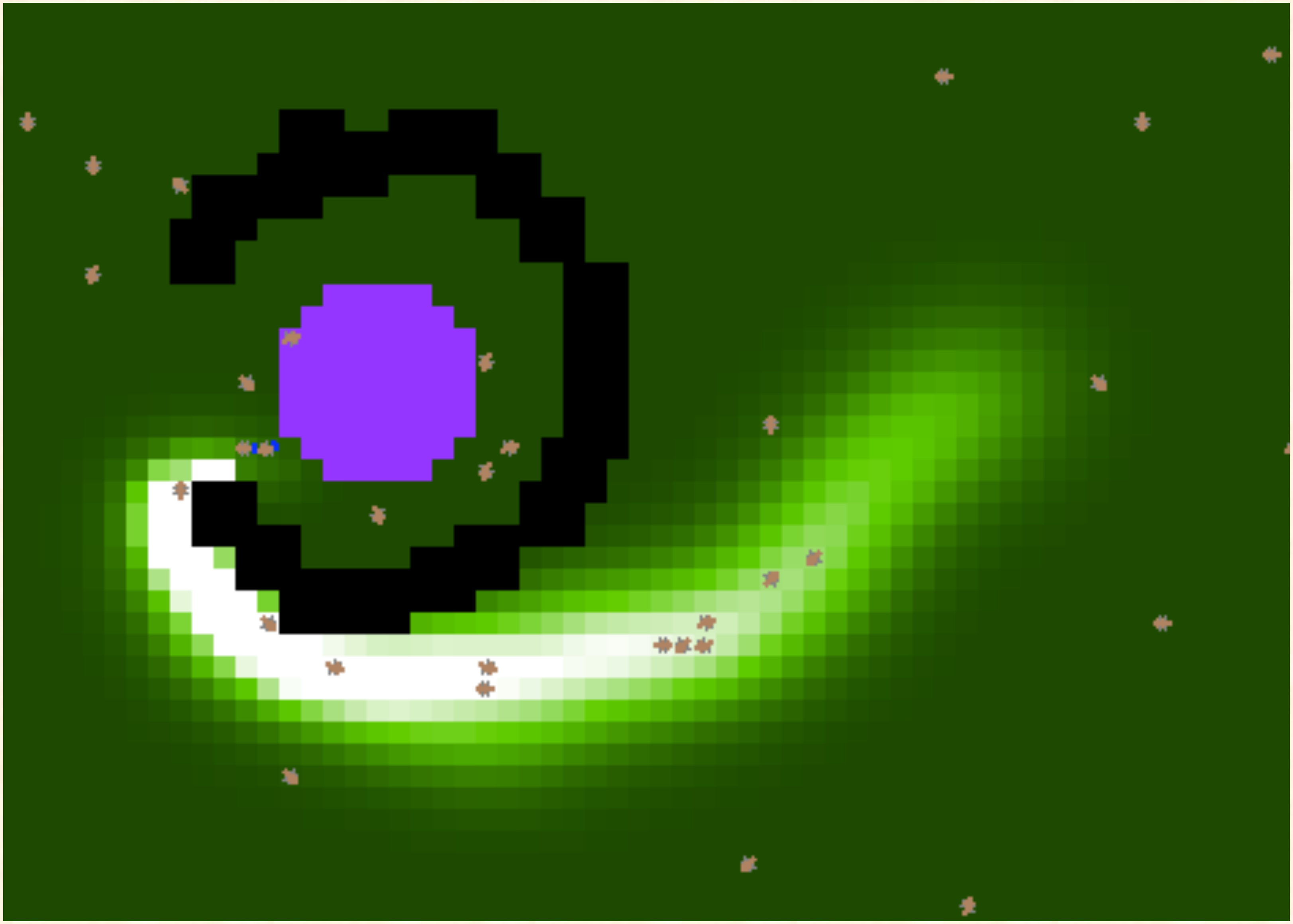












Ant Colony Optimization (ACO)

Developed in 1991 by Dorigo (PhD dissertation) in collaboration with Colorni & Maniezzo

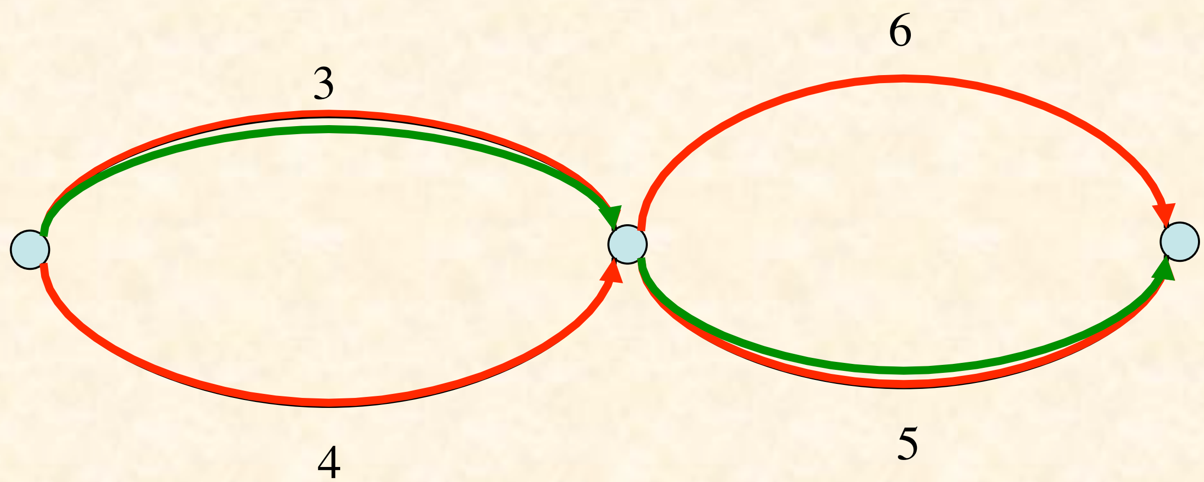
Basis of all Ant-Based Algorithms

- Positive feedback
- Negative feedback
- Cooperation

Positive Feedback

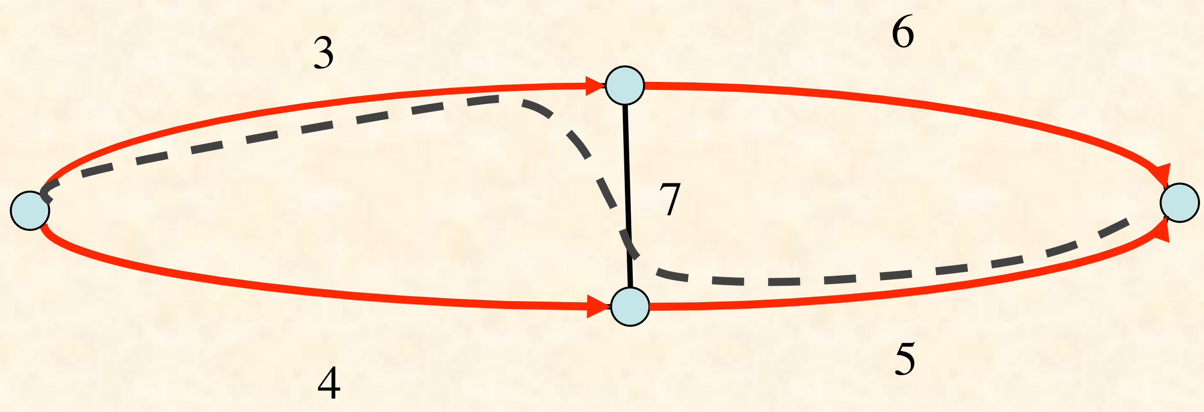
- To reinforce portions of good solutions that contribute to their goodness
- To reinforce good solutions directly
- Accomplished by *pheromone accumulation*

Reinforcement of Solution Components



Parts of good solutions *may* produce better solutions

Negative Reinforcement of Non-solution Components



Parts not in good solutions *tend* to be forgotten

Negative Feedback

- To avoid premature convergence (*stagnation*)
- Accomplished by *pheromone evaporation*

Cooperation

- For simultaneous exploration of different solutions
- Accomplished by:
  - *multiple ants* exploring solution space
  - *pheromone trail* reflecting multiple perspectives on solution space

Traveling Salesman Problem

- Given the travel distances between N cities
  - may be symmetric or not
- Find the shortest route visiting each city exactly once and returning to the starting point
- NP-hard
- Typical combinatorial optimization problem

Ant System for Traveling Salesman Problem (AS-TSP)

- During each iteration, each ant completes a tour
- During each tour, each ant maintains *tabu list* of cities already visited
- Each ant has access to
  - distance of current city to other cities
  - intensity of local pheromone trail
- Probability of next city depends on both

Transition Rule

- Let $\eta_{ij} = 1/d_{ij}$ = "nearness" of city j to current city i
- Let τ_{ij} = strength of trail from i to j
- Let J_i^k = list of cities ant k still has to visit after city i in current tour
- Then transition probability for ant k going from i to $j \in J_i^k$ in tour t is:

$$p_{ij}^k = \frac{\left[\tau_{ij}(t)\right]^\alpha \left[\eta_{ij}\right]^\beta}{\sum_{l \in J_i^k} \left[\tau_{il}(t)\right]^\alpha \left[\eta_{il}\right]^\beta}$$

Pheromone Deposition

- Let $T^k(t)$ be tour t of ant k
- Let $L^k(t)$ be the length of this tour
- After completion of a tour, each ant k contributes:

$$\Delta\tau_{ij}^k = \begin{cases} Q / L^k(t) & \text{if } (i,j) \in T^k(t) \\ 0 & \text{if } (i,j) \notin T^k(t) \end{cases}$$

Pheromone Decay

- Define total pheromone deposition for tour *t*:

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}(t)$$

- Let ρ be decay coefficient
- Define trail intensity for next round of tours:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t)$$

Number of Ants is Critical

- Too many:
  - suboptimal trails quickly reinforced
  - ∴ early convergence to suboptimal solution
- Too few:
  - don't get cooperation before pheromone decays
- Good tradeoff:
  number of ants = number of cities
  ($m = n$)

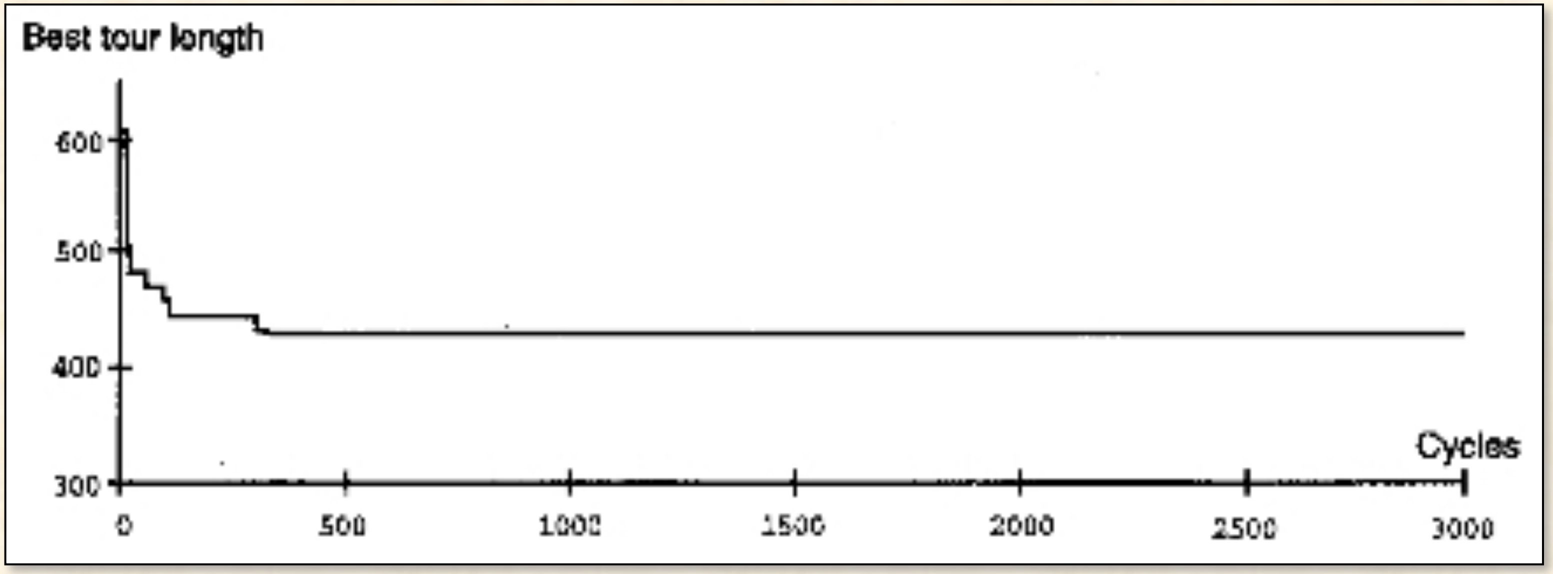
Improvement: "Elitist" Ants

- Add a few ($e \approx 5$) "elitist" ants to population
- Let T^+ be best tour so far
- Let L^+ be its length
- Each "elitist" ant reinforces edges in T^+ by Q/L^+
- Add e more "elitist" ants
- This applies accelerating positive feedback to best tour

Time Complexity

- Let t be number of tours
- Time is $\mathcal{O}(tn^2m)$
- If $m = n$ then $\mathcal{O}(tn^3)$
  - that is, cubic in number of cities

Convergence



- 30 cities ("Oliver30")
- Best tour length
- Converged to optimum in 300 cycles

fig. < Dorigo et al. (1996)

Evaluation

- Both "very interesting and disappointing"
- For 30-cities:
  - beat genetic algorithm
  - matched or beat tabu search & simulated annealing
- For 50 & 75 cities and 3000 iterations
  - did not achieve optimum
  - but quickly found good solutions
- I.e., does not scale up well
- Like all general-purpose algorithms, it is out-performed by special purpose algorithms

Improving Network Routing

1. Nodes periodically send *forward ants* to some recently recorded destinations
2. Collect information on way
3. Die if reach already visited node
4. When reaches destination, estimates time and turns into *backward ant*
5. Returns by same route, updating routing tables

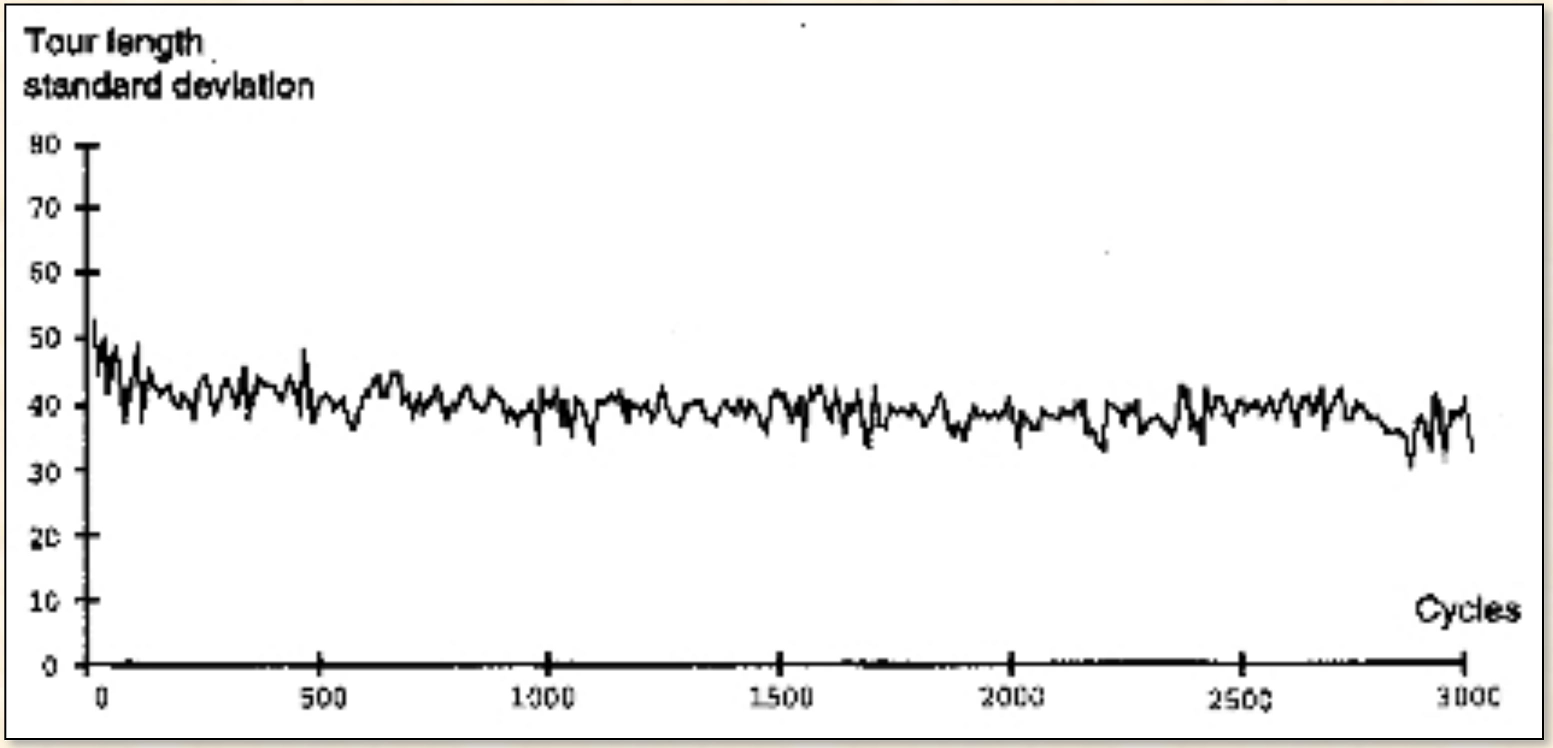
Some Applications of ACO

- Routing in telephone networks
- Vehicle routing
- Job-shop scheduling
- Constructing evolutionary trees from nucleotide sequences
- Various classic NP-hard problems
  - shortest common supersequence, graph coloring, quadratic assignment, …

Improvements as Optimizer

- Can be improved in many ways
- E.g., combine local search with ant-based methods
- As method of stochastic combinatorial optimization, performance is promising, comparable with best heuristic methods
- Much ongoing research in ACO
- But optimization is not a principal topic of this course

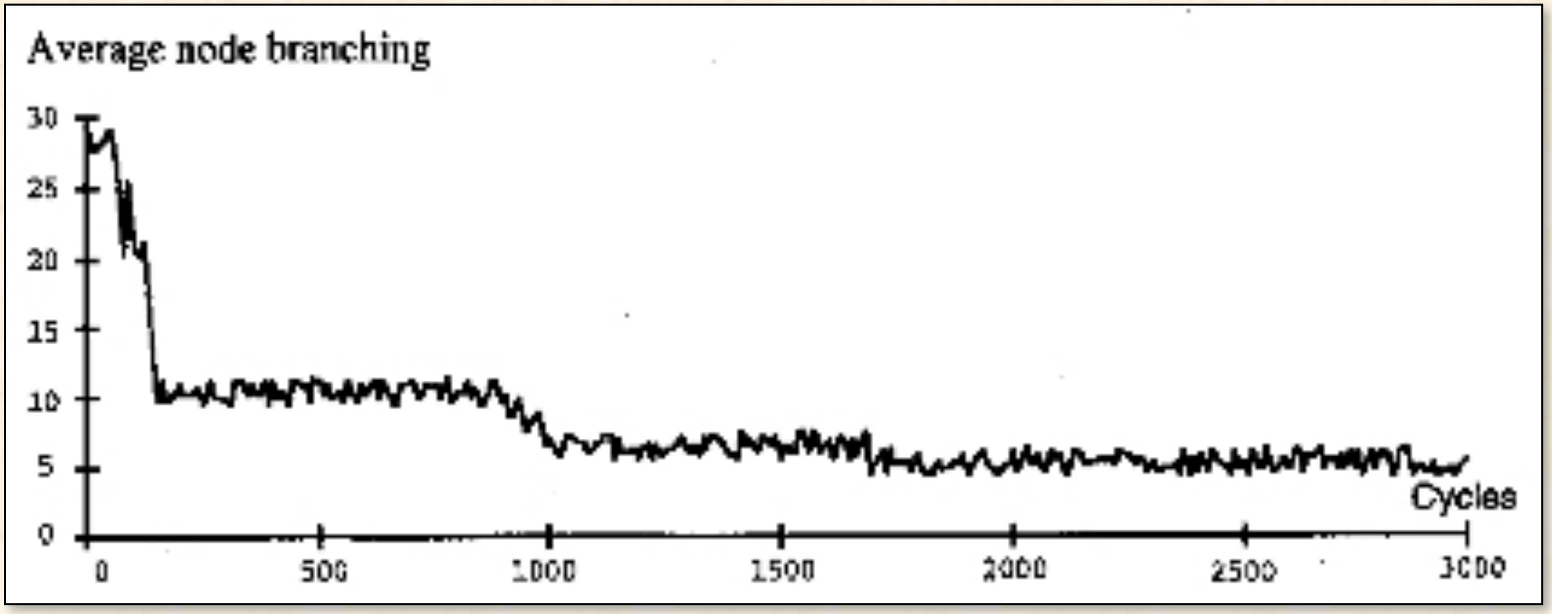
Nonconvergence



- Standard deviation of tour lengths
- Optimum = 420

fig. < Dorigo et al. (1996)

Average Node Branching Number



- Branching number = number of edges leaving a node with pheromone > threshold
- Branching number = 2 for fully converged solution

fig. < Dorigo et al. (1996)

The Nonconvergence Issue

- AS often does not converge to single solution
- Population maintains high diversity
- A bug or a feature?
- Potential advantages of nonconvergence:
  - avoids getting trapped in local optima
  - promising for dynamic applications
- Flexibility & robustness are more important than optimality in natural computation

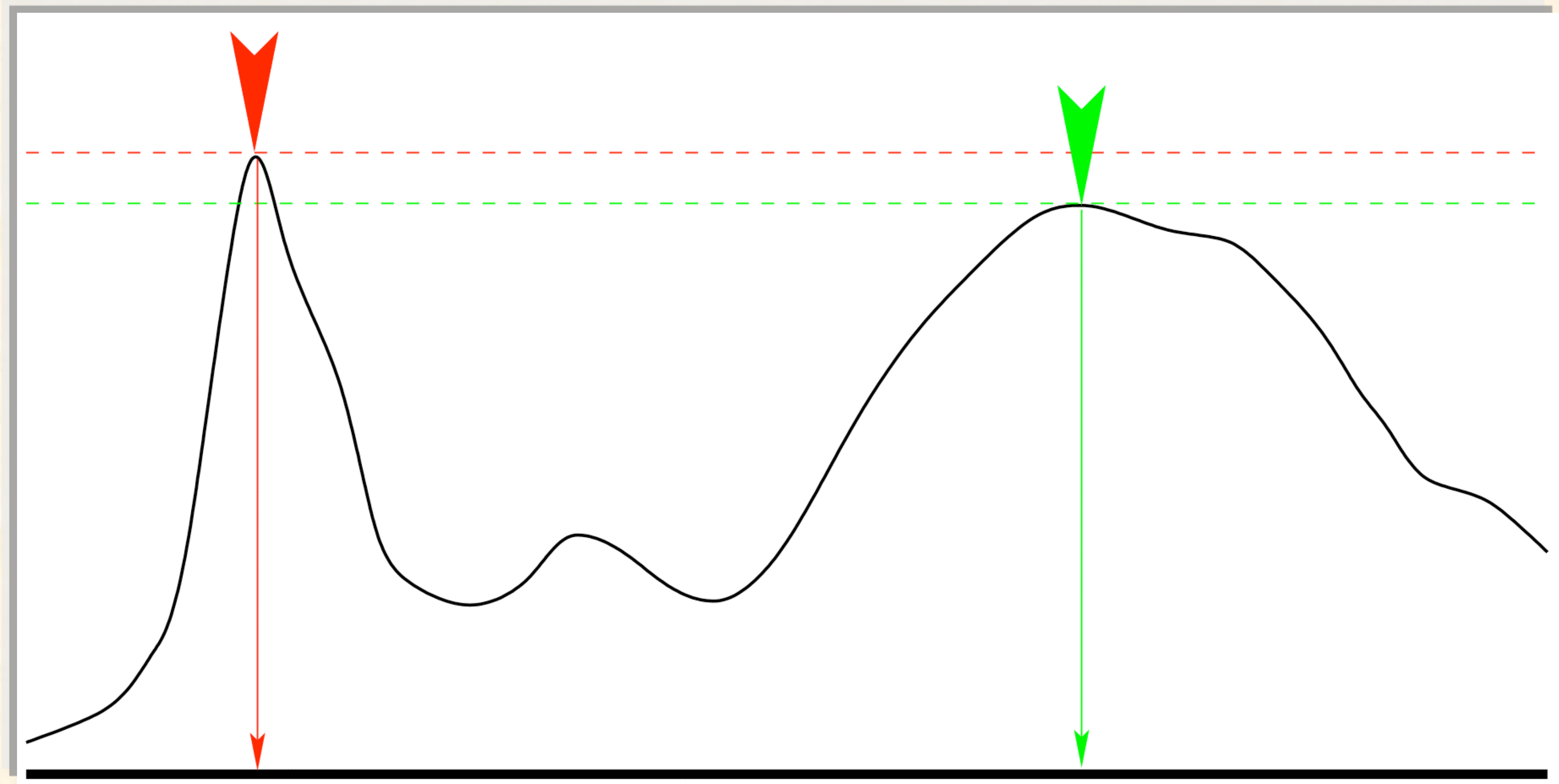
Natural Computation

Natural computation is computation that occurs in nature or is inspired by computation occurring in nature

Optimization in Natural Computation

- Good, but suboptimal solutions may be preferable to optima if:
  - suboptima can be obtained more quickly
  - suboptima can be adapted more quickly
  - suboptima are more robust
  - an ill-defined suboptimum may be better than a sharp optimum
- “The best is often the enemy of the good”

Robust Optima



Effect of Error/Noise