Identification of the Rotor Time Constant in Induction Machines without Speed Sensor

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***Abstract*— A differential-algebraic method is used to estimate the rotor time constant T_R of an induction motor without measurements of the rotor speed/position. The method consists of solving for the roots of a polynomial equation in T_R whose coefficients depend only on the stator currents, stator voltages, and their derivatives. Experimental results are presented.**

***Index Terms*— Rotor Time Constant, Sensorless Speed Observer, Induction Motor.**

I. INTRODUCTION

Induction motors are very attractive in many applications owing to their simple structure, low cost, and robust construction. Field-oriented control is now used to obtain high performance drive of the induction motor because it gives control characteristics similar to separately excited DC motors. Implementation of a (rotor-flux) field-oriented controller requires knowledge of the rotor speed and the rotor time constant T_R to estimate the rotor flux linkages. There has been considerable work done in the last several years to implement a field-oriented controller without the use of a speed sensor [1][2][3][4][5][6]. However, many of these methods still require the value of T_R, which can change with time due to ohmic heating; that is, to be able to update the value of T_R to the controller as it changes is valuable. The work presented here uses an algebraic approach to identify the rotor time constant T_R without the motor speed information. It is most closely related to the ideas described in [7][8][9][10][11]. Specifically, it is shown that T_R satisfies a polynomial equation whose coefficients are functions of the stator currents, the stator voltages, and their derivatives. A zero of this polynomial is the value of T_R. It is further shown T_R is not identifiable by this technique under steady-state conditions. It is also true (and shown here) that a standard least-squares approach cannot identify T_R under steady-state conditions. In [4], the speed ω and T_R are identified assuming constant speed but not (sinusoidal) steady state. In [12], the speed is assumed constant, but the flux magnitude is perturbed by a small amplitude sinusoidal signal to identify T_R.

The paper is organized as follows. Section II introduces a space vector model of the induction motor. Section III uses this model to develop a differential-algebraic equation that T_R must satisfy. Section IV shows that in steady state, T_R is not identifiable by either the differential-algebraic method nor a standard linear least-squares method. Section V presents the experimental results, while Section VI gives the conclusions and future work.

II. MATHEMATICAL MODEL OF INDUCTION MOTOR

The starting point of the analysis is a space vector model of the induction motor given by (see e.g., pp. 568 of [13])

$$\frac{d}{dt}\underline{i}_S = \frac{\beta}{T_R}\left(1 - jn_P\omega T_R\right)\underline{\psi}_R - \gamma\underline{i}_S + \frac{1}{\sigma L_S}\underline{u}_S \quad (1)$$

$$\frac{d}{dt}\underline{\psi}_R = -\frac{1}{T_R}\left(1 - jn_P\omega T_R\right)\underline{\psi}_R + \frac{M}{T_R}\underline{i}_S \quad (2)$$

$$\frac{d\omega}{dt} = \frac{n_p M}{J L_R}\operatorname{Im}\left\{\underline{i}_S\underline{\psi}_R^*\right\} - \frac{\tau_L}{J}, \quad (3)$$

where $\underline{i}_S \triangleq i_{Sa} + ji_{Sb}$, $\underline{\psi}_R \triangleq \psi_{Ra} + j\psi_{Rb}$, and $\underline{u}_S \triangleq u_{Sa} + ju_{Sb}$. Here, θ is the position of the rotor, $\omega = d\theta/dt$ is the rotor speed, n_p is the number of pole pairs, i_{Sa}, i_{Sb} are the (two-phase equivalent) stator currents, ψ_{Ra}, ψ_{Rb} are the (two-phase equivalent) rotor flux linkages, R_S, R_R are the stator and rotor resistances, respectively, M is the mutual inductance, L_S and L_R are the stator and rotor inductances, respectively, J is the moment of inertia of the rotor, and τ_L is the load torque. The symbols $T_R = \frac{L_R}{R_R}$, $\sigma = 1 - \frac{M^2}{L_S L_R}$, $\beta = \frac{M}{\sigma L_S L_R}$, $\gamma = \frac{R_S}{\sigma L_S} + \frac{\beta M}{T_R}$ have been used to simplify the expressions. T_R is referred to as the rotor time constant, while σ is called the total leakage factor.

III. DIFFERENTIAL-ALGEBRAIC APPROACH TO T_R ESTIMATION

The idea of the differential-algebraic approach is to solve (1) and (2) for T_R [14][15]. However, equations (1) and (2) are only four equations while there are six unknowns, namely ψ_{Ra}, ψ_{Rb}, $d\psi_{Ra}/dt$, $d\psi_{Rb}/dt$, ω, and T_R. Equation (3) is not used because it introduces the additional unknown τ_L. To find two more independent equations, equation (1) is differentiated to obtain

$$\begin{aligned}\frac{d^2}{dt^2}\underline{i}_S &= \frac{\beta}{T_R}\left(1 - jn_P\omega T_R\right)\frac{d}{dt}\underline{\psi}_R - jn_P\beta\underline{\psi}_R\frac{d\omega}{dt}\\ &\quad -\gamma\frac{d}{dt}\underline{i}_S + \frac{1}{\sigma L_S}\frac{d}{dt}\underline{u}_S. \end{aligned} \quad (4)$$

Using the (complex-valued) equations (1) and (2), one can solve for $\underline{\psi}_R$ and $\frac{d}{dt}\underline{\psi}_R$ in terms of ω, $\underline{i}_S$ and $\underline{u}_S$ and substitute the resulting expressions into (4) to obtain

$$\begin{aligned}\frac{d^2}{dt^2}\underline{i}_S = &-\frac{1}{T_R}\left(1-jn_P\omega T_R\right)\left(\frac{d}{dt}\underline{i}_S+\gamma\underline{i}_S-\frac{1}{\sigma L_S}\underline{u}_S\right)\\ &+\frac{\beta M}{T_R^2}\left(1-jn_P\omega T_R\right)\underline{i}_S-\gamma\frac{d}{dt}\underline{i}_S+\frac{1}{\sigma L_S}\frac{d}{dt}\underline{u}_S\\ &-\frac{jn_PT_R}{1-jn_P\omega T_R}\left(\frac{d}{dt}\underline{i}_S+\gamma\underline{i}_S-\frac{1}{\sigma L_S}\underline{u}_S\right)\frac{d\omega}{dt}.\end{aligned} \tag{5}$$

Solving (5) for $d\omega/dt$ gives

$$\frac{d\omega}{dt} = -\frac{\left(1-jn_P\omega T_R\right)^2}{jn_PT_R^2}+\frac{1-jn_P\omega T_R}{jn_PT_R}\times \frac{\frac{\beta M}{T_R^2}\left(1-jn_P\omega T_R\right)\underline{i}_S-\gamma\frac{d}{dt}\underline{i}_S+\frac{1}{\sigma L_S}\frac{d}{dt}\underline{u}_S-\frac{d^2}{dt^2}\underline{i}_S}{\frac{d}{dt}\underline{i}_S+\gamma\underline{i}_S-\frac{1}{\sigma L_S}\underline{u}_S}. \tag{6}$$

The left-hand side of (6) is real, so the right-hand side must also be real. Note by (1) that $d\underline{i}_S/dt+\gamma\underline{i}_S-\underline{u}_S/\left(\sigma L_S\right)=\frac{\beta}{T_R}\left(1-jn_P\omega T_R\right)\underline{\psi}_R$ so that the right-hand side of (6) is singular if and only if $\left|\underline{\psi}_R\right| = 0$. Other than at startup, $\left|\underline{\psi}_R\right| \neq 0$ in normal operation of the motor. Separating the right-hand side of (6) into its real and imaginary parts, the real part has the form

$$\begin{aligned}\frac{d\omega}{dt} \quad = \quad & a_2\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)\omega^2+a_1\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)\omega\\ &+a_0\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right).\end{aligned} \tag{7}$$

The expressions for $a_2\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)$, $a_1(u_{Sa},u_{Sb},i_{Sa},i_{Sb})$, and $a_0\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)$ are lengthy in terms of u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}, and their derivatives as well as of the machine parameters including T_R. As a consequence, they are not explicitly presented here. Their steady-state expressions are given in [6].

On the other hand, the imaginary part of the right-hand side of (6) must be zero. In fact, the imaginary part of (6) is a second degree polynomial equation in ω of the form

$$\begin{aligned}q(\omega) \quad \triangleq \quad & q_2(u_{Sa},u_{Sb},i_{Sa},i_{Sb})\omega^2+q_1(u_{Sa},u_{Sb},i_{Sa},i_{Sb})\omega\\ &+q_0(u_{Sa},u_{Sb},i_{Sa},i_{Sb})\end{aligned} \tag{8}$$

and, if ω is the speed of the motor, then $q(\omega) = 0$. The q_i are functions of u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}, and their derivatives as well as of the machine parameters including T_R. The expressions for $q_2(u_{Sa},u_{Sb},i_{Sa},i_{Sb})$, $q_1\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)$, and $q_0(u_{Sa},u_{Sb},i_{Sa},i_{Sb})$ are also lengthy and not explicitly presented here. (Their steady-state expressions are given in [6].) If the speed was measured, then (8) would be equal to zero and could then be solved for T_R. However, in the problem being considered, ω is not known. To eliminate ω, $q(\omega)$ in (8) is differentiated to obtain

$$\frac{d}{dt}q(\omega) = \left(2q_2\omega+q_1\right)\frac{d\omega}{dt}+\dot{q}_2\omega^2+\dot{q}_1\omega+\dot{q}_0 \tag{9}$$

where $dq(\omega)/dt \equiv 0$ if ω is equal to the motor speed. Next, $d\omega/dt$ in (9) is replaced by the right-hand side of (7) so that (9) may be written as

$$\begin{aligned}\frac{dq(\omega)}{dt} \quad = \quad & g(\omega) \triangleq 2q_2a_2\omega^3+\left(2q_2a_1+q_1a_2+\dot{q}_2\right)\omega^2\\ &+\left(2q_2a_0+q_1a_1+\dot{q}_1\right)\omega+q_1a_0+\dot{q}_0.\end{aligned} \tag{10}$$

$g(\omega)$ is a third-order polynomial equation in ω for which the speed of the motor is one of its zeros. Dividing[1] $g(\omega)$ in (10) by $q(\omega)^2$ in (8), $g(\omega)$ may be rewritten as

$$\begin{aligned}g(\omega) = \frac{1}{q_2}\Big(&\left(2q_2a_2\omega+2q_2a_1-q_1a_2+\dot{q}_2\right)q(\omega)\\ &+r_1\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)\omega+r_0\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)\Big)\end{aligned} \tag{11}$$

$$\begin{aligned}r_1\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right) \quad &\triangleq \quad 2q_2^2a_0-q_2q_1a_1+q_2\dot{q}_1-2q_2q_0a_2\\ &\qquad +q_1^2a_2-q_1\dot{q}_2 \qquad (12)\\ r_0\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right) \quad &\triangleq \quad q_2q_1a_0+q_2\dot{q}_0-2q_2q_0a_1\\ &\qquad +q_0q_1a_2-q_0\dot{q}_2. \qquad (13)\end{aligned}$$

If ω is equal to the speed of the motor, then both $g(\omega) = 0$ and $q(\omega) = 0$, and one obtains

$$r(\omega) \triangleq r_1\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)\omega+r_0\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right) = 0. \tag{14}$$

This is now a first-order polynomial equation in ω which uniquely determines the motor speed ω as long as r_1 (the coefficient of ω) is nonzero. (It is shown in Appendix VII-A that $r_1 \neq 0$ in steady state.) Solving for the motor speed ω using (14), one obtains

$$\omega = -r_0/r_1. \tag{15}$$

Next, replace ω in (8) by the expression in (15) to obtain

$$q_2r_0^2-q_1r_0r_1+q_0r_1^2 \equiv 0. \tag{16}$$

The expressions for q_i, r_i are in terms of motor parameters (including T_R) as well as the stator currents, voltages, and their derivatives. Expanding the expressions for q_0, q_1, q_2, r_0, and r_1, one obtains a twelfth-order polynomial equation in T_R, which can be written as

$$\sum_{i=0}^{12}C_i\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)T_R^i = 0. \tag{17}$$

Solving equation (17) gives T_R. The coefficients $C_i\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)$ of (17) contain third-order derivatives of the stator currents and second-order derivatives of the stator voltages making noise a concern. For short time intervals in which T_R does not vary, (17) must hold identically with T_R constant. In order to average out the effect of noise on the C_i, (17) is integrated over a time interval $[t_1,t_2]$ to obtain

$$\sum_{i=0}^{12}\left(\frac{1}{t_2-t_1}\int_{t_1}^{t_2}C_i\left(u_{Sa},u_{Sb},i_{Sa},i_{Sb}\right)dt\right)T_R^i = 0. \tag{18}$$

[1]Given the polynomials $g(\omega),q(\omega)$ in ω with $\deg\{g(\omega)\} = n_g, \deg\{q(\omega)\} = n_q$, the Euclidean division algorithm ensures that there are polynomials $\gamma(\omega),r(\omega)$ such that $g(\omega) = \gamma(\omega)q(\omega)+r(\omega)$ and $\deg\{r(\omega)\} \leq \deg\{q(\omega)\}-1 = n_q-1$. Consequently if, for example, ω_0 is a zero of both $g(\omega)$ and $q(\omega)$, then it must also be a zero of $r(\omega)$.

[2]$q_2 \neq 0$ if ω and the stator electrical frequency ω_S are nonzero, which hold under normal operating conditions. See [6][16].

There are 12 solutions satisfying (18). However, simulation results have always given 10 conjugate solutions. The remaining two solutions include the correct value of T_R while the other one was either negative or close to zero. The method is to compute the coefficients $\frac{1}{t_2-t_1}\int_{t_1}^{t_2} C_i dt$ and then compute the roots of (18). Among the positive real roots is the correct value of T_R. Experimental results using this method are presented in Section V.

IV. Identifiability of T_R in Steady State

A. Differential-algebraic approach

The polynomial (18) is now considered with the machine in steady state so that, in particular, the speed is constant. That is, $u_{Sa} + ju_{Sb} = \underline{U}_S e^{j\omega_S t}$ and $i_{Sa} + ji_{Sb} = \underline{I}_S e^{j\omega_S t}$ are substituted into (8) and (14) . In steady state, the motor speed in (15) becomes (see Appendix VII-A and [16])

$$\omega = -\frac{r_0}{r_1} = \frac{\omega_S(1-S)}{n_p} \tag{19}$$

where $S \triangleq (\omega_S - n_p\omega)/\omega_S$ is the normalized slip and ω_S is the electrical frequency. Substituting the steady-state expressions for q_2, q_1, and q_0 as well as the expression (19) for ω into (8) , one obtains $q_2\omega^2 + q_1\omega + q_0 =$

$$\begin{aligned}&\frac{n_p^2 T_R^2 |\underline{I}_S|^4 \omega_S^2 L_S (1-\sigma)^2 (1-S)}{\sigma(1+S^2\omega_S^2 T_R^2)}\left(\frac{\omega_S(1-S)}{n_p}\right)^2\\ &+\frac{n_p\omega_S |\underline{I}_S|^4 L_S(1-\sigma)^2\left(1-\omega_S^2 T_R^2(1-S)^2\right)}{\sigma(1+S^2\omega_S^2 T_R^2)}\\ &\times\left(\frac{\omega_S(1-S)}{n_p}\right) - \frac{|\underline{I}_S|^4\omega_S^2 L_S(1-\sigma)^2(1-S)}{\sigma(1+S^2\omega_S^2T_R^2)} \equiv 0.\end{aligned}$$

That is, in steady state (8) and (14) hold independent of the value of T_R and thus so does (17) making T_R unidentifiable in steady state by this method.

B. Linear least-squares approach

Vélez-Reyes et al [3][4] have used least-squares methods for simultaneous parameter and speed identification in induction machines. In the approach used herein, $d\omega/dt$ is taken to be zero so that a linear (in the parameters) regressor model can be obtained. Specifically, consider the mathematical model of the induction motor in (5). Assuming constant speed, $d\omega/dt = 0$ so that this equation reduces to

$$\begin{aligned}\frac{d^2}{dt^2}\underline{i}_S &= -\frac{1}{T_R}(1-jn_P\omega T_R)\left(\frac{d}{dt}\underline{i}_S + \gamma\underline{i}_S - \frac{1}{\sigma L_S}\underline{u}_S\right)\\ &\quad+\frac{\beta M}{T_R^2}(1-jn_P\omega T_R)\underline{i}_S - \gamma\frac{d}{dt}\underline{i}_S + \frac{1}{\sigma L_S}\frac{d}{dt}\underline{u}_S\end{aligned} \tag{20}$$

where $\underline{i}_S = i_{Sa} + ji_{Sb}$ and $\underline{u}_S = u_{Sa} + ju_{Sb}$. Decomposing equation (20) into its real and imaginary parts gives

$$\begin{aligned}\frac{d^2 i_{Sa}}{dt} &= \frac{1}{T_R}\left(-\frac{di_{Sa}}{dt} - \frac{R_S}{\sigma L_S}i_{Sa} + \frac{1}{\sigma L_S}u_{Sa}\right)\\ &\quad+n_p\omega\left(-\frac{di_{Sb}}{dt} - \frac{R_S}{\sigma L_S}i_{Sb} + \frac{1}{\sigma L_S}u_{Sb}\right)\\ &\quad-\gamma\frac{di_{Sa}}{dt} + \frac{1}{\sigma L_S}\frac{du_{Sa}}{dt}\end{aligned} \tag{21}$$

and

$$\begin{aligned}\frac{d^2 i_{Sb}}{dt} &= \frac{1}{T_R}\left(-\frac{di_{Sb}}{dt} - \frac{R_S}{\sigma L_S}i_{Sb} + \frac{1}{\sigma L_S}u_{Sb}\right)\\ &\quad-n_p\omega\left(-\frac{di_{Sa}}{dt} - \frac{R_S}{\sigma L_S}i_{Sa} + \frac{1}{\sigma L_S}u_{Sa}\right)\\ &\quad-\gamma\frac{di_{Sb}}{dt} + \frac{1}{\sigma L_S}\frac{du_{Sb}}{dt}.\end{aligned} \tag{22}$$

The goal here is to estimate T_R without knowledge of ω. So, it is now assumed the motor parameters are all known except for T_R. The set of equations (21) and (22) may then be rewritten in regressor form as

$$y(t) = W(t)K \tag{23}$$

where $K \in \mathbb{R}^2$, $y \in \mathbb{R}^2$, and $W \in \mathbb{R}^{2\times 2}$ are given by

$$K \triangleq \begin{bmatrix} 1/T_R \\ n_p\omega \end{bmatrix},$$

$$y(t) \triangleq \begin{bmatrix} \dfrac{du_{Sa}}{dt} - \sigma L_S\dfrac{d^2 i_{Sa}}{dt} - R_S\dfrac{di_{Sa}}{dt} \\[2ex] \dfrac{du_{Sb}}{dt} - \sigma L_S\dfrac{d^2 i_{Sb}}{dt} - R_S\dfrac{di_{Sb}}{dt} \end{bmatrix},$$

$$W(t) \triangleq \begin{bmatrix} L_S\dfrac{di_{Sa}}{dt} - u_{Sa} + R_S i_{Sa} & \sigma L_S\dfrac{di_{Sb}}{dt} - u_{Sb} + R_S i_{Sb} \\[2ex] L_S\dfrac{di_{Sb}}{dt} - u_{Sb} + R_S i_{Sb} & -\sigma L_S\dfrac{di_{Sa}}{dt} + u_{Sa} - R_S i_{Sa} \end{bmatrix}.$$

The regressor system (23) is linear in the parameters. The standard linear least-squares approach is to let (i.e., collect data at) $t = 0, T, 2T, \cdots, NT$, multiply (23) on the left by $W^T(nT)$, sum $W^T(nT)y(nT) = W^T(nT)W(nT)K$ from $t = 0$ to $t = NT$, and finally compute the solution to

$$R_W K = R_{YW} \tag{24}$$

where

$$R_W \triangleq \sum_{n=0}^{N} W^T(nT)W(nT), \quad R_{YW} \triangleq \sum_{n=0}^{N} W^T(nT)y(nT).$$

A unique solution to (24) exists if and only if R_W is invertible. However, R_W is never invertible in steady state as is now shown. To proceed, define

$$D(t) = \begin{bmatrix} i_{Sb}(t) & -i_{Sa}(t) \\ i_{Sa}(t) & i_{Sb}(t) \end{bmatrix}.$$

In steady state where $u_{Sa} + ju_{Sb} = \underline{U}_S e^{j\omega_S t}$ and $i_{Sa} + ji_{Sb} = \underline{I}_S e^{j\omega_S t}$, $\det(D(t)) = i_{Sa}^2(t) + i_{Sb}^2(t) = |\underline{I}_S|^2$, $D(t)^T D(t) = |\underline{I}_S|^2 I_{2\times 2}$. Multiply both sides of (23) on the left by $D(t)$ to obtain

$$D(t)y(t) = D(t)W(t)K$$

or

$$\begin{aligned}&\begin{bmatrix} R_S\omega_S|\underline{I}_S|^2 - \omega_S P \\ \sigma L_S\omega_S^2|\underline{I}_S|^2 - \omega_S Q \end{bmatrix} =\\ &\begin{bmatrix} -\omega_S L_S|\underline{I}_S|^2 + Q & R_S|\underline{I}_S|^2 - P \\ R_S|\underline{I}_S|^2 - P & \sigma L_S\omega_S|\underline{I}_S|^2 - Q \end{bmatrix} K\end{aligned} \tag{25}$$

where $P \triangleq u_{Sa}i_{Sa} + u_{Sb}i_{Sb}$ and $Q \triangleq u_{Sb}i_{Sa} - u_{Sa}i_{Sb}$ are the real and reactive powers, respectively, whose steady-state expressions are given by (30) and (31) in the Appendix. Using (30) and (31) to replace P and Q in (25), one obtains

$$\begin{aligned}\bar{D} &\triangleq D(t)\,W(t)\\ &= -\frac{|\underline{I}_S|^2(1-\sigma)\,\omega_S L_S}{1+S^2\omega_S^2T_R^2}\begin{bmatrix} S^2\omega_S^2T_R^2 & S\omega_S T_R\\ S\omega_S T_R & 1\end{bmatrix}\end{aligned} \quad (26)$$

$$\begin{aligned}\bar{Y} &\triangleq D(t)\,y(t)\\ &= -\omega_S\frac{|\underline{I}_S|^2(1-\sigma)\,\omega_S L_S}{1+S^2\omega_S^2T_R^2}\begin{bmatrix} S\omega_S T_R\\ 1\end{bmatrix}.\end{aligned} \quad (27)$$

That is, in steady state, $\bar{D} \triangleq D(t)\,W(t) \in \mathbb{R}^{2\times 2}$ and $\bar{Y} \triangleq D(t)\,y(t) \in \mathbb{R}^2$ are *constant* matrices. Further, it is easily seen that the determinant of $\bar{D} \triangleq D(t)\,W(t)$ is zero. Also,

$$\begin{aligned}R_{DW} &\triangleq \sum_{n=1}^{N}\left(D(nT)\,W(nT)\right)^T\left(D(nT)\,W(nT)\right)\\ &= |\underline{I}_S|^2\sum_{n=1}^{N}W^T(nT)W(nT) = |\underline{I}_S|^2 R_W.\end{aligned}$$

R_{DW} is singular because $D(t)\,W(t)$ is constant and singular. It then follows that R_W is also singular using steady-state data. Further,

$$\begin{aligned}R_{DWY} &\triangleq \sum_{n=1}^{N}\left(D(nT)\,W(nT)\right)^T\left(D(nT)\,y(nT)\right)\\ &= |\underline{I}_S|^2\sum_{n=1}^{N}W^T(nT)y(nT) = |\underline{I}_S|^2 R_{YW}.\end{aligned}$$

Thus R_W and R_{YW} are given by

$$\begin{aligned}R_W &= R_{DW}/|\underline{I}_S|^2 = N\bar{D}^T\bar{D}/|\underline{I}_S|^2\\ &= \frac{N|\underline{I}_S|^2(1-\sigma)^2\omega_S^2L_S^2}{1+S^2\omega_S^2T_R^2}\begin{bmatrix} S^2\omega_S^2T_R^2 & S\omega_S T_R\\ S\omega_S T_R & 1\end{bmatrix}\end{aligned} \quad (28)$$

$$\begin{aligned}R_{YW} &= R_{DWY}/|\underline{I}_S|^2 = N\bar{D}^T\bar{Y}/|\underline{I}_S|^2\\ &= \omega_S\frac{N|\underline{I}_S|^2(1-\sigma)^2\omega_S^2L_S^2}{1+S^2\omega_S^2T_R^2}\begin{bmatrix} S\omega_S T_R\\ 1\end{bmatrix},\end{aligned} \quad (29)$$

where again $\bar{D}$ and $\bar{Y}$ are from (26) and (27), respectively.

By inspection of (28) and (29), $K = [0\ \omega_S]^T$ is one solution to (24). The null space of R_W is generated by $[-1/T_R\ S\omega_S]^T$ so that all possible solutions are given by $[0\ \omega_S]^T + \alpha[-1/T_R\ S\omega_S]^T$ for some $\alpha \in \mathbb{R}$. In summary, solving (24) using steady-state data leads to an infinite set of solutions so that T_R is not identifiable using the linear regressor (23) with steady-state data.

V. Experimental Results

To demonstrate the viability of the speed sensorless estimator (18) for T_R, experiments were performed. A three-phase, 0.5 hp, 1735 rpm ($n_p = 2$ pole-pair) induction motor was driven by an Allen-Bradley PWM inverter to obtain the data. Given a speed command to the inverter, it produces PWM voltages to drive the induction motor to the commanded speed. Here a step speed command was chosen to bring the motor from standstill up to the rated speed of 188 rad/s. The stator currents and voltages were sampled at 10 kHz. The real-time computing system RTLAB from Opal-RT with a fully integrated hardware and software system was used to collect data [17]. Filtered differentiation (using digital filters) was used for the derivatives of the voltages and currents. Specifically, the signals were filtered with a third-order Butterworth filter whose cutoff frequency was 100 Hz. The voltages and currents were put through a $3-2$ transformation to obtain their two-phase equivalent values.

Using the data $\{u_{Sa}, u_{Sb}, i_{Sa}, i_{Sb}\}$ collected between 0.84 sec to 0.91 sec, which includes the time the motor accelerates, the quantities du_{Sa}/dt, du_{Sa}/dt, di_{Sa}/dt, di_{Sb}/dt, d^2i_{Sa}/dt^2, d^2i_{Sb}/dt^2, d^3i_{Sa}/dt^3, d^3i_{Sb}/dt^3 are calculated and used to evaluate the coefficients C_i, $i = 1, 2, \cdots, 12$ in equation (18). Solving (18), one obtains the 12 solutions

$$\begin{array}{ll} T_{R1} = +0.1064 & T_{R2} = -0.0186\\ T_{R3} = -0.0576 + j0.0593 & T_{R4} = -0.0576 - j0.0593\\ T_{R5} = -0.0037 + j0.0166 & T_{R6} = -0.0037 - j0.0166\\ T_{R7} = -0.0072 + j0.0103 & T_{R8} = -0.0072 - j0.0103\\ T_{R9} = +0.0125 + j0.0077 & T_{R10} = +0.0125 - j0.0077\\ T_{R11} = +0.0065 + j0.0018 & T_{R12} = +0.0065 - j0.0018. \end{array}$$

T_R must be a real positive number, so $T_R = 0.1064$ is the only possible choice. This value compares favorably with the value of $T_R = 0.11$ obtained using the method of Wang et al [18], which requires a speed sensor.

To illustrate the identified T_R, a simulation of the induction motor model was carried out using the measured voltages as input. Then the simulation's output [stator currents computed according to (1) and (2)] are used to compare with the measured (stator currents) outputs. Figure 1 shows the sampled two-phase equivalent current i_{Sb} and its simulated response i_{Sb-sim}. The phase a current i_{Sa} is similar, but shifted by $\pi/(2n_p)$. The resulting phase b current i_{Sb-sim} from the simulation corresponds well with the actual measured current i_{Sb}. Note that in equation (1) $\gamma = \dfrac{R_S}{\sigma L_S} + \dfrac{\beta M}{T_R}$ also depends on T_R.

VI. Conclusions and Future Work

This paper presented a differential-algebraic approach to the estimation of the rotor time constant of an induction motor without using a speed sensor. The experimental results demonstrated the practical viability of this method. Though the method is not applicable in steady state, neither is a standard linear least-squares approach. Future work includes studying an on-line implementation of the estimation algorithm and using such an online estimate in a speed sensorless field-oriented controller.

VII. Appendix: Steady-State Expressions

In the following, ω_S denotes the stator frequency and S denotes the normalized slip defined by $S \triangleq (\omega_S - n_p\omega)/\omega_S$. With $u_{Sa} + ju_{Sb} = \underline{U}_S e^{j\omega_S t}$ and $i_{Sa} + ji_{Sb} = \underline{I}_S e^{j\omega_S t}$, it is

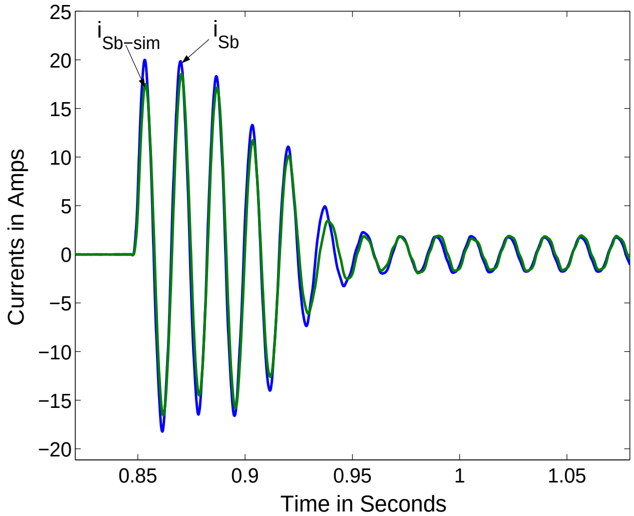

Fig. 1. Phase b current i_{Sb} and its simulated response i_{Sb-sim}.

shown in [19] that under steady-state conditions, the complex phasors $\underline{U}_S$ and $\underline{I}_S$ are related by ($S_p \triangleq \frac{R_R}{\sigma\omega_S L_R} = \frac{1}{\sigma\omega_S T_R}$)

$$\begin{aligned}\underline{I}_S &= \frac{\underline{U}_S}{R_S + j\omega_S L_S\left(\left(1+j\frac{S}{S_p}\right)/\left(1+j\frac{S}{\sigma S_p}\right)\right)}\\ &= \frac{\underline{U}_S}{\left(R_S + \frac{(1-\sigma)S\omega_S^2 L_S T_R}{1+S^2\omega_S^2 T_R^2}\right) + j\frac{\omega_S L_S\left(1+\sigma S^2\omega_S^2 T_R^2\right)}{1+S^2\omega_S^2 T_R^2}},\end{aligned}$$

and straightforward calculations (see [6]) give

$$\begin{aligned}P &\triangleq u_{Sa}i_{Sa} + u_{Sb}i_{Sb} = R_e\left(\underline{U}_S \underline{I}_S^*\right)\\ &= \left|\underline{I}_S\right|^2\left(R_S + \frac{(1-\sigma)S\omega_S^2 L_S T_R}{1+S^2\omega_S^2 T_R^2}\right) \qquad (30)\\ Q &\triangleq u_{Sb}i_{Sa} - u_{Sa}i_{Sb} = I_m\left(\underline{U}_S \underline{I}_S^*\right)\\ &= \left|\underline{I}_S\right|^2 \frac{\omega_S L_S\left(1+\sigma S^2\omega_S^2 T_R^2\right)}{1+S^2\omega_S^2 T_R^2}. \qquad (31)\end{aligned}$$

A. Steady-State Expression for r_1 and r_0

It is now shown that the steady-state value of r_1 in (12) is nonzero. Substituting the steady-state values of q_2, q_1, q_0, a_2, a_1, and a_0 shown in [6] (noting that $\dot{q}_1 \equiv 0$ and $\dot{q}_2 \equiv 0$ in steady state) into (12) gives

$$\begin{aligned}r_1 &= -\left|\underline{I}_S\right|^6\left(\frac{1}{1+S^2\omega_S^2 T_R^2}\right)^3 \frac{n_p^4(1-\sigma)^6 L_S^2}{\sigma^4}\times\\ &\quad \omega_S^3\left(1+T_R^2\omega_S^2(1-S)^2\right)^2\frac{1}{den}\\ r_0 &= \left|\underline{I}_S\right|^6\left(\frac{1}{1+S^2\omega_S^2 T_R^2}\right)^3 \frac{n_p^3(1-\sigma)^6 L_S^2}{\sigma^4}\times\\ &\quad \omega_S^4(1-S)\left(1+\omega_S^2 T_R^2\times(1-S)^2\right)^2\frac{1}{den}\end{aligned}$$

where

$$\begin{aligned}den &\triangleq n_p T_R\left|\underline{I}_S\right|^4\left(\left(\frac{(1-\sigma)}{\sigma T_R}\frac{1+S^2\omega_S^2 T_R^2 - S\omega_S^2 T_R^2}{1+S^2\omega_S^2 T_R^2}\right)^2\right.\\ &\quad \left. + \left(\frac{(1-\sigma)}{\sigma}\frac{\omega_S}{1+S^2\omega_S^2 T_R^2}\right)^2\right). \qquad (32)\end{aligned}$$

Recall from Section III [following (6)] that $den = 0$ if and only if $\left|\underline{\psi}_R\right| = 0$. It is then seen that $r_1 \neq 0$ in steady state.

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