Compensation of Non-Active Current in Power Systems –Definitions from Compensation Standpoint–

Fang Z. Peng, Senior Member, IEEE
Oak Ridge National Laboratory*
P.O. Box 2009, Bldg. 9102-1, Oak Ridge, TN 37831-8038
Phone: 865-576-7261, Fax: 241-6124, Email: pengfz@ornl.gov

Leon M. Tolbert, Senior Member, IEEE
Department of ECE, University of Tennessee, Knoxville
311 Ferris Hall, Knoxville, TN 37996-2100
Phone: 865-974-2881, Fax: 974-5483, Email: tolbert@utk.edu

Abstract: Many definitions have been formulated to characterize, detect, and measure active and non-active current and power for non-sinusoidal and non-periodic waveforms in electric systems. This paper presents definitions and compensation of non-active current from the compensation standpoint.

Keyword: non-active current, non-active power, reactive current, reactive power, harmonic current, harmonic power, active power, apparent power, var compensation

I. Introduction

Because of the widespread use of nonlinear loads and electronic power converters, non-sinusoidal and non-periodic loads and voltage distortion are becoming more common in today's electrical systems. Many papers have dealt with the definition, identification, characterization, detection, measurement, and compensation of such non-sinusoidal and non-periodic current and power [1–11]. Tolbert (co-author of this paper) and Habetler have compiled a comprehensive technical survey of the published literature on the topic [12].

Instantaneous active power is defined as the time rate of energy generation, transfer, or utilization. It is a physical quantity and satisfies the principle of conservation of energy. For a single-phase circuit, it is defined as the instantaneous product of voltage and current:

$$p(t) = v(t)i(t) . \tag{1}$$

Active power P is the time average of the instantaneous power over one period of the wave *p(t)*. For a polyphase circuit with M phases, each phase's instantaneous active power is still expressed as (1) and instantaneous total active power is the sum of the active powers of the individual phases:

$$p(t) = \sum_{i=1}^{M} p_i(t) = \sum_{i=1}^{M} v_i(t)i_i(t) . \tag{2}$$

Non-active power can be thought of as the useless power that causes increased line current and losses, greater generation requirements for utilities, and other effects/burdens to power systems and connected/related equipment.

For a single phase circuit with inductors, capacitors, and/or nonlinear elements, non-active power is the power that circulates back and forth between the source and loads and yields zero average active power over one period of the wave *p(t)*. Therefore, the non-active power for single phase circuits is based on average or rms values. For a polyphase circuit, non-active power is the power that circulates back and forth between the sources and loads and the power that circulates among the phases. The source-load circulating power yields zero average over one period of the wave *p(t)* because of unbalanced storage elements in the circuit. The phase circulating power contributes no total instantaneous active power to the circuits because of balanced storage elements. Therefore, the non-active power for polyphase circuits includes two components: one is based on average or rms values and the other is based on instantaneous value. Some theories are based on average values and restricted to frequency domain, while some others are formulated on time domain and instantaneous base. No matter what mathematical means are used, the goal of these theories is to improve the power factor and to minimize power losses and disturbances by identifying, measuring, and eliminating the useless power. This paper presents definitions of non-active power or non-active current from the compensation standpoint. Almost all existing non-active power theories and definitions can be extended and deduced from the definitions presented.

II. Compensation System and Definition of Non-Active Power

A. Compensation Systems

For a single or polyphase power system, a shunt compensator to minimize the useless power/current can be configured as in Fig. 1. Assuming that the shunt compensator only consists of passive (inductor and/or capacitor) and/or switching devices without any external power source and neglecting the compensator's power loss, then the active power of the com-

* Prepared by the Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-8038, managed by UT-Battelle, LLC for the U. S. Department of Energy under contract DE-AC05-00OR22725.

pensator should average zero according to the principle of conservation of energy. That is,

$$P_S(t) = P_L(t),\ P_C(t) = 0\text{, for } t\to\infty. \tag{3}$$

$$P_X(t) = \frac{1}{T_C}\int_{t-T_C}^{t} p_X(\tau)d\tau \text{ and } X = S, L, \text{or } C. \tag{4}$$

In (4), T_C is the averaging interval that can be zero, one fundamental cycle, one-half cycle, or multiple cycles, depending on compensation objectives and the passive components' energy storage capacity. Equations (3) and (4) must hold true regardless of single-phase or polyphase, passive compensation or active compensation. Based on these physical and practical limitations, non-active power and non-active current can be defined and formulated.

B. Definitions of Non-Active Power and Current

We extend Fryze's idea of non-active current/power [1] as follows:

$$i_p(t) = \frac{P(t)}{V_P^2(t)} v_P(t),\ i_q(t) = i(t) - i_p(t), \tag{5}$$

where

$$V_P(t) = \sqrt{\frac{1}{T_C}\int_{t-T_C}^{t} v_P^2(\tau)d\tau}. \tag{6}$$

$i_p(t)$ is the active current and $i_q(t)$ is the non-active current. $P(t)$ is the average active power over the interval $[t-T_C, t]$, which can be calculated from (4). $V_P(t)$ is the rms value of the voltage, $v_P(t)$ over the interval $[t-T_C, t]$, which is expressed by (6). $v_P(t)$ is the reference voltage that can be the voltage itself, $v_P(t)= v(t)$, or the fundamental component of $v(t)$, where $v(t)= v_f(t)+v_h(t)$ and $v_P(t)= v_f(t)$, or something else, depending on compensation objectives.. These definitions, (5) and (6) are valid for single- and poly-phase circuits. However, for polyphase circuits, voltages and currents are expressed as a vector, e.g., for a three-phase system, $v = [v_a, v_b, v_c]^T$, $i = [i_a, i_b, i_c]^T$, and $v^2 = [v_a, v_b, v_c]\cdot[v_a, v_b, v_c]^T$.

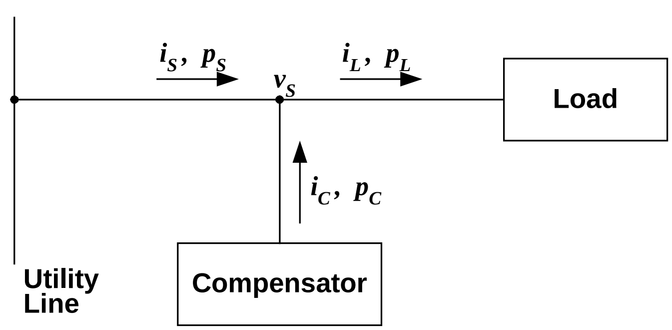


Fig. 1. A shunt compensator configuration.

III. Discussion, Deduction, and Compensation

Equation (5) provides the basic definitions of active and non-active current, from which most of the existing non-active power theories and definitions based on time-domain can be extended and deduced. The following discusses some deductions and compensation examples.

A. Sinusoidal Single-Phase Circuits

For a single phase circuit with sinusoidal waveforms, e.g., $v_S=V_S\sin(\omega t)$ and $i_L=I_L\sin(\omega t-\alpha)$, the active and non-active currents are consistent with the traditional active and reactive currents and can be derived from (4), (5) and (6) by the following steps: (i) choose T_C to be one or half fundamental cycle, $T_C=2\pi/\omega$ or $T_C=\pi/\omega$; (ii) calculate the average active power, P_L according to (4); (iii) calculate the rms value of the voltage v_S, according to (6); (iv) calculate the active current and non-active current, i_{Lp} and i_{Lq} according to (5). The result is

$$i_{Lp}(t) = I_L\cos\alpha\sin(\omega t),\ i_{Lq}(t) = -I_L\sin\alpha\cos(\omega t). \tag{7}$$

Equation (7) clearly shows the consistence with the traditional reactive power theory. An algorithm for active compensators can be easily implemented because the definitions as formulated in (4), (5) and (6) are all real-time based. In addition, it is easy to show that the compensator needs zero average power when it injects the non-active current because $P_C=0$ when $i_C= i_{Lq}$. After compensation, the source surrent, i_S will only contain the load active current, i_{Lp}.

B. Non-Sinusoidal Single-Phase Circuits

For a single-phase circuit with non-sinusoidal waveforms, the active and non-active currents, which can be derived from (4), (5) and (6) using the steps described in section III.A, are consistent with the traditional Fryze's active and non-reactive currents when choosing $v_P=v_S$ in (5) and (6). For example, if $v_S=v_f(t)+v_h(t)=V_{Sf}\sin(\omega t)+V_{Sh}\sin(\omega_h t+\beta_h)$ and $i_L=I_{Lf}\sin(\omega t-\alpha)+ I_{Lh}\sin(\omega_h t+\beta_h-\alpha_h)$ then

$$i_{Lp}(t) = \frac{V_{Sf}I_{Lf}\cos\alpha + V_{Sh}I_{Lh}\cos\alpha_h}{V_{Sf}^2 + V_{Sh}^2}\cdot \left[V_{Sf}\sin(\omega t) + V_{Sh}\sin(\omega_h t+\beta_h)\right], \tag{8}$$

$$i_{Lq}(t) = -I_{Lf}\sin\alpha\cos(\omega t) - I_{Lh}\sin\alpha_h\cos(\omega_h t+\beta_h) + \frac{V_{Sh}^2 I_{Lf}\cos\alpha - V_{Sf}V_{Sh}I_{Lh}\cos\alpha_h}{V_{Sf}^2+V_{Sh}^2}\cdot\sin(\omega t) + \frac{V_{Sf}^2 I_{Lh}\cos\alpha_h - V_{Sf}V_{Sh}I_{Lf}\cos\alpha}{V_{Sf}^2+V_{Sh}^2}\cdot\sin(\omega_h t+\beta_h). \tag{9}$$

Again, the average power of i_{Lq} is zero, which satisfies the requirements in (3) for a compensator. However, it is observed from (8) that the active current contains harmonics because of the voltage distortion, which means that the source current will not become sinusoidal after the compensator injects (compensates) the non-active current expressed in (9). In most cases, it is desirable that compensation of non-active current results in a pure sine wave source current. In order to achieve that, one should choose $v_P = v_{Sf}$ in (5) and (6). By doing so, one has

$$i_{Lp}(t) = \left(I_{Lf}\cos\alpha + \frac{V_{Sh} I_{Lh}\cos\alpha_h}{V_{Sf}} \right)\cdot\sin(\omega t)\,, \tag{10}$$

$$i_{Lq}(t) = -I_{Lf}\sin\alpha\cos(\omega t) - \frac{V_{Sh} I_{Lh}}{V_{Sf}}\cos\alpha_h \cdot \sin(\omega t) + I_{Lh}\sin(\omega_h t + \beta_h - \alpha_h). \tag{11}$$

Equation (10) shows that the active current is a sine wave, and (11) shows that the non-active current contains all harmonic current and fundamental reactive current. After compensation, the source current will become sinusoidal and active. In addition, it is noticeable that the active and non-active currents expressed in (10) and (11) still meet the compensation energy conservation requirements in (3). This is a very important property of the definitions given in (4), (5), and (6), which is also necessary in order to implement compensation in Fig. 1.

C. Single-Phase Circuits with Non-Sinusoidal and Non-Periodic Current

It is more convenient using simulations to study a load current with non-sinusoidal and non-periodic waveform. Fig. 2 shows a case study, where the load generates a non-periodic pulse current. The calculation is based on (4), (5), and (6) with T_C = one-half fundamental cycle. The simulation results clearly demonstrate the following points: (1) the active load current, ILp, is sinusoidal and in phase with the voltage although the load current, IL, is highly distorted and non-periodic; (2) the calculated load non-active current, ILq, is highly distorted and out of phase with the source voltage; (3) the average load power, PLq, generated from the load non-active current is zero. Therefore, when a compensator is used to compensate for the load non-active current it will consume average zero power and maintain the requirement in (3). In addition, the source current will become sinusoidal and in phase with the voltage.

D. Three-Phase Circuits

The definitions described in (5) and (6) are valid for a three-phase system as well regardless of whether the voltage and current waveforms are sinusoidal or non-sinusoidal, periodic or non-periodic, and balanced or unbalanced. Results are similar to those in the previous subsections. For polyphase (*M*-phase) systems, there is one interesting concept, instantaneous reactive (or non-active) power and current, which do not exist in single phase situations. This instantaneous non-active power theory [2, 3, 10] can be deduced from (5) and (6) as well.

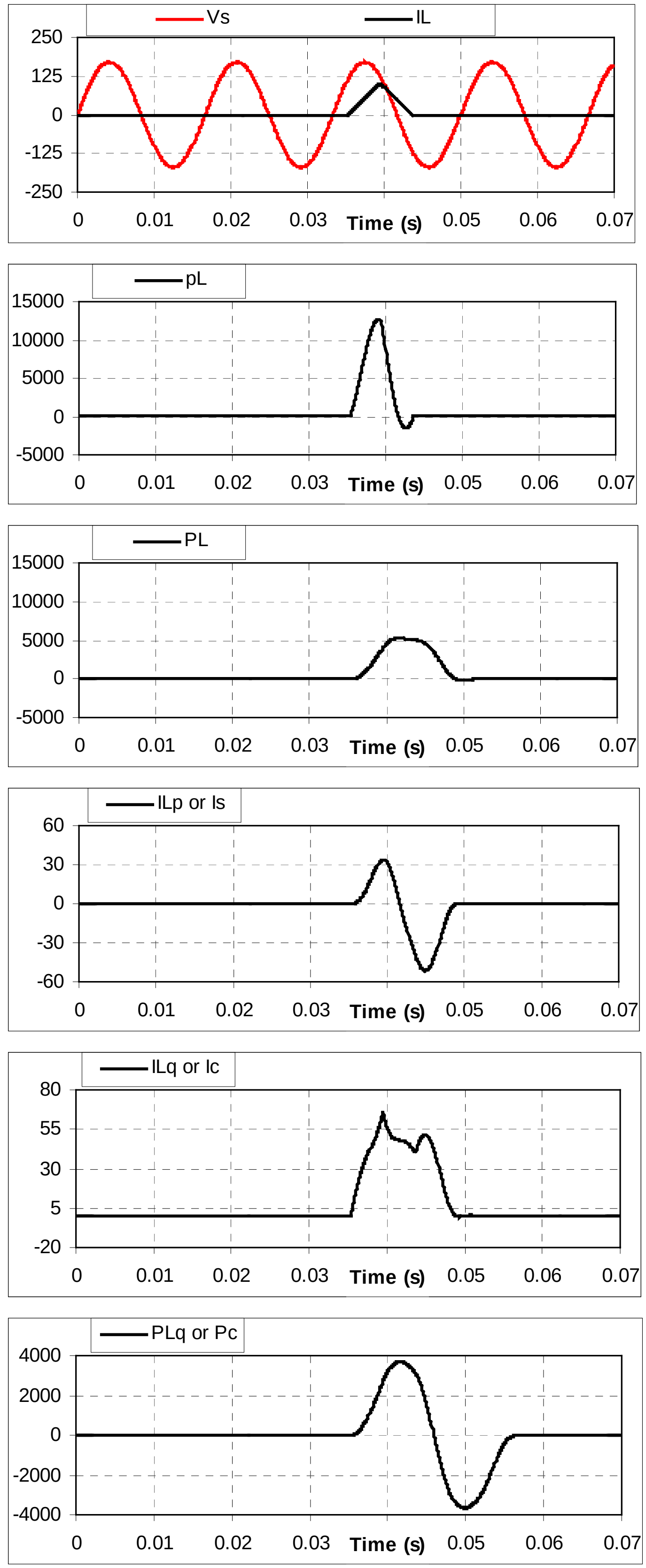


Fig. 2. Simulation results for non-periodic current compensation.

In (4) and (6), choosing $T_C \rightarrow 0$ yields the instantaneous non-active current (power) theory. That is,

$$i_p(t) = \frac{p(t)}{V_P^2(t)} v_P(t),\ i_q(t) = i(t) - i_p(t), \tag{12}$$

where

$$v_P(t) = [v_{1P}, v_{2P}, \cdots\ v_{MP}\ ]^T,\ i(t) = [i_1, i_2, \cdots\ i_M\ ]^T, \text{ and} \tag{13}$$

$$V_P^2(t) = v_P^2(t) = v_{1P}^2(t) + v_{2P}^2(t) + \cdots + v_{MP}^2(t)\,. \tag{14}$$

Again, the definitions given in (4), (5) and (6) have great flexibility to meet all compensation objectives. Regardless of whether the voltage and current are sinusoidal or non-sinusoidal, balanced or unbalanced, and periodic or non-periodic, the definitions give means to calculate any non-active current component that requires compensation. Consider a three phase four wire system (Fig. 3): $v_S = [v_{Sa}, v_{Sb}, v_{Sc}]^T$, $i_L = [i_{La}, i_{Lb}, i_{Lc}]^T$, and $i_C = [i_{Ca}, i_{Cb}, i_{Cc}]^T$. If the compensation objective is to make the source current sinusoidal and balanced, one can calculate compensation current as follows:

- Separate the voltage into four components: fundamental positive-sequence, v_{Sf+}; fundamental negative-sequence, v_{Sf-}; zero sequence, v_{S0}; and harmonic component, v_{Sh}, i.e.,

  $v_S = v_{Sf+} + v_{Sf-} + v_{S0} + v_{Sh}$, and $v_{S0} = \frac{1}{3}(v_{Sa} + v_{Sb} + v_{Sc}) \cdot [1, 1, 1]^T$. (15)

- Choose T_C=one-half or one fundamental cycle and $v_P = v_{Sf+}$ in (4), (5) and (6), i.e.,

  $$T_C = T/2\text{, or } T_C = T \text{ and } v_P = v_{Sf+}\,. \tag{16}$$

- Calculate the load non-active current, i_{Lq}, as (5) and let $i_C = i_{Lq}$.

From (4), (5), (6), (15), and (16), it is easy to show that the compensator consumes zero average power, P_C=0, and satisfies the requirements in (3). Fig. 4 shows simulation results of a case in which both the source voltage and load current are distorted and unbalanced and contain zero-sequence. The results clearly show that: (1) the source current, Isa, Isb, and Isc, becomes sinusoidal; (2) the source current becomes balanced as indicated by zero source neutral current, Isn; and (3) the average power of the compensator, Pc, equals zero.

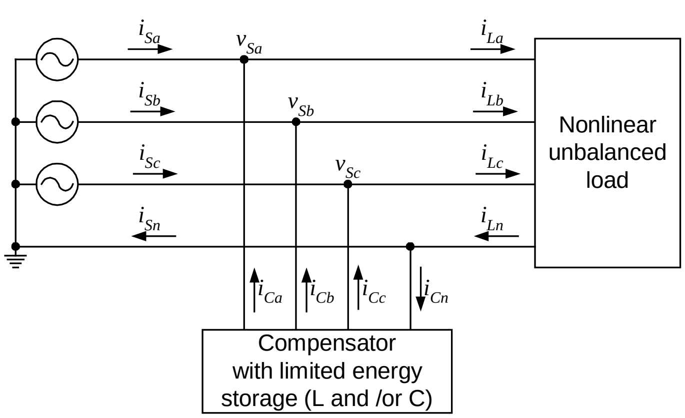


Fig. 3. A compensation system for three-phase four-wire system.

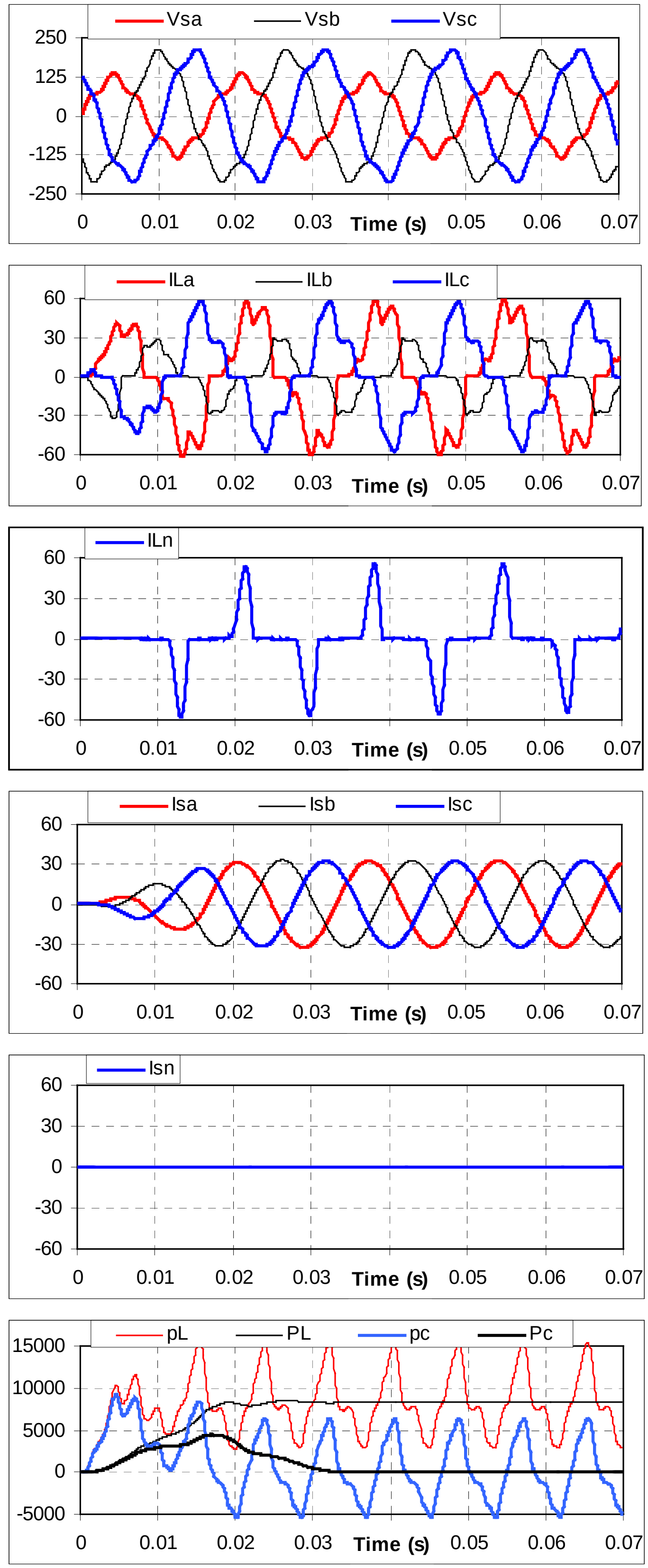


Fig. 4. Simulation results for a distorted and unbalanced three-phase four-wire system.

IV. CONCLUSIONS

In this paper, definitions of active and non-active power and current have been given from the compensation standpoint. Their definitions are consistent with the traditional reactive and non-active concept for single phase circuits. In addition, the instantaneous reactive power theories can be deduced from the proposed definitions for polyphase systems. The definitions also have the flexibility that any compensation objective can be achieved by choosing an appropriate averaging interval (T_C) and reference voltage.

REFERENCES

[1] S. Fryze, "Active, Reactive, and Apparent Power in Non-Sinusoidal Systems," Przeglad Elektrot, no. 7, pp. 193-203, 1931. (in Polish)

[2] H. Akagi, Y. Kanazawa, A. Nabae, "Instantaneous Reactive Power Compensators Comprising Switching Devices without Energy Storage Components," *IEEE Trans. Ind. Appl.*, vol.20, pp.625–630, May/June 1984.

[3] Jacques L. Willems, "A New Interpretation of the Akagi-Nabae Power Components of Nonsinusoidal Three-Phase Situations," *IEEE Trans. Instrum. Meas.*, vol.41, no.4, August 1992.

[4] A. Ferrero and G. Superti-Furga, "A New Approach to the Definition of Power Components in Three-Phase Systems Under Nonsinusoidal Conditions," *IEEE Trans. Instrum. Meas.*, vol.40, no.3, June 1991.

[5] L. S. Czarnecki, "Orthogonal Decomposition of the Currents in a 3-phase Nonlinear Asymmetrical Circuit with a Nonsinusoidal Voltage Source," *IEEE Trans. Instrum. Meas.*, vol.37, no.1, March 1988.

[6] ——, "Scattered and Reactive Current, Voltage, and Power in Circuit with Nonsinusoidal Waveforms and Their Compensation," *IEEE Trans. Instrum. Meas.*, vol.40, no.3, June 1991.

[7] A. Nabae, et al, "Reactive Power Compensation Based on Neutral Line Current Separating and Combining Method," Transactions of IEE of Japan, Vol. 114-D, No.7/8, July/August 1994. pp. 800–801.(in Japanese)

[8] A. Nabae and T. Tanaka, "A New Definition of Instantaneous Active-Reactive Current and Power Based on Instantaneous Space Vectors on Polar Coordinates in Three-Phase Circuits," IEEE/PES Winter Meeting, Paper No. 96 WM 227-9 PWRD, 1996.

[9] L. Rossetto and P. Tenti, "Evaluation of Instantaneous Power Terms in Multi-Phase Systems: Techniques and Application to Power Conditioning Equipment," *ETEP* Vol. 4, No. 6, November/December 1994.

[10] F. Z. Peng and J. S. Lai, "Generalized Instantaneous Reactive Power Theory for Three-Phase Power Systems." *IEEE Trans. Instrum. Meas.*, vol.45, no.1, pp.293–297, February 1996.

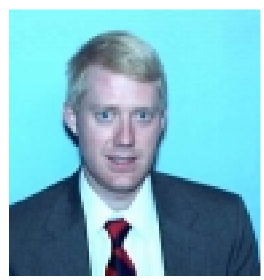
[11] F. Z. Peng and J. S. Lai, "Reactive Power and Harmonic Compensation Based on the Generalized Instantaneous Reactive Power Theory for Three-Phase Power Systems," Proceedings of the 7th International Conference on Harmonics and Quality of Power, pp. 83–89, Las Vegas, NV, October 16–18, 1996.

[12] L. M. Tolbert and T. G. Habetler, "Survey of Active and Non-Active Power Definition," IEEE International Power Electronics Congress, Acapulco Mexico, 2000.

V. BIOGRAPHIES

Fang Z. Peng (IEEE Member'93, Senior Member'96) received the B.S. degree in electrical engineering from Wuhan University of Hydraulic and Electrical Engineering, China, in 1983 and the M.S. and Ph.D. Degrees in electrical engineering from Nagaoka University of Technology, Japan, in 1987 and 1990, respectively.

He joined Toyo Electric Manufacturing Company, Ltd., from 1990 to 1992 as a research scientist, was engaged in research and development of active power filters, flexible ac transmission systems (FACTS) applications and motor drives. From 1992 to 1994, he worked with Tokyo Institute of Technology as a Research Assistant Professor, initiated a multilevel inverter program for FACTS applications and a speed-sensorless vector control project. From 1994 to 1997, he was a Research Assistant Professor at University of Tennessee, working for Oak Ridge National Laboratory (ORNL). Since 1997, he has been a staff member at ORNL, Lead (principal) Scientist of the Power Electronics and Electric Machinery Research Center. Dr. Peng received many awards including the 1996 First Prize Paper Award and the 1995 Second Prize Paper Award of Industrial Power Converter Committee in IEEE/IAS Annual Meeting; the 1996 Advanced Technology Award of the Inventors Clubs of America, Inc., the International Hall of Fame; the 1991 First Prize Paper Award in IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS; and the 1990 Best Paper Award in the Transactions of the IEE of Japan, the Promotion Award of Electrical Academy.



Leon M. Tolbert (S'88-M'91-SM'98) received the B.E.E., M.S., and Ph.D. degrees in electrical engineering from the Georgia Institute of Technology, Atlanta.

He joined the Engineering Division of Lockheed Martin Energy Systems in 1991 and worked on several electrical distribution projects at the three U.S. Department of Energy plants in Oak Ridge, TN. In 1997, he became a research engineer in the Power Electronics and Electric Machinery Research Center at the Oak Ridge National Laboratory. In 1999, he was appointed as an assistant professor in the Department of Electrical and Computer Engineering at the University of Tennessee, Knoxville.

Dr. Tolbert is a Registered Professional Engineer in the state of Tennessee.