Eliminating Harmonics in a Multilevel Converter using Resultant Theory

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Abstract— **A method is given to determine conditions for which the switching angles in a multilevel converter can be chosen to produce the required fundamental voltage while at the same time cancel out higher order harmonics. A complete analysis is given for a 7– level converter where it is shown that for a range of the modulation index m_I, the switching angles can be chosen to produce the desired fundamental $V_1 = m_I(s4V_{dc}/\pi)$ while making the 5th and 7th harmonics identically zero.**

Keywords— **multilevel inverter, multilevel converter, resultants, hybrid electric vehicle, motor drive, cascade inverter**

I. Introduction

Designs for heavy duty hybrid-electric vehicles (HEVs) that have large electric drives such as tractor trailers, transfer trucks, or military vehicles will require advanced power electronic inverters to meet the high power demands (> 100 kW) required of them. Development of large electric drive trains for these vehicles will result in increased fuel efficiency, lower emissions, and likely better vehicle performance (acceleration and braking).

Transformerless multilevel inverters are uniquely suited for this application because of the high VA ratings possible with these inverters [9]. The multilevel voltage source inverter's unique structure allows it to reach high voltages with low harmonics without the use of transformers or series-connected, synchronized-switching devices. The general function of the multilevel inverter is to synthesize a desired voltage from several levels of dc voltages. For this reason, multilevel inverters can easily provide the high power required of a large electric traction drive. For parallel-configured HEVs, a cascaded H-bridges inverter can be used to drive the traction motor from a set of batteries, ultracapacitors, or fuel cells. The use of a cascade inverter also allows the HEV drive to continue to operate even with the failure of one level of the inverter structure [14][15][16].

Multilevel inverters also have several advantages with respect to hard-switched two-level pulse width modulation (PWM) adjustable-speed drives (ASDs). Motor damage and failure have been reported by industry as a result of some ASD inverters' high voltage change rates (dV/dt), which produced a common-mode voltage across the motor windings. High-frequency switching can exacerbate the problem because of the numerous times this common mode voltage is impressed upon the motor each cycle. The main problems reported have been "motor bearing failure" and "motor winding insulation breakdown" because of circulating currents, dielectric stresses, voltage surge, and corona discharge [1][4][13].

II. Cascaded H-bridges

Cascade multilevel inverter consists of a series of H-bridge (single-phase full-bridge) inverter units. The general function of this multilevel inverter is to synthesize a desired voltage from several separate dc sources (SDCSs), which may be obtained from batteries, fuel cells, or ultracapacitors in a HEV. Figure 1 shows a single-phase structure of a cascade inverter with SDCSs [9]. Each SDCS is con-
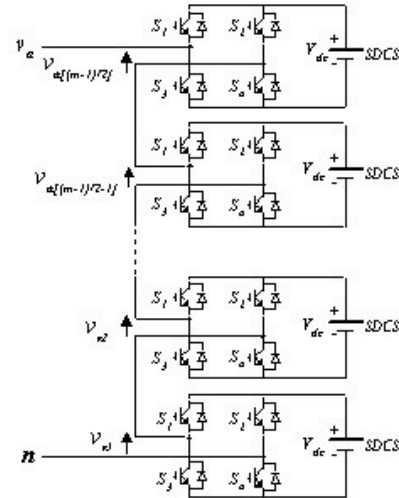

Fig. 1.

nected to a single-phase full-bridge inverter. Each inverter level can generate three different voltage outputs, $+V_{dc}, 0$ and $-V_{dc}$ by connecting the dc source to the ac output side by different combinations of the four switches, S_1, S_2, S_3 and S_4. The ac output of each level's full-bridge inverter is connected in series such that the synthesized voltage waveform is the sum of all of the individual inverter outputs. The number of output phase voltage levels in a cascade multilevel inverter is then $2s+1$, where s is the number of dc sources. An example phase voltage waveform for an 11-level cascaded multilevel inverter with five SDCSs ($s = 5$) and five full bridges is shown in Figure 2. The output phase voltage is given by $v_{an} = v_{a1} + v_{a2} + v_{a3} + v_{a4} + v_{a5}$.
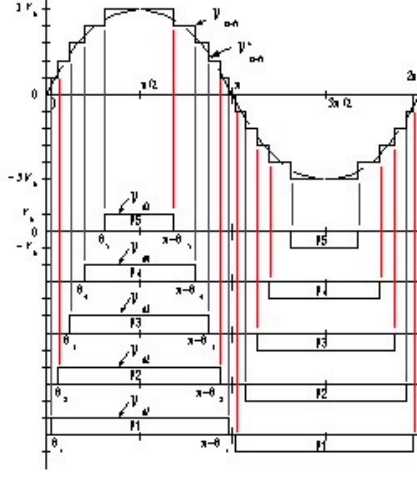
Fig. 2.

With enough levels and an *appropriate* switching algorithm, the multilevel inverter results in an output voltage that is almost sinusoidal. For the 11 - level example shown in Figure 2, the waveform has less than 5% THD with each of the active devices of the H-bridges active devices switching only at the fundamental frequency. Each H-bridge unit generates a quasi-square waveform by phase-shifting its positive and negative phase legs' switching timings. Each switching device always conducts for $180°$ (or $\frac{1}{2}$ cycle) regardless of the pulse width of the quasi-square wave so that this switching method results in equalizing the current stress in each active device.

III. Switching Algorithm for the Multilevel Converter

The Fourier series expansion of the (stepped) output voltage waveform of the multilevel inverter as shown in Figure 2 is [14][15][16]

$$V(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty}$$

$$\frac{4V_{dc}}{n\pi} \left(\cos(n\theta_1) + \cos(n\theta_2) + \cdots + \cos(n\theta_s)\right) \sin(n\omega t)$$
(1)

where s is the number of dc sources. Ideally, given a desired fundamental voltage V_1, one wants to determine the switching angles $\theta_1, \cdots, \theta_n$ so that (1) becomes $V(\omega t) = V_1 \sin(\omega t)$. In practice, one is left with trying to do this approximately. Two predominate methods in choosing the switching angles $\theta_1, \cdots \theta_n$ are (1) eliminate the lower frequency dominant harmonics, or (2) minimize the total harmonic distortion. The more popular and straightforward of the two techniques is the first, that is, eliminate the lower dominant harmonics and filter the output to remove the higher residual frequencies. Here, the choice is also to eliminate the lower frequency harmonics.

The goal here is to choose the switching angles $0 \leq \theta_1 < \theta_2 < \cdots < \theta_s \leq \pi/2$ so as to make the first harmonic equal to the desired fundamental voltage V_1 and specific higher harmonics of $V(\omega t)$ equal to zero. As the application of interest here is a three-phase motor drive, the triplen harmonics in each phase need not be canceled as they automatically cancel in the line-to-line voltages. Consequently, the desire here is to cancel the $5^{th}, 7^{th}, 11^{th}, 13^{th}$ order harmonics as they dominate the total harmonic distortion.

The mathematical statement of these conditions is then

$$\frac{4V_{dc}}{\pi}\left(\cos(\theta_1) + \cos(\theta_2) + \cdots + \cos(\theta_s)\right) = V_1$$
$$\cos(5\theta_1) + \cos(5\theta_2) + \cdots + \cos(5\theta_s) = 0$$
$$\cos(7\theta_1) + \cos(7\theta_2) + \cdots + \cos(7\theta_s) = 0 \quad (2)$$
$$\cos(11\theta_1) + \cos(11\theta_2) + \cdots + \cos(11\theta_s) = 0$$
$$\cos(13\theta_1) + \cos(13\theta_2) + \cdots + \cos(13\theta_s) = 0.$$

This is a system of 5 transcendental equations in the unknowns $\theta_1, \theta_2, \cdots, \theta_s$ so that at least 5 steps are needed ($s = 5$) if there is to be any chance of a solution. One approach to solving this set of nonlinear transcendental equations (2) is to use an iterative method such as the Newton-Raphson method [3][14][15][16]. The correct solution to the conditions (2) would mean that the output voltage of the $11-$level inverter would not contain the $5^{th}, 7^{th}, 11^{th}$ and 13th order harmonic components.

The fundamental question is "When does the set of equations (2) have a solution?". As will be shown below, it turns out that a solution exists for only specific ranges of the modulation index[1] $m_I \triangleq V_1/(s4V_{dc}/\pi)$. This range does not include the low end or the high end of the modulation index. A method is now presented to find the solutions when they exist. This method is based on the theory of resultants of polynomials [5]. To proceed, let $s = 5$, and define

$$x_1 = \cos(\theta_1)$$
$$x_2 = \cos(\theta_2)$$
$$x_3 = \cos(\theta_3)$$
$$x_4 = \cos(\theta_4)$$
$$x_5 = \cos(\theta_5).$$

Using the trigonometric identities

$$\cos(5\theta) = 5\cos(\theta) - 20\cos^3(\theta) + 16\cos^5(\theta)$$
$$\cos(7\theta) = -7\cos(\theta) + 56\cos^3(\theta) - 112\cos^5(\theta)$$
$$+ 64\cos^7(\theta)$$
$$\cos(11\theta) = -11\cos(\theta) + 220\cos^3(\theta) - 1232\cos^5(\theta) +$$
$$2816\cos^7(\theta) - 2816\cos^9(\theta) + 1024\cos^{11}(\theta)$$
$$\cos(13\theta) = 13\cos(\theta) - 364\cos^3(\theta) + 2912\cos^5(\theta) -$$
$$9984\cos^7(\theta) + 16640\cos^9(\theta) -$$
$$13312\cos^{11}(\theta) + 4096\cos^{13}(\theta)$$

[1]Each inverter has a dc source of V_{dc} so that the maximum output voltage of the multilevel inverter is sV_{dc}. A square wave of amplitude sV_{dc} results in the maximum fundamental output possible of $V_{1\,max} = 4sV_{dc}/\pi$. The modulation index is therefore $m_I \triangleq V_1/V_{1\,max} = V_1/(s4V_{dc}/\pi)$.

the conditions (2) become

$$p_1(x) \triangleq x_1 + x_2 + x_3 + x_4 + x_5 - m = 0$$

$$p_5(x) \triangleq \sum_{i=1}^{5} \left(5x_i - 20x_i^3 + 16x_i^5\right) = 0$$

$$p_7(x) \triangleq \sum_{i=1}^{5} \left(-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7\right) = 0$$

$$p_{11}(x) \triangleq \sum_{i=1}^{5} \left(-11x_i + 220x_i^3 - 1232x_i^5 + \right.$$
$$\left. 2816x_i^7 - 2816x_i^9 + 1024x_i^{11}\right) = 0$$

$$p_{13}(x) \triangleq \sum_{i=1}^{5} \left(13x_i - 364x_i^3 + 2912x_i^5 - 9984x_i^7 \right.$$
$$\left. +16640x_i^9 - 13312x_i^{11} + 4096x_i^{13}\right) = 0$$

$$(3)$$

where $x = (x_1, x_2, x_3, x_4, x_5)$ and $m \triangleq V_1/(4V_{dc}/\pi)$. This is a set of five equations in the five unknowns x_1, x_2, x_3, x_4, x_5. The interest here is to find solutions x for $m \in [0, s]$ which satisfy $0 \le x_5 < \cdots < x_2 < x_1 \le 1$. This development has resulted in a set of polynomial equations rather than trigonometric equations. Though the degree is high, the theory of resultants of polynomials [5] provides a systematic way to determine *all* the zeros of the set of polynomials (3).

A. Seven Level Case

To illustrate the procedure of using the theory of resultants to solve the system (3), the seven level case is considered. The conditions are

$$p_1(x) \triangleq x_1 + x_2 + x_3 - m = 0, \quad m \triangleq \frac{V_1}{4V_{dc}/\pi} = sm_I$$

$$p_5(x) \triangleq \sum_{i=1}^{3} \left(5x_i - 20x_i^3 + 16x_i^5\right) = 0 \qquad (4)$$

$$p_7(x) \triangleq \sum_{i=1}^{3} \left(-7x_i + 56x_i^3 - 112x_i^5 + 64x_i^7\right) = 0.$$

Substitute $x_3 = m - (x_1 + x_2)$ into p_5, p_7 to get

$$p_5(x_1, x_2) = 5x_1 - 20x_1^3 + 16x_1^5 + 5x_2 - 20x_2^3 + 16x_2^5$$
$$+5(m - x_1 - x_2) - 20(m - x_1 - x_2)^3$$
$$+16(m - x_1 - x_2)^5$$

$$p_7(x_1, x_2) = -7x_1 + 56x_1^3 - 112x_1^5 + 64x_1^7 - 7x_2$$
$$+56x_2^3 - 112x_2^5 + 64x_2^7 - 7(m - x_1 - x_2)$$
$$+56(m - x_1 - x_2)^3 - 112(m - x_1 - x_2)^5$$
$$+64(m - x_1 - x_2)^7$$

The goal here is to find solutions of

$$p_5(x_1, x_2) = 0$$
$$p_7(x_1, x_2) = 0.$$

For each fixed x_1, $p_5(x_1, x_2)$ can be viewed as a polynomial of (at most) degree 5 in x_2 whose coefficients are polynomials of (at most) degree 5 in x_1. For example[2],

$$p_5(x_1, x_2) = 5m - 20m^3 + 16m^5 + 60m^2 x_1 - 80m^4 x_1$$
$$-60mx_1^2 + 160m^3 x_1^2 - 160m^2 x_1^3 + 80mx_1^4$$
$$+ \left[60m^2 - 80m^4 - 120mx_1 + 320m^3 x_1 \right.$$
$$\left. +60x_1^2 - 480m^2 x_1^2 + 320mx_1^3 - 80x_1^4\right] x_2$$
$$+ \left[-60m + 160m^3 + 60x_1 \right.$$
$$\left. -480m^2 x_1 + 480mx_1^2 - 160x_1^3\right] x_2^2$$
$$+ \left[-160m^2 + 320mx_1 - 160x_1^2\right] x_2^3$$
$$+[80m - 80x_1]x_2^4$$

This is often written as $p_5(x_1, x_2) \in \Re[x_1](x_2)$ to emphasize that p_5 is being viewed as a polynomial in x_2 whose coefficients are in the ring of polynomials $\Re[x_1]$. Similarly, $p_7(x_1, x_2) \in \Re[x_1](x_2)$ is a polynomial of degree 7 in x_2 whose coefficients are polynomials of (at most) degree 7 in x_1.

A pair (x_{10}, x_{20}) is a simultaneous solution of $p_5(x_{10}, x_{20}) = 0, p_7(x_{10}, x_{20}) = 0$, if and only if the corresponding resultant polynomial $r_{5,7}(x_{10}) = 0$. (The reader is referred to [5] for an explanation of resultants and their computation.) Consequently, finding the roots of the resultant polynomial $r_{5,7}(x_1) = 0$ gives candidate solutions for x_1 to check for common zeros of $p_5 = p_7 = 0$. Here, the resultant polynomial $r_{5,7}(x_1)$ of the pair $\{p_5(x_1, x_2), p_7(x_1, x_2)\}$ was found with MATHEMATICA® using the `Resultant` command. The polynomial $r_{5,7}(x_2)$ turned out to be a 22nd order polynomial. The algorithm is as follows:

Algorithm for the 7 Level Case
1. Given m, find the roots of $r_{5,7}(x_1) = 0$.
2. Discard any roots that are less than zero, greater than 1 or that are complex. Denote the remaining roots as $\{x_{1i}\}$.
3. For each fixed zero x_{1i} in the set $\{x_{1i}\}$, substitute it into p_5 and solve for the roots of $p_5(x_{1i}, x_2) = 0$.
4. Discard any roots (in x_2) that are complex, less than zero or greater than one. Denote the pairs of remaining roots as $\{(x_{1j}, x_{2j})\}$.
5. Compute $m - x_{1j} - x_{2j}$ and discard any pair (x_{1j}, x_{2j}) that makes this quantity negative or greater than one. Denote the triples of remaining roots as $\{(x_{1k}, x_{2k}, x_{3k})\}$.
6. Discard any triple for which $x_{3k} < x_{2k} < x_{1k}$ does not hold. Denote the remaining triples as $\{(x_{1l}, x_{2l}, x_{3l})\}$. The switching angles that are a solution to the three level system (4) are

$$\{(\theta_{1l}, \theta_{2l}, \theta_{3l})\} = \{\left(\cos^{-1}(x_{1l}), \cos^{-1}(x_{2l}), \cos^{-1}(x_{3l})\right)\}.$$

A.1 Minimization of the 5th and 7th Harmonic Components

For those values of m for which $p_5(x_1, x_2), p_7(x_1, x_2)$ do not have common zeros satisfying $0 \le x_1 \le 1, 0 \le x_2 \le 1$,

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[2]In this case, it turns out that the coefficient of the x_2^5 is zero so that $p_5(x_1, x_2)$ has degree 4 in x_2.

the next best thing is to minimize the error

$$c(x_1, x_2) = p_5^2(x_1, x_2)/25 + p_7^2(x_1, x_2)/49.$$

This was accomplished by simply computing the values of $c(j\Delta x, k\Delta y)$ for $j, k = 0, 1, 2, ..., 1000$ with $\Delta x = .001, \Delta y = .001$ and then choosing the minimum value.

A.2 Results for the 7 Level Inverter

The results are summarized in Figures 3, 4 and 5. These three figures show the switching angles $\theta_1, \theta_2, \theta_3$ vs. m for those values of m in which the system (4) has a solution. Note that for m in the range from approximately 1.49 to 1.85, there are two different sets of solutions that solve (4). On the other hand, for $m \in [0, 0.8]$, $m \in [0.83, 1.15]$ and $m \in [2.52, 2.77]$ there are *no* solutions to (4). Interestingly, for $m \approx 0.8, m \approx 0.82$ and $m \approx 2.76$ there are (isolated) solutions.
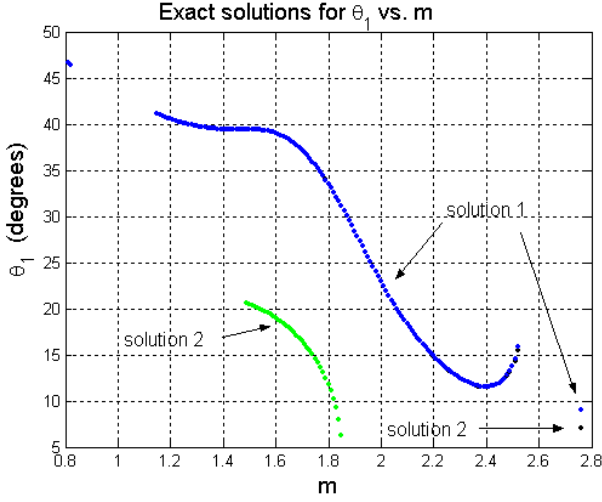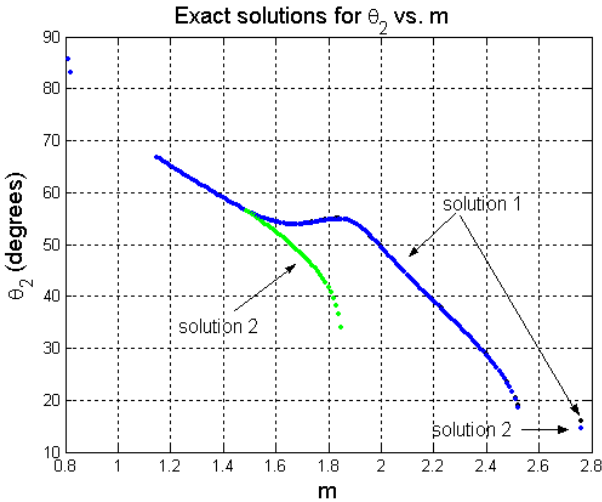
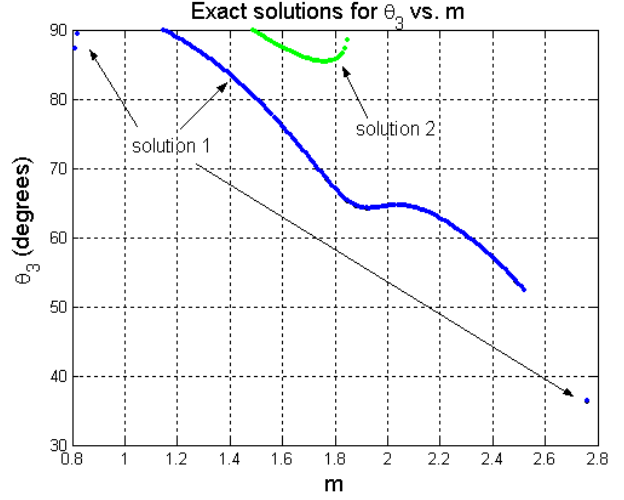Fig. 5. θ_3 vs m

Fig. 3. θ_1 vs m

Fig. 4. θ_2 vs m

As pointed out above, for $m \in [0, 0.8]$, $m \in [0.83, 1.15]$, $m \in [2.52, 2.77]$ and $m \in [2.78, 3]$ there are no solutions satisfying the conditions (4). Consequently, for these ranges of m, the switching angles were determined by minimizing $\sqrt{(p_5/5)^2 + (p_7/7)^2}$. Figure 6 shows a plot of the resulting minimum error $\sqrt{(p_5/5)^2 + (p_7/7)^2}$ vs. m for these values of m. As Figure 6 shows, when $m \approx 0.81$ and $m \approx 2.76$, the error is zero corresponding to the isolated solutions to (4) for those values of m. For $m = 1.15$ and $m = 2.52$, the error goes to zero because these values correspond to the boundary of the exact solutions of (4). However, note, e.g., when $m = 0.25$, the error is about 0.25, that is, the error is the same size as m. Other than close to the endpoints of the two intervals $[0, 0.8], [2.78, 3]$ the minimum error $\sqrt{(p_5/5)^2 + (p_7/7)^2}$ is too large to make the corresponding switching angles for this interval of any use. Consequently, for m in this interval, one must use some other approach (e.g., PWM) in order to get reduced harmonics. For the other two intervals $[0.83, 1.15], [2.52, 2.77]$, the minimum error $\sqrt{(p_5/5)^2 + (p_7/7)^2}$ is around 5% or less so that it might be satisfactory to use the corresponding switching angles for these intervals.

IV. Experimental Work

A prototype three-phase 11-level wye-connected cascaded inverter has been built using 100 V, 70 A MOS-FETs as the switching devices [19]. A battery bank of 15 SDCSs of 48 Volts DC each feed the inverter (5 SDCSs per phase). In the experimental study here, this prototype system was configured to be a 7-level (3 SDCSs per phase) converter with each level being 12 Volts. A 50 pin ribbon cable provides the communication link between the gate driver board and the real-time processor. In this work, the OPALRT[R] real-time computing platform [8] was used to interface the computer (which generates the logic signals) to this cable. The OPALRT[R] system allows one to write
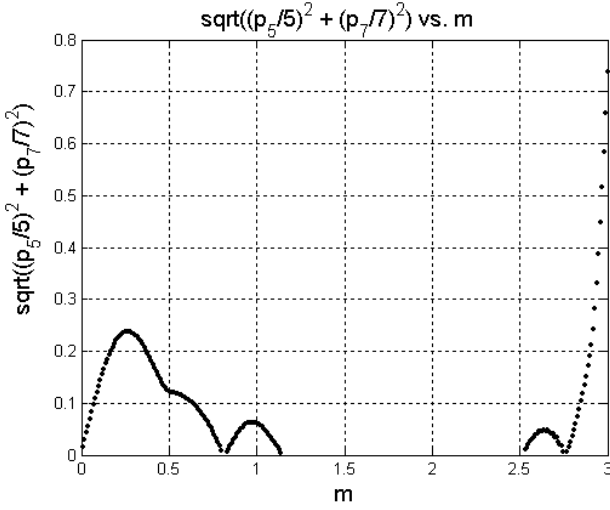
Fig. 6. Error $= \sqrt{(p_5/5)^2 + (p_7/7)^2}$ vs. m

of the 5th harmonic and a 0.15 normalized magnitude of the 7th harmonic for a total normalized distortion of $\sqrt{(0.225)^2 + (0.15)^2} = 0.27$ due to these two harmonics. Figure 6 shows an error of about 0.125 at $m = 0.5$ for a normalized magnitude of $0.125/0.5 = 0.25$ because of these two harmonics, which is in close agreement.
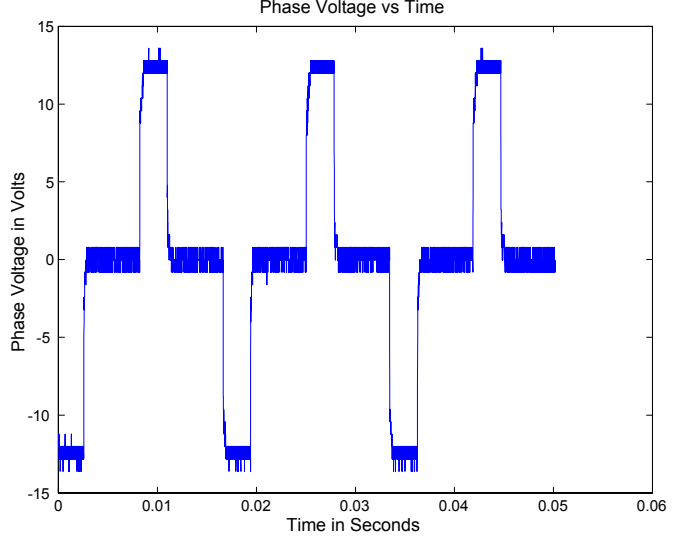

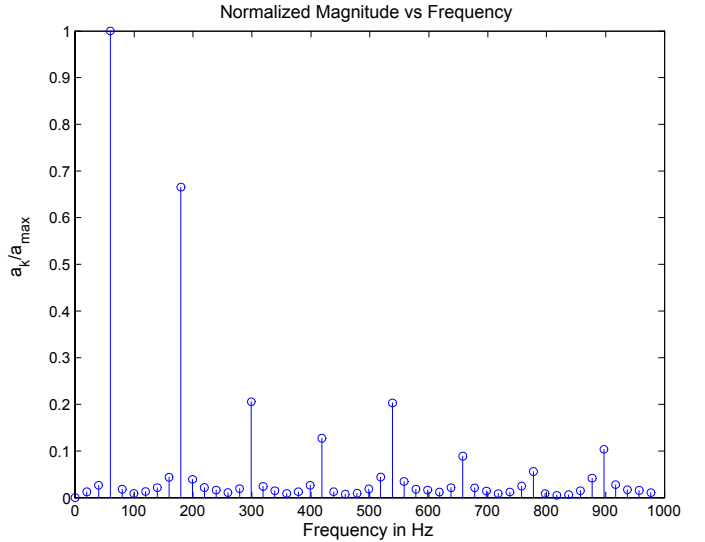Fig. 7. Phase voltage when $m = 0.5$

the switching algorithm in SIMULINK$^{\circledR}$ which is then converted to C code using RTW$^{\circledR}$. The OPALRT$^{\circledR}$ software provides icons to interface the SIMULINK$^{\circledR}$ model to the digital I/O board and converts the C code into executables. Using the XHP$^{\circledR}$ (extra high performance) option in OPALRT$^{\circledR}$ as well as the multiprocessor option to spread the computation between two processors, an execution time of 16 microseconds was achieved.

Experiments were performed to validate the theoretical results of section III-A.2. Due to space limitations, only data for $m = 0.5$ and 2 are presented. The first value $m = 0.5$ corresponds to the case where the 5th and 7th harmonics cannot be eliminated while the second value $m = 2$ is a case in which these harmonics can be eliminated. In this set of data, the angles were chosen by taking θ_1, θ_2 according to the *upper* curves in Figures 3 and 4, respectively and the corresponding θ_3 from the *lower* curve in Figure 5. The frequency was set to 60 Hz in each case and the program was run in real time with a 16 microseconds sample period, i.e., the logic signals were updated to the gate driver board every 16 microseconds.

The voltage was measured using a high speed data acquisition oscilloscope every $T = 5$ microseconds resulting in the data $\{v(nT), n = 1, ..., N\}$ where $N = 3(1/60)/(5 \times 10^{-6}) = 10000$ samples corresponding to three periods of the 60 Hz waveform. A fast Fourier transform was performed on this voltage data to get $\{\hat{v}(k\omega_0), k = 1, ..., N\}$ where the frequency increment is $\omega_0 = (2\pi/T)/N = 2\pi(20)$ rad/sec or 20 Hz. The number $\hat{v}(k\omega_0)$ is simply the Fourier coefficient of the kth harmonic (whose frequency is $k\omega_0$ with $\omega_0 = \frac{2\pi}{N}\frac{1}{T}$) in the Fourier series expansion of the phase voltage signal $v(t)$. With $a_k = |\hat{v}(k\omega_0)|$ and $a_{\max} = \max_k \{|\hat{v}(k\omega_0)|\}$, the data that is plotted is the normalized magnitude $a_k/a_{\max}$.

Figure 7 is the plot of the phase voltage for $m = 0.5$ and the corresponding FFT of this signal is given in Figure 8. Figure 8 show a 0.225 normalized magnitude

Fig. 8. Normalized FFT $a_k/a_{\max}$ vs frequency for $m = 0.5$

Figure 9 is the plot of the phase voltage for $m = 2$. The corresponding FFT of this signal is given in Figure 10. Figure 10 shows 5th and 7th harmonics are zero as predicted in Figure 6.

V. CONCLUSIONS AND FURTHER WORK

A full solution to the problem eliminating the 5th and 7th harmonics in a seven level multilevel inverter has been given. Specifically, resultant theory was used to completely
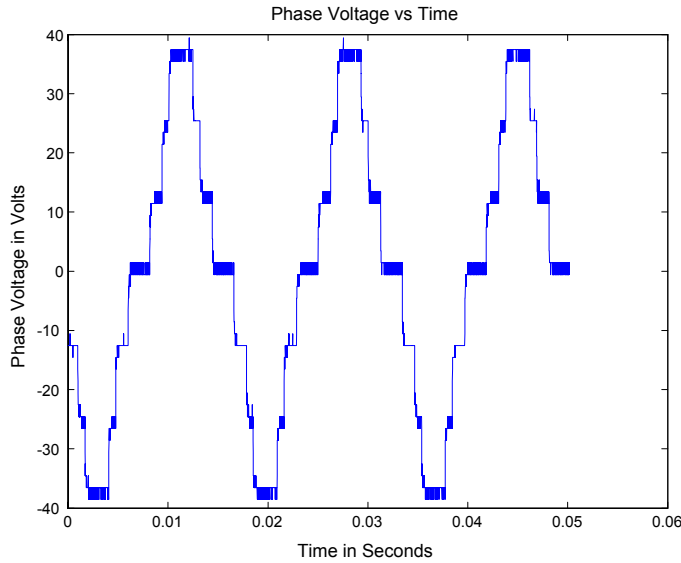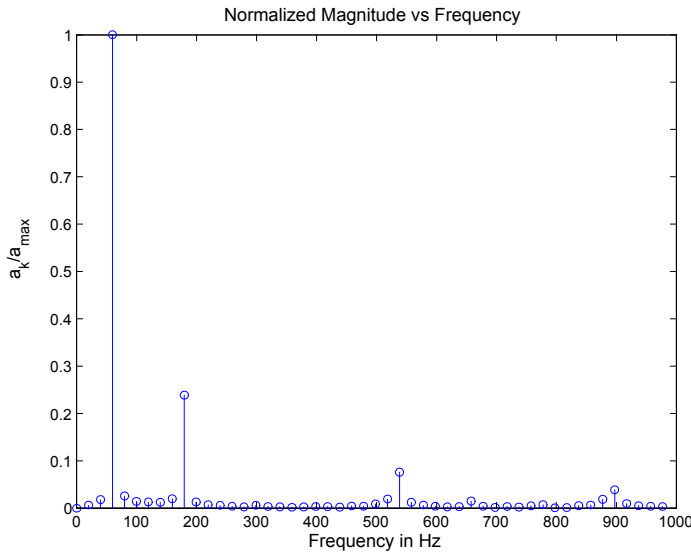
Fig. 9. Phase voltage when $m = 2$

Fig. 10. Normalized FFT a_k/a_{max} vs frequency for $m = 2$

characterize for each m when a solution existed and when it did not (in contrast to numerical techniques such as Newton-Raphson). Futher, it was shown that for a range of values of m, there were two sets of solutions and these values were also completely characterized. The solution set that happened to minimize the 11th and 13th harmonics was chosen. Experimental results were also presented and corresponded well to the theoretically predicted results. Further work is now underway to consider the case studied by Cunnyngham [3] where the separate dc sources do not all provide equal voltages V_{dc}.

VI. Acknowledgements

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