1. The real number $y$ is the derivative of the function $f$ at the point $\bar{x}$ iff (if and only if) for every real number $\epsilon > 0$ there is a real number $\delta > 0$ such that for every real number $x$ with $0 < |x - \bar{x}| < \delta$,

$$\left| \frac{f(x) - f(\bar{x})}{x - \bar{x}} - y \right| < \epsilon.$$ 

Write the negation of the proposition that $y$ is the derivative of the function $f$ at the point $\bar{x}$. Your answer should not contain the not-sign ($\neg$) nor the words “not,” “no,” “none,” etc. (You can use either words or symbols.) Hint: Look at Example 2.50 in Grimaldi, p. 99.

For each of the following, you are to prove the proposition, but:

(1) Show how you have devised your proof by the naming the methods (Forward-Backward, Construction, Choose, etc.) that you are using and labeling the steps A1, A2, . . . , B1, B2, . . . as we have done in class. You will not receive full credit if you do not identify the methods and label the steps!

(2) After you have invented a proof, execute it by writing it out clearly in Euclidean order.

2. A function $f$ is bounded above iff there is a real number $y$ such that, for all real numbers $x$, $f(x) \leq y$. Prove that $f(x) = -x^2 + 2x$ is bounded above.

3. If $x$ is a real number satisfying $x^3 + 3x^2 - 9x - 27 \geq 0$, then $|x| \geq 3$. Hint: Express the conclusion as a disjunction.