Lecture 13: Midterm, Dynamics and Control (cont.)

ECE 481: Power Electronics
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Chapter 7. AC Equivalent Circuit Modeling

7.1 Introduction

7.2 The basic AC modeling approach

7.3 State-space averaging

7.4 Circuit averaging and averaged switch modeling

7.5 The canonical circuit model

7.6 Modeling the pulse-width modulator

7.7 Summary of key points
Neglecting the switching ripple

Suppose the duty cycle is modulated sinusoidally:
\[ d(t) = D + D_p \cos \omega_m t \]
where \( D \) and \( D_p \) are constants, \( |D_p| \ll D \) and the modulation frequency \( \omega_m \) is much smaller than the converter switching frequency \( \omega_s = 2\pi f_s \).

The resulting variations in transistor gate drive signal and converter output voltage:

The actual waveform \( v(t) \) including ripple, versus the averaged waveform \( \bar{v}(t) \) with ripple neglected.
Output voltage spectrum
with sinusoidal modulation of duty cycle

Contains frequency components at:
- Modulation frequency and its harmonics
- Switching frequency and its harmonics
- Sidebands of switching frequency

With small switching ripple, high-frequency components (switching harmonics and sidebands) are small.
If ripple is neglected, then only low-frequency components (modulation frequency and harmonics) remain.

Chapter 7: AC equivalent circuit modeling
Averaging to remove switching ripple

Average over one switching period to remove switching ripple:

\[
\begin{align*}
L \frac{d\langle i(t) \rangle_{rs}}{dt} &= \langle v_s(t) \rangle_{rs} \\
C \frac{d\langle v_c(t) \rangle_{rs}}{dt} &= \langle i_s(t) \rangle_{rs}
\end{align*}
\]

where

\[
\langle x(t) \rangle_{rs} = \frac{1}{T_s} \int_{0}^{T_s} x(t) \, dt
\]

Note that, in steady-state,

\[
\langle v_s(t) \rangle_{rs} = 0, \quad \langle i_s(t) \rangle_{rs} = 0
\]

by inductor volt-second balance and capacitor charge balance.

\[
\langle v_s \rangle \cdot v_s = v_{gs} \cdot D^* v
\]

Small-signal modeling of the diode

- \( \bar{I} \) is a small-signal value, \( \bar{I} = \bar{I}_0 + \Delta \theta \bar{I}_0 \)
- \( \bar{I} \ll \bar{I} \)

Nonlinear diode, driven by current source having a DC and small AC component

\[
i = I_s \bar{I}
\]

Small-signal AC model

\[
\bar{I} = \frac{1}{2} \left( \bar{v} \right)
\]

Linearization of the diode \( i \cdot v \) characteristic about a quiescent operating point

Actual nonlinear characteristic

Quiescent operating point

Linearized characteristic

\( \bar{I} \ll \bar{I} \)
Buck-boost converter: nonlinear static control-to-output characteristic

\[ V = V_s D(1 - D) \]

Quiescent operating point

\[ \dot{V} = C_{mol} \omega \dot{\omega} \]

Linearized function

Actual nonlinear characteristic

Example: linearization at the quiescent operating point

\[ D = 0.5 \]

Result of averaged small-signal ac modeling

Small-signal ac equivalent circuit model

\[ G_m(s) = \frac{V(s)}{\dot{\omega}(s)} \]

buck-boost example
7.2. The basic AC modeling approach

Buck-boost converter example

\[ \dot{V}(t) = V_0, \quad \dot{I}(t) = -\frac{V(t)}{R}, \quad \dot{Q}(t) = i(t) \]

\[ \langle V(t) \rangle = L \frac{d\dot{Q}(t)}{dt} = \frac{1}{T} \int_0^T v(t) dt + \frac{1}{T} \int_0^T v(t) dt \]

\[ \langle V(t) \rangle \approx d(t) v(t) + d'(t) v(t) \]  

Large signal averaged over \( T > \) nonlinear