Lecture 18: \(Z_{\text{out}}\) Closed Loop Converters

ECE 481: Power Electronics
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Announcements

• No office hours this afternoon
• HW#7 due, solutions and HW#8 this afternoon
ECE 482: Power Electronic Circuits

Course reformatted for Spring 2014
- Design-oriented introduction to the analysis, modeling, and testing of power electronics
- Fabrication of the multiple switched-mode power converters
- Analog and digital control systems
  - Realize a functioning, sub-kW electric vehicle
  - Compete to achieve best performance of EV drive train
ECE482 Schedule

Deviation of exact curve from magnitude asymptotes

\[ |G(j\omega)| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega_0}{\omega_b}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega_0}{\omega_b}\right)^2}} \]

At \( \omega = \omega_0 \), the exact magnitude is

\[ |G(j\omega_0)| = Q \]

or, in dB:

\[ |G(j\omega_0)|_{dB} = |Q|_{dB} \]

The exact curve has magnitude \( Q \) at \( f = f_0 \). The deviation of the exact curve from the asymptotes is \( |Q|_{dB} \).

\[ -40 \text{ dB/decade} \]
Asymptotes for Complex Poles

The Low-Q Approximation

\[ f_1 = \frac{Qf_0}{F(Q)} \]

\[ f_2 = \frac{f_0F(Q)}{Q} \]

Magnitude

Phase

Summary: Asymptotes for Complex Poles

Magnitude

Phase

\[ f_1 = \frac{Qf_0}{F(Q)} \]

\[ f_2 = \frac{f_0F(Q)}{Q} \]

\[ f_0 = 10^{-1/2Q}f_0 \]

\[ f_0 = 10^{-1/2Q}f_0 \]
8.2.3. Physical origins of the right half-plane zero

\[ G(s) = \left( 1 - \frac{s}{\omega_n} \right) \]

- phase reversal at high frequency
- transient response: output initially tends in wrong direction

\[ y(t) \rightarrow u_{in}(t) \rightarrow \frac{s}{\omega_n} \rightarrow \begin{cases} \text{RHP zero influence} \end{cases} \rightarrow y(t) \]
Two converters whose CCM control-to-output transfer functions exhibit RHP zeroes

\[ \langle i_d \rangle_L = d \langle i_i \rangle_L = (1 - d) \frac{C}{R} \]

*Boost*

\[ v_o = \frac{v_i}{1 - d} \]

*Buck-boost*

\[ v_o = \frac{v_i}{d} \]

Fundamentals of Power Electronics Chapter 8: Converter Transfer Functions

Waveforms, step increase in duty cycle

- Increasing \( d(t) \) causes the average diode current to initially decrease.
- As inductor current increases to its new equilibrium value, average diode current eventually increases.
Buck-Boost Output Impedance

\[ D = 0.6 \]
\[ R = 10\Omega \]
\[ V_s = 30V \]
\[ L = 160\mu H \]
\[ C = 160\mu F \]
\[ L/D^2 = 1000\mu H \]
\[ f_o = 400\text{ Hz} \]

\[ Z_{out} = R + \frac{1}{\frac{j}{Z_{in}} \frac{1}{Z_{in}}} \]

Impedance graph paper

\[ C = 1000\mu F \]
\[ f_o = 400\text{ Hz} \]
Nonideal Output Impedance

\[ D = 0.6 \]
\[ R = 10 \Omega \]
\[ V_s = 30V \]
\[ L = 160\mu H \]
\[ C = 160\mu F \]
\[ L/D^2 = 1000\mu H \]
\[ f_o = 400 \text{ Hz} \]
\[ R_L = 316m\Omega = -10\text{dB}\Omega \]
\[ ESR = 10m\Omega = -40\text{dB}\Omega \]
Transfer functions predicted by canonical model

\[ H_e = \frac{R L_e}{s^2 L_e + s L_e + 1} \]

Output impedance \( Z_{\text{out}} \): set sources to zero

\[ Z_{\text{out}} = Z_1 \parallel \frac{1}{sC} \]

\[ Z_{\text{out}} = \parallel \]
Graphical construction of output impedance

\[ \frac{1}{\omega C} \]
\[ ||Z_f|| = \omega L_e \]
\[ R \]
\[ Q = \frac{R}{R_0} \]
\[ ||Z_{out}|| \]

Graphical construction of filter effective transfer function

\[ \frac{\omega L_e}{\omega L_e} \]
\[ Q = \frac{R}{R_0} \]
\[ f_0 \]
\[ \log \left( \frac{X}{X_1} \right) = \log(X_1) - \log(X_2) \]
Chapter 9. Controller Design

9.1. Introduction

9.2. Effect of negative feedback on the network transfer functions

9.2.1. Feedback reduces the transfer function from disturbances to the output

9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

9.3. Construction of the important quantities \( 1/(1+T) \) and \( T/(1+T) \) and the closed-loop transfer functions
Controller design

9.4. Stability
   9.4.1. The phase margin test
   9.4.2. The relation between phase margin and closed-loop damping factor
   9.4.3. Transient response vs. damping factor

9.5. Regulator design
   9.5.1. Lead (PD) compensator
   9.5.2. Lag (PI) compensator
   9.5.3. Combined (PID) compensator
   9.5.4. Design example

9.6. Measurement of loop gains
   9.6.1. Voltage injection
   9.6.2. Current injection
   9.6.3. Measurement of unstable systems

9.7. Summary of key points
Negative feedback: a switching regulator system

Negative feedback

Often just a gain, may have some filtering.
9.2. Effect of negative feedback on the network transfer functions

Small signal model: open-loop converter

Output voltage can be expressed as

\[ v(s) = G_{o}(s) \, \tilde{a}(s) + G_{f}(s) \, \tilde{r}(s) - Z_{a}(s) \, \tilde{i}_{\text{load}}(s) \]

where

\[ G_{o}(s) = \left. \frac{d}{dt} \right| _{t_{\text{load}} = 0} \]

\[ G_{f}(s) = \left. \frac{d}{dt} \right| _{t_{\text{load}} = 0} \]

\[ Z_{a}(s) = \left. \frac{R(s)}{t_{\text{load}}(s)} \right| _{t_{\text{load}} = 0} \]

Voltage regulator system small-signal model

- Use small-signal converter model
- Perturb and linearize remainder of feedback loop:
  - \( v_{\text{ref}}(t) = V_{\text{ref}} + \tilde{v}_{\text{ref}}(t) \)
  - \( v_{i}(t) = V_{i} + \tilde{v}_{i}(t) \)
  - etc.
Regulator system small-signal block diagram

Closed Loop Reference-to-Output
Closed Loop Disturbance-to-Output

\[ \varphi = \varphi_{ref} - \frac{G_s G_{d_1}/V_u}{1 + HG_s G_{d_1} / V_u} + \varphi_T - \frac{G_s}{1 + HG_s G_{d_1} / V_u} - \frac{i_{load}}{1 + HG_s G_{d_1} / V_u} \]

which is of the form

\[ \varphi = \varphi_{ref} \left( \frac{1}{H} \right) \frac{G_s}{V_u} + \varphi_T - \frac{Z_{out}}{V_u} \]

with \( T(s) = H(s) G(s) G_d(s) / V_u \) = "loop gain"

\[ \varphi = \text{ref} \frac{1}{H} \frac{G_s}{V_u} \]

Loop gain \( T(s) \) = products of the gains around the negative feedback loop.
9.2.1. Feedback reduces the transfer functions from disturbances to the output

Original (open-loop) line-to-output transfer function:

\[ G_v(s) = \left. \frac{V(s)}{V_i(s)} \right|_{r_{vd}=0} \]

With addition of negative feedback, the line-to-output transfer function becomes:

\[ \left. \frac{V(s)}{V_i(s)} \right|_{r_{vd}=0} = \frac{G_v(s)}{1 + T(s)} \]

Feedback reduces the line-to-output transfer function by a factor of

\[ \frac{1}{1 + T(s)} \]

If \( T(s) \) is large in magnitude, then the line-to-output transfer function becomes small.

Closed-loop output impedance

Original (open-loop) output impedance:

\[ Z_{vo}(s) = -\left. \frac{V(s)}{I_{vo}(s)} \right|_{r_{vo}=0} \]

With addition of negative feedback, the output impedance becomes:

\[ -\left. \frac{V(s)}{I_{vo}(s)} \right|_{r_{vo}=0} = \frac{Z_{vo}(s)}{1 + T(s)} \]

Feedback reduces the output impedance by a factor of

\[ \frac{1}{1 + T(s)} \]

If \( T(s) \) is large in magnitude, then the output impedance is greatly reduced in magnitude.
9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop.

Closed-loop transfer function from \( v_{ref} \) to \( v(s) \) is:

\[
\left. \frac{v(s)}{v_{ref}(s)} \right|_{t \to \infty} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}
\]

If the loop gain is large in magnitude, i.e., \( \| T \| \gg 1 \), then \((1 + T) \approx T\) and \(T/(1+T) = T/T = 1\). The transfer function then becomes

\[
\left. \frac{v(s)}{v_{ref}(s)} \right|_{t \to \infty} = \frac{1}{H(s)}
\]

which is independent of the gains in the forward path of the loop.

This result applies equally well to dc values:

\[
\frac{V}{V_{ref}} = \frac{1}{H(0)} \frac{T(0)}{1 + T(0)} = \frac{1}{H(0)}
\]

9.3. Construction of the important quantities \( 1/(1+T) \) and \( T/(1+T) \)

\[
T(s) = T_0 \left( \frac{1 + \frac{s}{\omega_1}}{1 + \frac{s}{\omega_1} + \left( \frac{s}{\omega_1} \right)^2} \right) \left( 1 + \frac{s}{\omega_2} \right)
\]

At the crossover frequency \( f_c \), \( \| T \| = 1 \).
Approximating $1/(1+T)$ and $T/(1+T)$

$$\frac{T}{1+T} \approx \begin{cases} \frac{1}{T} & \text{for } \|T\| >> 1 \\ T & \text{for } \|T\| << 1 \end{cases}$$

$$\frac{1}{1+T(s)} \approx \begin{cases} \frac{1}{T(s)} & \text{for } \|T\| >> 1 \\ 1 & \text{for } \|T\| << 1 \end{cases}$$

Example: construction of $T/(1+T)$

Example graph showing frequency response with crossover frequencies $f_1$ and $f_2$.
Example: analytical expressions for approximate reference to output transfer function

At frequencies sufficiently less that the crossover frequency, the loop gain $T(s)$ has large magnitude. The transfer function from the reference to the output becomes

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{T(s)}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{1}{H(s)}$$

This is the desired behavior: the output follows the reference according to the ideal gain $1/H(s)$. The feedback loop works well at frequencies where the loop gain $T(s)$ has large magnitude.

At frequencies above the crossover frequency, $\| T \| < 1$. The quantity $T(1+T)$ then has magnitude approximately equal to 1, and we obtain

$$\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)} \approx \frac{T(s)}{H(s)}$$

$$\frac{G_v(s)G_w(s)}{V_M}$$

This coincides with the open-loop transfer function from the reference to the output. At frequencies where $\| T \| < 1$, the loop has essentially no effect on the transfer function from the reference to the output.

Same example: construction of $1/(1+T)$