Lecture 20: Feedback Loop Compensation

ECE 481: Power Electronics
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Fall 2013

9.5.4. Design example
Quiescent operating point

Input voltage \( V_i = 28\, \text{V} \)
Output \( V = 15\, \text{V}, \, \iota_{\text{load}} = 5\, \text{A}, \, R = 3\, \Omega \)
Quiescent duty cycle \( D = 15/28 = 0.536 \)
Reference voltage \( V_{\text{ref}} = 5\, \text{V} \)
Quiescent value of control voltage \( V_c = D V_i = 2.14\, \text{V} \)
Gain \( H(s) \) \( H = V_{\text{ref}}/V = 5/15 = 1/3 \)
Small-signal model

\[ \frac{\Delta i(t)}{\Delta v(t)} = \frac{1}{sC} + \frac{1}{R} \]

\[ \frac{\Delta v(t)}{\Delta i(t)} = \frac{1}{sC} - \frac{1}{R} \]

\[ H(s) = \frac{1}{sC} + \frac{1}{R} \]

\[ H(s) = \frac{1}{sC} - \frac{1}{R} \]
Open-Loop Control-to-Output Transfer Function

\[ G(s) = \frac{V}{D} \left( 1 + \frac{1}{R} + s^2 LC \right) \]

standard form:

\[ G(s) = G_0 \left( 1 + \frac{s}{Q_0} + \left( \frac{s}{\omega_0} \right)^2 \right) \]

salient features:

\[ G_0 = \frac{V}{D} \left( \frac{28}{2\pi f_c} \right) \]
\[ \omega_0 = \frac{Q_0}{\sqrt{LC}} \left( \frac{1}{2\pi} \right) \]
\[ Q_0 = R \sqrt{\frac{C}{L}} = 2.5 \rightarrow 12.5 \text{dB} \]
Open-loop line-to-output transfer function and output impedance

\[ G_o(s) = D \frac{1}{1 + sL + s^2LC} \]

- same poles as control-to-output transfer function standard form:

\[ G_o(s) = G_p \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \]

Output impedance:

\[ Z_o(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{1}{sC + \frac{sL}{1 + \frac{s}{R} + s^2LC}} \]
**System block diagram**

\[ T(s) = G_c(s) \left( \frac{1}{V_m} \right) G_d(s) H(s) \]

Design \( G_c \) to get:
- High \( V_c \)
- Large \( \Delta \)

First, look at \( G_c = 1 \)

Uncompensated loop gain (with \( G_c = 1 \))

\[
\sqrt{V} = \frac{1}{4} \frac{1}{1 + \frac{1}{f_c}} V_{\text{ref}}
\]

\[
V = \frac{1}{1 + \frac{2.33}{10^3}} \times 10V
\]

\( V = 15V \)

With \( G_c = 1 \) the loop gain is

\[ T(s) = \frac{1}{1 + \frac{1}{Q_0 \omega_0} + \left( \frac{s}{\omega_0} \right)^2} \]

\[ T(s) = \frac{H}{D} \frac{V_m}{V_m} = 2.33 \approx 7.4 \text{dB} \]

\[ f_c = 1.8 \text{ kHz}, \quad \theta_m = 5^\circ \]

\[ \theta_m \approx \phi \]
**Q vs. $\phi_m$**

![Graph showing Q vs. $\phi_m$](image1)

**Lead compensator design**

To optimally obtain a compensator phase lead of $\theta$ at frequency $f_c$, the pole and zero frequencies should be chosen as follows:

$$f_p = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}}$$

$$f_z = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}}$$

If it is desired that the magnitude of the compensator gain at $f_c$ be unity, then $G_\text{p}_0$ should be chosen as

$$G_\text{p}_0 = \sqrt{\frac{f_c}{f_p}}$$
Lead compensator design

- Obtain a crossover frequency of 5 kHz, with phase margin of 52°.
- \(T_u\) has phase of approximately –180° at 5 kHz, hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of +52° at 5 kHz.
- \(T_u\) has magnitude of –20.6 dB at 5 kHz.
- Lead compensator gain should have magnitude of +20.6 dB at 5 kHz.
- Lead compensator pole and zero frequencies should be
  \[ f_p = (5\text{kHz}) \sqrt{\frac{1 - \sin (52^\circ)}{1 + \sin (52^\circ)}} = 1.7\text{kHz} \]
  \[ f_z = (5\text{kHz}) \sqrt{\frac{1 + \sin (52^\circ)}{1 - \sin (52^\circ)}} = 14.5\text{kHz} \]
- Compensator dc gain should be
  \[ G_o = \left(\frac{f_z}{f_p}\right)^2 \sqrt{\frac{T_u}{f_z}} \left(\frac{f}{f_p}\right) \approx 3.7 \Rightarrow 11.3\text{dB} \]

Lead compensator Bode plot
Loop gain, with lead compensator

\[ T(s) = \frac{1 + \frac{s}{\omega_c}}{1 + \frac{s}{\omega_n} + \left(\frac{s}{\omega_b}\right)} \]

- \[ T_n = 8.6 \Rightarrow 18.7 \text{ dB} \]
- \[ Q_n = 9.5 \Rightarrow 19.5 \text{ dB} \]

- \[ f_c = 1.1 \text{ kHz} \]
- \[ f_c = 17 \text{ kHz} \]
- \[ f_c = 1.7 \text{ kHz} \]
- \[ f_c = 5 \text{ kHz} \]

\[ f = 1 \text{ Hz} \]
\[ f = 10 \text{ Hz} \]
\[ f = 100 \text{ Hz} \]
\[ f = 1 \text{ kHz} \]
\[ f = 10 \text{ kHz} \]
\[ f = 100 \text{ kHz} \]

1/(1+T), with lead compensator

- need more low-frequency loop gain
- hence, add inverted zero (PID controller)
Improved compensator (PID)

- \[ G_c(s) = G_m \left( \frac{1 + \frac{s}{\omega_0}}{1 + \frac{s}{\omega_0}} \right) \left( 1 + \frac{\omega_n}{s} \right) \]

- Add inverted zero to PD compensator, without changing dc gain or corner frequencies
- Choose \( f_z \) to be \( f/10 \), so that phase margin is unchanged

\[ T(s) \text{ and } \frac{1}{1 + T(s)} \text{, with PID compensator} \]
Line-to-output transfer function

\[ \frac{\hat{v}_c}{\hat{v}_s} \]

Open-loop \[ G_{av} \]

\[ G_{av}(0) = D \]

20 dB/decade

20 dB

-20 dB

-40 dB

-60 dB

-80 dB

-100 dB

1 Hz 10 Hz 100 Hz 1 kHz 10 kHz 100 kHz

Closed-loop \[ \frac{G_{av}}{1 + \tau} \]

\[ Q_h \]

\[ f_0 \]

\[ f_1 \]

\[ f_2 \]

\[ f_c \]

-40 dB/decade