Lecture 20: Feedback Loop Compensation

ECE 481: Power Electronics
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9.5.4. Design example

[Diagram showing a feedback loop with components such as an inductor (L), capacitor (C), resistor (R), and a pulse-width modulator (V_m = 4 V), with labels for input voltage (v), error signal, and sensor gain (H(s)).]
Quiescent operating point

- Input voltage: $V_s = 28$V
- Output: $V = 15$V, $I_{load} = 5$A, $R = 3\Omega$
- Quiescent duty cycle: $D = 15/28 = 0.536$
- Reference voltage: $V_{ref} = 5$V
- Quiescent value of control voltage: $V_c = DV_{ref} = 2.14$V
- Gain $H(s)$: $H = V_{ref}/V = 5/15 = 1/3$

Small-signal model
Open-Loop Control-to-Output Transfer Function

- $V = 15V$, $I_{load} = 5A$, $R = 3\Omega$
- $D = 15/28 = 0.536$
Open-loop control-to-output transfer function $G_{vo}(s)$

$$G_{vo}(s) = \frac{V}{D} \frac{1}{1 + \frac{R}{L} s^2 + s^4LC}$$

standard form:

$$G_{vo}(s) = G_m \frac{1}{1 + \frac{\frac{3}{Q_o R_0}}{Q_o} + \left(\frac{s}{\omega_0}\right)^2}$$

salient features:

$G_m = \frac{V}{D} = 28V$

$\omega_0 = \frac{2\pi}{2\pi LC} = 1kHz$

$Q_o = R \sqrt{\frac{L}{C}} = 9.5 \rightarrow 19.5\text{dB}$

Open-loop line-to-output transfer function and output impedance

$$G_{ao}(s) = \frac{D}{1 + \frac{L}{R} s + s^4LC}$$

same poles as control-to-output transfer function

standard form:

$$G_{ao}(s) = G_m \frac{1}{1 + \frac{s}{Q_o \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Output impedance:

$$Z_{ao}(s) = R \parallel \frac{1}{sC} \parallel sL = \frac{HL}{1 + \frac{s^2}{R^2} + s^2LC}$$
System block diagram

\[ T(s) = \frac{G_c(s)}{V_o} \left( \frac{1}{\frac{1}{H(s)}} \right) G_{v}(s) H(s) \]

\[ T(s) = \frac{G(s) H(s)}{1 + \frac{s}{\omega_n} + \left( \frac{s}{\omega_n} \right)^2} \]

\[ V_o = \frac{4}{V_o} \]

\[ \hat{V}_o(t) = \hat{V}(t) \]

\[ H = \frac{1}{2} \]

Converter power stage

Uncompensated loop gain (with \( G_c = 1 \))

With \( G_c = 1 \), the loop gain is

\[ T(s) = T_{lo} \frac{1}{1 + \frac{s}{\omega_n} + \left( \frac{s}{\omega_n} \right)^2} \]

\[ T_{lo} = \frac{H V_o}{V_o} = 2.33 \Rightarrow 7.4 \text{ dB} \]

\[ f_c = 1.8 \text{ kHz}, \ \phi_m = 5^\circ \]
Lead compensator design

- Obtain a crossover frequency of 5 kHz, with phase margin of 52°
- $T_v$ has phase of approximately – 180° at 5 kHz; hence lead (PD) compensator is needed to increase phase margin.
- Lead compensator should have phase of + 52° at 5 kHz
- $T_v$ has magnitude of – 20.6 dB at 5 kHz
- Lead compensator gain should have magnitude of + 20.6 dB at 5 kHz
- Lead compensator pole and zero frequencies should be
  \[ f_c = (5 \text{kHz}) \sqrt{\frac{1 - \sin (52^\circ)}{1 + \sin (52^\circ)}} = 1.7 \text{kHz} \]
  \[ f_p = (5 \text{kHz}) \sqrt{\frac{1 + \sin (52^\circ)}{1 - \sin (52^\circ)}} = 14.5 \text{kHz} \]
- Compensator dc gain should be $G_o = \left( \frac{f_c}{f_p} \right)^3 \sqrt{\frac{f_c}{f_p}} = 3.7 \Rightarrow 11.3 \text{dB}$
Loop gain, with lead compensator

\[ T(s) = T_a \cdot \frac{1 + \frac{s}{\omega_p}}{1 + \frac{s}{\omega_p} \left(1 + \frac{s}{Q \omega_b} + \frac{\omega_b}{\omega_p}\right)} \]

\[ T_a = 8.6 \Rightarrow 18.7 \text{ dB} \]
\[ \omega_p = 9.5 \Rightarrow 19.5 \text{ dB} \]

1/(1+T), with lead compensator

- need more low-frequency loop gain
- hence, add inverted zero (PID controller)
Improved compensator (PID)

\[ G(s) = \frac{G_m \left( 1 + \frac{s}{\omega_n} \right) \left( 1 + \frac{\omega_n}{\omega_z} \right)}{1 + \frac{s}{\omega_z}} \]

- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose \( f_c \) to be \( f_c/10 \), so that phase margin is unchanged

\[ T(s) \text{ and } \frac{1}{1+T(s)}, \text{ with PID compensator} \]
Line-to-output transfer function

$G_m(0) = D$

$G_m$ (Open-loop) $\| G_m $ (Closed-loop)

$-40 \text{ dB/decade}$

$20 \text{ dB/decade}$

$\frac{D}{T_p G_m}$

$f_0$ $f_1$ $f_2$ $f_c$

$1 \text{ Hz}$ $10 \text{ Hz}$ $100 \text{ Hz}$ $1 \text{ kHz}$ $10 \text{ kHz}$ $100 \text{ kHz}$

$f$
Another Compensator Design Example

Buck Averaged Small-Signal Model

<table>
<thead>
<tr>
<th>Power stage parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Switching frequency: $f_s = 1\text{MHz}$</td>
</tr>
<tr>
<td>• $V_{\text{ref}} = 1.8\text{V}$</td>
</tr>
<tr>
<td>• $I_{\text{out}} = 0\text{ to } 5\text{A}$</td>
</tr>
<tr>
<td>• $V_g = 5\text{V}$</td>
</tr>
<tr>
<td>• $L = 1\mu\text{H}$</td>
</tr>
<tr>
<td>• $R_L = 30\text{m\Omega}$</td>
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<tr>
<td>• $C = 200\mu\text{F}$</td>
</tr>
<tr>
<td>• $R_{\text{esr}} = 0.8\text{m\Omega}$</td>
</tr>
<tr>
<td>• $V_M = 1\text{V}$</td>
</tr>
<tr>
<td>• $H = 1$</td>
</tr>
</tbody>
</table>

Pair of poles:

\[
\frac{f_s}{2\pi\sqrt{CL}} = 11\text{kHz}
\]

\[
Q_{\text{loss}} = \frac{\sqrt{L/C}}{R_{\text{esr}} + R_L} = 2.3 \rightarrow 7.2\text{dB} \quad Q_{\text{load}} = \frac{R}{\sqrt{L/C}} > 5
\]

\[
Q = Q_{\text{loss}} \parallel Q_{\text{load}} = \frac{Q_{\text{loss}}Q_{\text{load}}}{Q_{\text{loss}} + Q_{\text{load}}} < 2.3 \rightarrow 7.2\text{dB}
\]
Uncompensated loop gain $T_u$

$T_u(s) = H_{\text{sense}}(\frac{1}{V_M})G_{vd}(s)$

Plot magnitude and phase responses of $T_u(s)$ to plan how to design $G_c(s)$

Magnitude and phase Bode plots of $T_u$

$T_u(s) = H_{\text{sense}}(\frac{1}{V_M})G_{vd}(s)$

$T_u = G_{dc}(\frac{1}{V_M})H_{\text{sense}} = 5 \rightarrow 14 \text{dB}$

$Q = 2.3 \rightarrow 7.2 \text{dB}$

$T_u = \left(\frac{f_c}{f_T}\right)^2$

$T_u = 1 \text{MHz}$

$\text{esr} = \frac{1}{10} f_{esr}$

$\frac{1}{10} f_{esr} = 10 \text{Hz}$

$-90^\circ$

$-180^\circ$
Magnitude and phase Bode plots of $T_u$

Uncompensated loop gain, $T_u = G_{vd}H_{sense}(1/VM)$

Exact magnitude and phase responses (MATLAB)

Target cross-over frequency $f_c = f_s/10 = 100 \text{ kHz}$

No phase margin: a lead (PD) compensator is required

Lead (PD) compensator design

1. Choose: $f_c = 100 \text{ kHz}$
   $\theta = \phi_m = 53^\circ$

2. Compute:
   
   $f_p = f_c \sqrt{\frac{1 - \sin(\theta)}{1 + \sin(\theta)}} = 33 \text{ kHz}$
   $f_p = f_c \sqrt{\frac{1 + \sin(\theta)}{1 - \sin(\theta)}} = 300 \text{ kHz}$

3. Find $G_{co}$ to position the crossover frequency:
   
   $T_u \left(\frac{f_p}{f_c}\right)^2 G_{co} \sqrt{\frac{f_p}{f_c}} = 1$
   $G_{co} = \frac{1}{T_u \left(\frac{f_p}{f_c}\right)^2} \sqrt{\frac{f_p}{f_c}} = 5.45 \rightarrow 15 \text{ dB}$
Lead (PD) compensator summary

$$G_c(s) = G_{co} \frac{1}{1 + \frac{s}{\omega_z}} \frac{1}{1 + \frac{s}{\omega_{p1}}} \frac{1}{1 + \frac{s}{\omega_{p2}}}$$

- $G_{co} = 5.45 \rightarrow 15\,\text{dB}$
- $f_z = 33\,\text{kHz}$
- $f_{p1} = 300\,\text{kHz}$
- $f_c = 100\,\text{kHz} \quad (=1/10\,\text{of}\,f_z)$

High-frequency gain of the lead compensator: $G_{co} f_{p1}/f_z = 49\,(34\,\text{dB})$

Added high-frequency pole: $f_{p2} = 1\,\text{MHz} \quad (=f_{\text{esr}} = f_s\,\text{in this example})$

Practical implementation would require an op-amp with a gain bandwidth product (GBW) of at least $49 f_{p2} = 49\,\text{MHz}$

Loop gain with lead (PD) compensator

Loop gain $T_{m} G_{co} = 28.7 \rightarrow 29.7\,\text{dB}$

- $f_z = 100\,\text{kHz}$
- $f_{p1} = 300\,\text{kHz}$
- $f_{p2} = 1\,\text{MHz}$
- $\phi_m = 53^\circ$
Add lag (PI) compensator

\[ G(s) = G_c \left(1 + \frac{\omega_n}{\omega_c}\right) \]

Integrates low-frequency loop gain and regulation

Choose \( 10f_L < f_c \) so that phase margin stays approximately the same: \( f_L = 8 \text{ kHz} \)

Keep the same cross-over frequency: \( G_{\infty} = G_{cm} = 5.45 \rightarrow 15 \text{ dB} \)

Adding PI Compensator
Complete analog PID compensator: summary

\[ G_c(s) = G_m \left( \frac{1 + \frac{s}{\omega_L}}{1 + \frac{s}{\omega_p}} \right) \]

\[ f_L = 8 \text{ kHz} \]

\[ f_z = 33 \text{ kHz} \]

\[ f_{p1} = 300 \text{ kHz} \]

\[ f_{p2} = 1 \text{ MHz} \]

Crossover frequency: \( f_c = 100 \text{ kHz} \) (=1/10 of \( f_z \))

Phase margin: \( \varphi_m = 53^\circ \)

Magnitude and phase Bode plots of \( T \)
Verification: exact loop gain magnitude and phase responses (MATLAB)

Analog PID compensator implementation

$\omega_1 = \frac{R_2}{R_1}$

$\omega_p = \frac{1}{2\pi R_2 C_2}$

$\omega_m = \frac{1}{2\pi R_4 C_4}$

$f_L = \frac{1}{2\pi R_2 C_2}$

$f_z = \frac{1}{2\pi (R_1 + R_4) C_4}$

$f_p = \frac{1}{2\pi R_4 C_4}$

$f_{\omega 2} = \frac{1}{2\pi R_2 C_3}$
Verification of closed-loop responses

Closed-loop reference-to-output response

Closed-loop transfer function from \( \hat{v}_{\text{in}} \) to \( \hat{v}(s) \) is:

\[
\frac{\hat{v}(s)}{\hat{v}_{\text{in}}(s)} \bigg|_{s=0} = \frac{T(s)}{H(s)(1 + T(s))}
\]

Closed-loop output impedance

\[
\frac{\hat{v}(s)}{-I_{\text{load}}(s)} \bigg|_{s=0} = \frac{Z_{\text{out}}(s)}{1 + T(s)}
\]

and step-load transient response

Construction of closed-loop \( T/(1+T) \) response

Closed-loop reference-to-output response \( v/v_{\text{ref}} = T/(1+T) \)

Closed-loop BW = \( f_c \)
Closed-loop reference-to-output response

Small-signal step-reference response
Small-signal step-reference response

10 mV step (1.79 V to 1.8 V) in \( v_{ref} \n\)

**Output impedance**

Synchronous buck open-loop output impedance

\[
Z_{out}(s) = \left( R_{err} + \frac{1}{sC} \right) \left( R_L + sL \right)
\]

- \( L = 1 \ \mu H \)
- \( R_L = 30 \ \text{m\Omega} \)
- \( C = 200 \ \mu F \)
- \( R_{err} = 0.8 \ \text{m\Omega} \)
Open-loop output impedance: algebra on the graph

Construction of $1/(1+T)$
Construction of closed-loop output impedance

\[ Z_{\text{out,CL}} = \frac{Z_{\text{out}}}{1 + T} \]

Closed-loop output impedance \( Z_{\text{out,CL}} \)

\[ Z_{\text{out,CL}} = \frac{Z_{\text{out}}}{1 + T} \]

\[ 8 \text{ m}\Omega, -42 \text{ dB}\Omega \]
Verification: closed-loop output impedance

Step-load transient responses

2.5-5 A step-load transient
Step-load transient responses

2.5-5A step-load transient

$\Delta v \approx 15$ mV

$\approx 10$ μs settling time