9.7. Summary of key points

1. Negative feedback causes the system output to closely follow the reference input, according to the gain $L/H(s)$. The influence on the output of disturbances and variation of gains in the forward path is reduced.

2. The loop gain $\gamma(s)$ is equal to the products of the gains in the forward and feedback paths. The loop gain is a measure of how well the feedback system works: a large loop gain leads to better regulation of the output. The crossover frequency $f_c$ is the frequency at which the loop gain $\gamma$ has unity magnitude, and is a measure of the bandwidth of the control system.
Summary of key points

3. The introduction of feedback causes the transfer functions from disturbances to the output to be multiplied by the factor \((1/(1+TF(s)))\). At frequencies where \(T\) is large in magnitude (i.e., below the crossover frequency), this factor is approximately equal to \(1/T(s)\). Hence, the influence of low-frequency disturbances on the output is reduced by a factor of \(1/(1+TF(s))\). At frequencies where \(T\) is small in magnitude (i.e., above the crossover frequency), the factor is approximately equal to 1. The feedback loop then has no effect. Closed-loop disturbance-to-output transfer functions, such as the line-to-output transfer function or the output impedance, can easily be constructed using the algebra-on-the-graph method.

4. Stability can be assessed using the phase margin test. The phase of \(T\) is evaluated at the crossover frequency, and the stability of the important closed-loop quantities \(1/(1+TF(s))\) and \(1/(1+T)\) is then deduced. Inadequate phase margin leads to ringing and overshoot in the system transient response, and peaking in the closed-loop transfer functions.

Summary of key points

5. Compensators are added in the forward paths of feedback loops to shape the loop gain, such that desired performance is obtained. Lead compensators, or \(PD\) controllers, are added to improve the phase margin and extend the control system bandwidth. \(PI\) controllers are used to increase the low-frequency loop gain, to improve the rejection of low-frequency disturbances and reduce the steady-state error.

6. Loop gains can be experimentally measured by use of voltage or current injection. This approach avoids the problem of establishing the correct quiescent operating conditions in the system, a common difficulty in systems having a large dc loop gain. An injection point must be found where interstage loading is not significant. Unstable loop gains can also be measured.
Part III. Magnetics

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14 Inductor Design
15 Transformer Design

Chapter 13 Basic Magnetics Theory

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Chapter 13  Basic Magnetics Theory

13.5  Several Types of Magnetic Devices, Their $B$–$H$ Loops, and Core vs. Copper Loss

13.5.1  Filter inductor
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13.6  Summary of Key Points

13.1  Review of Basic Magnetics

13.1.1  Basic relationships

- Faraday’s law
- Ampere’s law
- Terminal characteristics
- Core characteristics
Basic quantities

**Magnetic quantities**
- Length $l$
- Magnetic field $H$
- MMF $\mathcal{F} = HI$

**Electrical quantities**
- Length $l$
- Electric field $E$
- Voltage $V = EL$

Total flux $\Phi$
Flux density $B$
- $\Phi = \int B \cdot dA$
- $B = \frac{\Phi}{A}$

Total current $I$
Current density $J$
- $I = \int J \cdot dA$
- $J = \frac{I}{A}$

Magnetic field $H$ and magnetomotive force $\mathcal{F}$

Magnetomotive force (MMF) $\mathcal{F}$ between points $x_1$ and $x_2$ is related to the magnetic field $H$ according to

$$\mathcal{F} = \int_{x_1}^{x_2} H \cdot dl$$

**Example: uniform magnetic field of magnitude $H$**
- Length $l$
- Magnetic field $H$
- MMF $\mathcal{F} = HL$

**Analogous to electric field of strength $E$, which induces voltage (EMF) $V$:***
- Length $l$
- Electric field $E$
- Voltage $V = EL$
Flux density $B$ and total flux $\Phi$

The total magnetic flux $\Phi$ passing through a surface of area $A_c$ is related to the flux density $B$ according to:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{A}$$

**Example: uniform flux density of magnitude $B$**

- Total flux $\Phi$
- Flux density $B$
- Surface $S$ with area $A_c$

**Analogous to electrical conductor current density of magnitude $J$, which leads to total conductor current $I$**

- Total current $I$
- Current density $J$
- Surface $S$ with area $A_c$

Faraday’s law

Voltage $v(t)$ is induced in a loop of wire by change in the total flux $\Phi(t)$ passing through the interior of the loop, according to:

$$v(t) = \frac{d\Phi(t)}{dt}$$

For uniform flux distribution $\Phi(t) = B(t) A_c$, and hence:

$$v(t) = A_c \frac{dB(t)}{dt}$$

**Assumption:**
- Voltage $v(t)$ is induced with respect to position
- Uniform with respect to position

**Diagram:**
- Area $A_c$
- Flux $\Phi(t)$
- Flux density $B(t)$
- Voltage $v(t)$
- Surface $S$
Lenz’s law

The voltage \( \nu(t) \) induced by the changing flux \( \Phi(t) \) is of the polarity that tends to drive a current through the loop to counteract the flux change.

**Example: a shorted loop of wire**
- Changing flux \( \Phi(t) \) induces a voltage \( \nu(t) \) around the loop.
- This voltage, divided by the impedance of the loop conductor, leads to current \( i(t) \).
- This current induces a flux \( \Phi'(t) \), which tends to oppose changes in \( \Phi(t) \).

Ampere’s law

The net MMF around a closed path is equal to the total current passing through the interior of the path:

\[
\mathcal{F} = \oint H \cdot dl = \text{total current passing through interior of path}
\]

**Example: magnetic core. Wire carrying current \( i(t) \) passes through core window.**
- Illustrated path follows magnetic flux lines around interior of core.
- For uniform magnetic field strength \( H(t) \), the integral (MMF) is \( H(t) l_m \). So
  \[
  \Phi(t) = H(t) l_m = i(t)
  \]

\[ A(t) l_m = \nu(t) \]
Ampere’s law: discussion

- Relates magnetic field strength $H(t)$ to winding current $i(t)$
- We can view winding currents as sources of MMF
- Previous example: total MMF around core, $\mathcal{F}(t) = H(t)i_c$, is equal to the winding current MMF $i(t)$
- The total MMF around a closed loop, accounting for winding current MMF’s, is zero

\[ \oint \mathbf{E} \cdot d\mathbf{l} = \text{line voltage} \]

\[ \oint \mathbf{H} \cdot d\mathbf{l} = \text{line current} \]

Core material characteristics: the relation between $B$ and $H$

- $B = \mu H$

\[ \mu_0 = \text{permeability of free space} \]
\[ = 4\pi \times 10^{-7}\text{ Henries per meter} \]

\[ H = \frac{\Delta I}{\Delta t} \]
\[ H_{sat} = \frac{\Delta I_{sat}}{\Delta t} \]

$B$ vs. $H$ graph:
- Free space: $B = \mu_0 H$
- Magnetic core material:
  - Linear model: $B = MH$ if $B < B_{sat}$
  - $B > B_{sat}$ and hysteresis

Highly nonlinear, with hysteresis and saturation
Piecwise-linear modeling of core material characteristics

No hysteresis or saturation
\[ B = \mu H \]
\[ \mu = \mu_r \mu_0 \]

Typical \( \mu_r = 10^3 \) to \( 10^5 \)

Saturation, no hysteresis
\[ B = B_{sat} \]
\[ H = H \]

Typical \( B_{sat} \):
- \( 0.3 \) to \( 0.5 \) T, ferrite
- \( 0.5 \) to \( 1 \) T, powdered iron
- \( 1 \) to \( 2 \) T, iron laminations

Units

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<th>MKS</th>
<th>unamplified</th>
<th>conversions</th>
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<td>( B = \mu \mu_0 H )</td>
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<td>Tesla</td>
<td>Gauss</td>
<td>( 1 ) T = ( 10^5 ) G</td>
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<td>( H )</td>
<td>Ampere / meter</td>
<td>Oersted</td>
<td>( 1 ) A/m = ( 4\pi \times 10^{-7} ) Ot</td>
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<td>( \Phi )</td>
<td>Weber</td>
<td>Maxwell</td>
<td>( 1 ) Wb = ( 10^8 ) Mx</td>
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<tr>
<td></td>
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<td></td>
<td>( 1 ) T = ( 1 ) Wb / ( m^2 )</td>
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</tbody>
</table>
Example: a simple inductor

Faraday’s law:
For each turn of wire, we can write
\[ v_{\text{turn}}(t) = \frac{d\Phi(t)}{dt} \]

Total winding voltage is
\[ v(t) = n v_{\text{turn}}(t) = n \frac{d\Phi(t)}{dt} \]

Express in terms of the average flux density \( B(t) = \mathcal{B}(t)/A_c \)
\[ v(t) = n A_c \frac{dB(t)}{dt} \]
Inductor example: Ampere’s law

Choose a closed path which follows the average magnetic field line around the interior of the core. Length of this path is called the mean magnetic path length $\ell_m$.

For uniform field strength $H(t)$, the core MMF around the path is $H \ell_m$.

Winding contains $n$ turns of wire, each carrying current $i(t)$. The net current passing through the path interior (i.e., through the core window) is $n i(t)$.

From Ampere’s law, we have

$$H(t) \ell_m = n i(t)$$

Inductor example: core material model

$$B = \begin{cases} 
B_{sat} & \text{for } H \geq \frac{B_{sat}}{\mu} \\
\mu H & \text{for } \mid H \mid < \frac{B_{sat}}{\mu} \\
-B_{sat} & \text{for } H \leq -\frac{B_{sat}}{\mu}
\end{cases}$$

Find winding current at onset of saturation: substitute $i = I_{sat}$ and $H = B_{sat}/\mu$ into equation previously derived via Ampere’s law. Result is

$$I_{sat} = \frac{B_{sat} l_m}{\mu H}$$
Electrical terminal characteristics

We have:
\[ v(t) = nA \frac{dB(t)}{dt} \]
\[ H(t) \ell_m = n \, i(t) \]
\[ B = \begin{cases} \frac{B_{sat}}{\mu} & \text{for } H \geq \frac{B_{sat}}{\mu} \\ \mu H & \text{for } |H| < \frac{B_{sat}}{\mu} \\ -\frac{B_{sat}}{\mu} & \text{for } H \leq -\frac{B_{sat}}{\mu} \end{cases} \]

Eliminate \( B \) and \( H \), and solve for relation between \( v \) and \( i \). For \( |i| < I_{sat} \):
\[ v(t) = \mu n^2 A_c \frac{di(t)}{dt} \]
which is of the form
\[ v(t) = L \frac{di(t)}{dt} \]  \( L = \frac{\mu n^2 A_c}{\ell_m} \)

For \( |i| > I_{sat} \), the flux density is constant and equal to \( B_{sat} \). Faraday’s law then predicts
\[ v(t) = nA_c \frac{dB_{sat}}{dt} = 0 \]
—saturation leads to short circuit

13.1.2 Magnetic circuits

Uniform flux and magnetic field inside a rectangular element:
\[ \Phi = H \ell \]

MMF between ends of element is
\[ \mathcal{F} = H \ell \]

Since \( H = B / \mu \) and \( B = \Phi / A_c \), we can express \( \mathcal{F} \) as
\[ \mathcal{F} = \Phi / \mu A_c \]  with
\[ \mathcal{R} = \frac{\ell}{\mu A_c} \]

A corresponding model:
\[ \Phi \] \( \mathcal{R} \) — reluctance of element
Magnetic circuits: magnetic structures composed of multiple windings and heterogeneous elements

- Represent each element with reluctance
- Windings are sources of MMF
- MMF $\rightarrow$ voltage, flux $\rightarrow$ current
- Solve magnetic circuit using Kirchoff's laws, etc.

Magnetic analog of Kirchoff’s current law

Divergence of $B = 0$
Flux lines are continuous and cannot end
Total flux entering a node must be zero
**Magnetic analog of Kirchhoff’s voltage law**

Follows from Ampere’s law:

$$\oint_{\text{closed path}} H \cdot dl = \text{total current passing through interior of path}$$

Left-hand side: sum of MMF’s across the reluctances around the closed path

Right-hand side: currents in windings are sources of MMF’s. An \( n \)-turn winding carrying current \( i(t) \) is modeled as an MMF (voltage) source, of value \( n(t) \).

Total MMF’s around the closed path add up to zero.

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**Example: inductor with air gap**

\[ L = \frac{\mu n^2 A_c}{I_g} \]

Cross-sectional area \( A_c \)

Core permeability \( \mu \)

Air gap length \( I_g \)

Magnetic path length \( I_m \)
Magnetic circuit model

\[ \Phi_c + \Phi_g = ni \]
\[ ni = \Phi \left( R_c + R_g \right) \]
\[ R_c = \frac{l_c}{\mu A_c} \]
\[ R_g = \frac{l_g}{\mu_0 A_c} \]

Solution of model

Faraday’s law: \[ v(t) = n \frac{d\Phi(t)}{dt} \]
Substitute for \( \Phi \): \[ v(t) = \frac{n^2}{R_c + R_g} \frac{di(t)}{dt} \]
Hence inductance is \[ L = \frac{n^2}{R_c + R_g} \]
Effect of air gap

\[ ni = \Phi \left( \mathcal{R}_c + \mathcal{R}_s \right) \]

\[ L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_s} \]

\[ \Phi_{sat} = B_{sat}A_z \]

\[ I_{sat} = \frac{B_{sat}A_z}{n} \left( \mathcal{R}_c + \mathcal{R}_s \right) \]

Effect of air gap:
- Decrease inductance
- Increase saturation current
- Inductance is less dependent on core permeability