Lecture 4: Steady-State Averaged Modeling

ECE 481: Power Electronics
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Chapter 3. Steady-State Equivalent Circuit Modeling, Losses, and Efficiency

3.1. The dc transformer model
3.2. Inclusion of inductor copper loss
3.3. Construction of equivalent circuit model
3.4. How to obtain the input port of the model
3.5. Example: inclusion of semiconductor conduction losses in the boost converter model
3.6. Summary of key points
**Ideal Transformer Model**

\[ \frac{V_1}{n_1} = \frac{V_2}{n_2} \left( = \frac{V_3}{n_3} = \frac{V_4}{n_4} = \ldots \right) \]

\[ \sum I_{n_1} + I_{n_1} = 0 \]

\[ + I_{n_3} n_3 + I_{n_4} n_4 + \ldots \]

- Dot Notation
  - \( P_{es} \): term of voltage
  - Current in one dot comes into the other dot

\[ V_4 = n V_1 \]

\[ I_2 = \frac{1}{n} I_1 \]

\[ P_1 = V_1 I_1 \]

\[ P_2 = V_2 I_2 \]

\[ P_2 = n V_1 \times I_1 \]

\[ P_1 = V_1 I_1 = P_1 \]

**Simplifying Circuits With Ideal XF**

- "Pushing" or "reflecting" through a XF
Simplifying Circuits With Ideal XF

![Circuit Diagram]

\[ Z_{eq} = \frac{V_{i+1}}{I_{load}} \]

\[ I_f = -I_{load} \]

\[ I_i = nI_f = -nI_{load} \]

\[ V_i = -I_i Z_a = Z_a n I_{load} \]

\[ V_c = nV_i = n^2 Z_a I_{load} \]

\[ V_2 = V_{load} \]

\[ Z_{eq} = \frac{n^2 Z_a I_{load}}{I_{load}} \]

\[ Z_{eq} \times \frac{1}{n^2} \]

Simplifying Circuits With Ideal XF

![Circuit Diagram]

\[ Z_c = \frac{1}{n C_a} \]

\[ Z_c' = \frac{n}{sC_a} \Rightarrow \frac{C_e}{n^2} \]
3.1. The dc transformer model

Basic equations of an ideal dc-dc converter:

\[
P_{\text{in}} = P_{\text{out}} \quad (\eta = 100\%)
\]

\[
V = M(D) \frac{V_i}{I_i} \quad (\text{ideal conversion ratio})
\]

\[
I_i = M(D) I_o
\]

\[
V_o = n V_i
\]

Those equations are valid in steady-state. During transients, energy storage within filter elements may cause

\[
P_{\text{in}} \neq P_{\text{out}}
\]

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Equivalent circuits corresponding to ideal dc-dc converter equations

\[
P_{\text{in}} = P_{\text{out}} \quad V_i I_i = V I
\]

\[
V = M(D) V_i \quad I_o = M(D) I_i
\]

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Dependent sources

**DC transformer**

**Power input**

**Power output**

**Control input**

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The DC transformer model

Models basic properties of ideal dc-dc converter:
- conversion of dc voltages and currents, ideally with 100% efficiency
- conversion ratio \( M = \frac{V_2}{V_1} \) controllable via duty cycle

- Solid line denotes ideal transformer model, capable of passing dc voltages and currents
- Time-invariant model (no switching) which can be solved to find dc components of converter waveforms

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Example: use of the DC transformer model

1. Original system
2. Insert dc transformer model
3. Push source through transformer
4. Solve circuit

\[ V = M(D) \frac{V_1}{R + M(D) R_1} \]
3.2. Inclusion of inductor copper loss

Dc transformer model can be extended, to include converter nonidealities.

Example: inductor copper loss (resistance of winding):

Insert this inductor model into boost converter circuit:

![Image of inductor circuit]

Analysis of nonideal boost converter

![Image of boost converter circuit]

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Nonideal Boost Converter

\[ \langle v_2 \rangle = \int_0^1 V_2 - I_L R_L - D' \frac{V}{R} \text{d}t \]

\[ \langle i_2 \rangle = \int_0^1 D' I_L - \frac{V}{R} \text{d}t \]

Circuit equations, switch in position 1

Inductor current and capacitor voltage:

\[ v_L(t) = V_L - i(t) R_L \]
\[ i(t) = -\frac{v(t)}{R} \]

Small ripple approximation:

\[ v_L(t) = V_L - I R_L \]
\[ i(t) = -\frac{V}{R} \]
Circuit equations, switch in position 2

\[ v_i(t) = \frac{V_s - \dot{v}(t) R_L - v(t) - V_s - I R_L}{1} = V \]

\[ i_i(t) = \dot{v}(t) - v(t) / R = I - V / R \]

Inductor voltage and capacitor current waveforms

Average inductor voltage:

\[ \langle v_i(t) \rangle = \frac{1}{T_s} \int_{0}^{T_s} v_i(t) \, dt \]

\[ = D(V_s - I R_L) + D'(V_s - I R_L - V) \]

Inductor volt-second balance:

\[ 0 = V_s - I R_L - DT_s V \]

Average capacitor current:

\[ \langle i_c(t) \rangle = D \left( -V / R \right) + D' \left( I - V / R \right) \]

Capacitor charge balance:

\[ 0 = DT_s V - V / R \]
Solution for output voltage

We now have two equations and two unknowns:

\[ 0 = V_c - I R_c - D V \]
\[ 0 = DI + V / R \]

Eliminate \( I \) and solve for \( V_c \):

\[ M(\frac{v}{c}) = \frac{V_c}{\left[ \frac{1 + R_c}{V} \right]} \]
\[ M = M_{\text{ideal}} = \left( \frac{1 + R_c}{D} \right) \]

3.3. Construction of equivalent circuit model

Results of previous section (derived via inductor volt-sec balance and capacitor charge balance):

1. \( \langle v_c \rangle = 0 = V_c - I R_c - D V \)
2. \( \langle l_c \rangle = 0 = DI - V / R \)

View these as loop and node equations of the equivalent circuit. Reconstruct an equivalent circuit satisfying these equations.
Inductor voltage equation

\[ \langle v_i \rangle = 0 = V_e - I R_L - \frac{dV}{dt} \]

- Derived via Kirchhoff's voltage law, to find the inductor voltage during each subinterval.
- Average inductor voltage then set to zero.
- This is a loop equation: the dc components of voltage around a loop containing the inductor sum to zero.

- \( I R_L \) term: voltage across resistor of value \( R_L \) having current \( I \).
- \( \frac{dV}{dt} \) term: for now, leave as dependent source.

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Capacitor current equation

\[ \langle i_C \rangle = 0 = \frac{dI}{dt} - V / R \]

- Derived via Kirchhoff's current law, to find the capacitor current during each subinterval.
- Average capacitor current then set to zero.
- This is a node equation: the dc components of current flowing into a node connected to the capacitor sum to zero.

- \( V / R \) term: current through load resistor of value \( R \) having voltage \( V \).
- \( \frac{dI}{dt} \) term: for now, leave as dependent source.
Complete equivalent circuit

The two circuits, drawn together:

The dependent sources are equivalent to a \( D : 1 \) transformer:

Dependent sources and transformers

- sources have same coefficient
- reciprocal voltage/current dependence

Solution of equivalent circuit

Converter equivalent circuit

Refer all elements to transformer secondary:

Solution for output voltage using voltage divider formula:

\[
V = \frac{V_1}{D} \cdot \frac{R}{R + \frac{K_1}{D^2}} = \frac{V_1}{D} \cdot \frac{1}{1 + \frac{K_1}{D^2} R}
\]

\[
M_{\text{ideal}} = \frac{4}{D} \left( 1 + \frac{K_1}{D^2} \right) = M_{\text{ideal}}
\]
Solution for input (inductor) current

\[ I = \frac{V_s}{D^2 R + R} = \frac{V_s}{D^2} \left( 1 + \frac{R_s}{D^2 R} \right) \]

Solution for converter efficiency

\[ P_o = (V_s) (I) \]
\[ P_{in} = (V_s) (D I) \]
\[ \eta = \frac{P_o}{P_{in}} = \frac{(V_s) (D I)}{(V_s) (D I)} = \frac{V_s D I}{V_s D I} = \frac{M}{M_{ideal}} \]
\[ \eta = \frac{1}{1 + \frac{R_s}{D^2 R}} \]

\[ M = M_{ideal} \]
3.4. How to obtain the input port of the model

Buck converter example. — use procedure of previous section to derive equivalent circuit

1. $i_g = i_c$
2. $i_2 = \psi$

Average inductor voltage and capacitor current:

$$\langle v_L \rangle = 0 = D V_i - I_c L - V_c$$
$$\langle i_L \rangle = 0 = I_c - V_c / R$$
Construct equivalent circuit as usual

\[ \langle v_i \rangle = 0 = DV_i - I_r R_i - V_C \]

\[ \langle i_o \rangle = 0 = I_r - V_C / R \]

What happened to the transformer?

* Need another equation

Add 1 eq: Averaging of input port

Modeling the converter input port

Input current waveform \( i_i(t) \):

\[ i_i(t) = \begin{cases} \overline{i_i} & 0 \leq t < T_s \\ 0 & T_s \leq t \end{cases} \]

Dc component (average value) of \( i_i(t) \) is

\[ \overline{i_i} = \frac{1}{T_s} \int_0^{T_s} i_i(t) \, dt = DI_i \]

\[ \mathbb{I}_D < D I_i \]
Input port equivalent circuit

\[ I_s = \frac{1}{T_s} \int_{0}^{T_s} i_s(t) \, dt = DI_L \]

Complete equivalent circuit, buck converter

Input and output port equivalent circuits, drawn together:

Replace dependent sources with equivalent DC transformer:

volt-sec balance
amp-ohm balance
3.5. Example: inclusion of semiconductor conduction losses in the boost converter model

Boost converter example

Models of on-state semiconductor devices:
- MOSFET: on-resistance $R_{on}$
- Diode: constant forward voltage $V_f$ plus on-resistance $R_D$

Insert these models into subinterval circuits

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