Operating States of a Power System

Power systems operate in one of three operating states:

**Normal state:**
Loads = Generation - Losses
Operational constraints are NOT violated.

- Secure normal: No Action
- Insecure normal: Preventive control action (SCOPF)

**Emergency state:**
Operating constraints are violated
Requires immediate corrective action.

**Restorative state:**
Load versus generation balance is to be restored
Requires restorative control actions.
Operating States of a Power System

- **NORMAL STATE**
- **SECURE or INSECURE**
- **RESTORATIVE STATE**
- **PARTIAL OR TOTAL BLACKOUT**
- **EMERGENCY STATE**
- **OPERATIONAL LIMITS ARE VIOLATED**
Classical Role of State Estimation
Facilitating Static Security Analysis

Security Analysis:

Monitoring the system, identifying its operating state, determining necessary preventive actions to make it secure.

Monitoring involves RTU's to measure and telemeter various quantities and a state estimator

Measured quantities:

**Flows**: line power flows
**Phasor Magnitude**: bus voltage and line current magnitudes
**Phasor Angle**: phase angle for bus voltage and line current
**Injections**: generator outputs and loads
**Status**: circuit breaker and switch status information, transformer tap positions
State Estimation Functions

**Topology processor:**
Creates one-line diagram of the system using the detailed circuit breaker status information.

**Observability analysis:**
Checks to make sure that state estimation can be performed with the available set of measurements.

**State estimation:**
Estimates the system state based on the available measurements.

**Bad data processing:**
Checks for bad measurements. If detected, identifies and eliminates bad data.

**Parameter and structural error processing:**
Estimates unknown network parameters, checks for errors in circuit breaker status.
State Estimation and Related Functions

Weighted Least Squares (WLS) Estimator

- Analog Measurements: $P_i, Q_i, P_f, Q_f, V, I, \theta_k, \delta_{ki}$
- Topology Processor
- Network Observability Analysis
- State Estimator (WLS)
- Load Forecasts
- Generation Schedules
- Pseudo Measurements: [injections: $P_i, Q_i$]
- Bad Data Processor
- Circuit Breaker Status: Assumed or Monitor
Communication Infrastructure

SCADA / EMS Configuration

© Ali Abur
Energy Management System Applications
SCADA / EMS Configuration

Measurements → State Estimation → Security Monitoring → Contingency Analysis → On-line Power Flow

Emergency Control → Security Monitoring
Restorative Control → On-line Power Flow

Secure
Y → STOP
N → Security Constrained OPF → Preventive Action

Topology Processor → Load Forecasting → External Equivalents
Problem Statement

• \([z]\) : Measurements
  P-Q injections
  P-Q flows
  V magnitude, I magnitude

• \([x]\) : States
  V, \(\theta\), Taps (parameters)

**EXAMPLE:**

• \([z]\) = \([ P12; P13; P23; P1; P2; P3; V1; Q12; Q13; Q23; Q1; Q2; Q3 ]\)
  \(m = 13\) (no. of measurements)

• \([x]\) = \([ V1; V2; V3; \theta 2; \theta 3 ]\)
  \(n = 5\) (no. of states)
Network Model

Bus/branch and bus/breaker Models

Bus/Busbreaker

Topology Processor

Bus/Branch
Measurements

Bus/branch and bus/breaker Models

Bus/branch

Bus/Breaker

© Ali Abur
Measurement Model

\[ [z_m] = [h([x])] + [e] \]

\( z_i \) : true measurement
\( e_i \) : measurement error

\( e_i = e_s + e_r \)

systematic \hspace{1cm} \text{random}
Assumptions

• $e_i \sim N(0, \sigma_i^2)$
• Holds true if:
  
  $e_s = 0$, $e_r \sim N(0, \sigma_i^2)$

• If $e_s \neq 0$, then $E(e_i) \neq 0$, i.e. SE will be biased!
Consider the random variables $X_1, X_2, \ldots, X_n$ with a p.d.f of $f(X | \theta)$, where $\theta$ is unknown.

The joint p.d.f of a set of random observations $x = \{ x_1, x_2, \ldots , x_n \}$ will be expressed as:

$$f_n( x | \theta) = f(x_1 | \theta) f(x_2 | \theta) \cdots f(x_n | \theta)$$

This joint p.d.f is referred to as the **Likelihood Function**.

The value of $\theta$, which will maximize the function $f_n( x | \theta)$ will be called the **Maximum Likelihood Estimator (MLE)** of $\theta$. 
Maximum Likelihood Estimator (MLE)

Normal (Gaussian) Density Function, $f(z)$

$$f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{1}{2} \left( \frac{z - \mu}{\sigma} \right)^2 \right\}$$

Likelihood Function, $f_m(z)$

$$f_m(z) = f_m(z_1)f_m(z_2) \cdots f_m(z_m)$$

Log-Likelihood Function, $L$

$$L = \log f_m(z) = \sum_{i=1}^{m} \log f(z_i)$$

$$= -\frac{1}{2} \sum_{i=1}^{m} \left( \frac{z_i - \mu_i}{\sigma_i} \right)^2 - \frac{m}{2} \log 2\pi - \sum_{i=1}^{m} \log \sigma_i$$
Maximum Likelihood Estimator (MLE)

Weighted Least Squares (WLS) Estimator

Given the set of observations $z_1, z_2, \ldots, z_n$, MLE will be the solution to the following:

\[
\text{Maximize } f_m(z) \\
\text{OR} \\
\text{Minimize } \sum_{i=1}^{m} \left( \frac{z_i - \mu_i}{\sigma_i} \right)^2
\]

Defining a new variable "r", measurement residual:

\[
\text{Minimize } \sum_{i=1}^{m} W_{ii} r_i^2 \\
\text{Subject to } z_i = h_i(x) + r_i \quad i = 1, \ldots, m \\
\mu_i = E(z_i) = h_i(x)
\]

The solution of the above optimization problem is called the weighted least squares (WLS) estimator for $x$. 
Measurement Model

Given a set of measurements, \([z]\) and the correct network topology/parameters:

\[ [z] = [h ([x])] + [e] \]

- Measurements:
  - Known!
  - They are measured
  - Contain errors

- True System States:
  - Unknown!
  - Can be measured or estimated

- Measurement Errors:
  - Unknown!
  - Can not be directly measured or computed
Measurement Model

Following the state estimation, the estimated state will be denoted by $\hat{x}$:

$$[z] = [h (\hat{x})] + [r]$$

- **Measurements**: They are measured.
- **Estimated System States**: Contain errors.
- **Measurement Residuals**: Computed.
Simple Example

\[ Z = h \theta + e \]

\[
\begin{align*}
\text{SLOPE} &= \theta^* \\
\text{ESTIMATED MEASUREMENT} &\quad \text{MEASURED VALUE} \\
r_i &\quad \text{MEASUREMENT RESIDUAL} = Z - h \theta^*
\end{align*}
\]
Weighted Least Squares (WLS) Estimation

Minimize \( \omega_1 r_1^2 + \omega_2 r_2^2 + \omega_3 r_3^2 + \omega_4 r_4^2 \)

What are weights, \( w_i \) ?

\[ \omega_i = \frac{1.0}{\sigma_i^2} \]

How are they chosen?

\[ \sigma_i^2 \] Assumed error variance of measurement “\( i \)”. 
Network Observability

Definitions

Fully observable network:

A power system is said to be fully observable if voltage phasors at all system buses can be uniquely estimated using the available measurements.
Network Observability

Necessary and Sufficient Conditions

\[ \mathbf{Z}_p = \mathbf{H} \cdot \mathbf{\theta} \]

\[
\begin{bmatrix}
  z_1 \\
  z_2 \\
  z_3 \\
  \vdots \\
  z_m
\end{bmatrix} =
\begin{bmatrix}
  H_{11} & \cdots & H_{1n} \\
  H_{21} & \cdots & H_{2n} \\
  H_{31} & \cdots & \vdots \\
  \vdots & \cdots & \vdots \\
  H_{m1} & \cdots & H_{mn}
\end{bmatrix}
\begin{bmatrix}
  \theta_1 \\
  \vdots \\
  \theta_n
\end{bmatrix}
\]

State Vector = \([\theta_2 \ \theta_3]\]

\[ m \geq n \Rightarrow \text{NECESSARY BUT “NOT” SUFFICIENT} \]

EXAMPLE: \[ m = 2, n = 2, \text{UNOBSERVABLE SYSTEM} \]

\[ \text{Rank}(\mathbf{H}) = n \Rightarrow \text{SUFFICIENT} \]
Measurement Classification

Types of Measurements

1. **CRITICAL MEASUREMENTS**
   
   *WHEN REMOVED, THE SYSTEM BECOMES UNOBSERVABLE*

2. **REDUNDANT MEASUREMENTS**
   
   *CAN BE REMOVED WITHOUT AFFECTING NETWORK OBSERVABILITY*
CRITICAL MEASUREMENTS

• If they have gross errors, such errors cannot be detected

• Measurement residuals will always be equal to zero, i.e. critical measurements will be perfectly satisfied by the estimated state

• If they are lost or temporarily unavailable, the system will no longer be observable, thus state estimation cannot be executed
Network Observability

Definitions

Unobservable branch:

• If the system is found not to be observable, it will imply that there are *unobservable* branches whose power flows can not be determined.

Observable island:

• *Unobservable* branches connect *observable* islands of an *unobservable* system. State of each observable island can be estimated using any one of the buses in that island as the reference bus.
Network Observability

Definitions

RED LINES: Unobservable Branches
If the system is found unobservable, use pseudo-measurements in order to merge observable islands.

Pseudo-measurements:
• Forecasted bus loads
• Scheduled generation

Select pseudo-measurements such that they are critical.

Errors in critical measurements do not propagate to the residuals of the other (redundant) measurements.
Observable Islands

Unobservable Branches

- ISLAND 1
- ISLAND 2
- ISLAND 3
• Fixed Topology:
  – Assume a fixed network topology
  – Place meters to observe the network

• Robust Strategies:
  – (N-1) Security
    • Loss of measurements
    • Outage of branches (loss of lines, transformers)
Meter Placement

Robust Against Breaker Operations

© Ali Abur
Meter Placement

Robust Against Breaker Operations

© Ali Abur
Given a measurement configuration \( \{Z\} \) and the network topology \( \{T\} \):

- Can the state \([X]\) be estimated?
- Can \([X]\) be estimated from the same \(\{Z\}\) for different topologies \(\{T1\}, \{T2\}, \ldots \text{etc}\)?
- Can \([X]\) be estimated for the same \(\{T\}\) if one or more measurements are lost \(\{Z\} \rightarrow \{Z'\}\)?
Meter Placement

Robust Measurement Design

ROBUST DESIGN

NON-ROBUST DESIGN
Place measurements at optimal locations so that:

- The system will not contain any critical measurements
- Network will remain observable in case of line/transformer outages or measurement loss and failures
Optimal Meter Placement

30-Bus System Example
If an estimator remains insensitive to a finite number of errors in the measurements, then it is considered to be **robust**.

Example: Given \( z = \{ 0.9, 0.95, 1.05, 1.07, 1.09 \} \), estimate \( z \) using the following estimators:

1. \( \hat{X}_a = mean\{z_i\} = \frac{1}{5} \sum_{i=1}^{5} z_i \)

2. \( \hat{X}_b = median\{z_i\}, \quad i = 1, \ldots, 5 \)

Solution:
Replace \( z_5 = 1.09 \) by an infinitely large number \( z'_5 = \infty \).

The new estimate will then be:
\[
\hat{X}'_a = \frac{1}{5} \sum_{i=1}^{5} z_i = \infty
\]

This estimator is NOT robust.

Replace both \( z_5 \) and \( z_4 \) by infinity.

The new estimate will then be:
\[
\hat{X}'_b = 1.05 \quad \text{(finite)}
\]

This is a more robust estimator than the one above.
Consider the problem:

Minimize \( \sum_{i=1}^{m} \rho(r_i) \)

Subject to \( z = h(x) + r \)

Where \( \rho(r_i) \) is a chosen function of the measurement residual

In the special case of the WLS state estimation:

\[ \rho(r_i) = \frac{r_i^2}{\sigma_i^2} \]
Robust Estimation

M-Estimators

Some Examples of M-Estimators

Quadratic-Constant

\[ \rho(r_i) = \begin{cases} \frac{r_i^2}{\sigma_i^2} & \frac{r_i}{\sigma_i} \leq a \\ \frac{a_i^2}{\sigma_i^2} & \text{otherwise} \end{cases} \]

Quadratic-Linear

\[ \rho(r_i) = \begin{cases} \frac{r_i^2}{\sigma_i^2} & \frac{r_i}{\sigma_i} \leq a \\ 2a \sigma_i \ |r_i| - a^2 \sigma_i^2 & \text{otherwise} \end{cases} \]

Least Absolute Value (LAV)

\[ \rho(r_i) = |r_i| \]
Robust Estimation

*LAV Estimator Example*

**Measurement Model:**

\[ z_i = A_{i1}x_1 + A_{i2}x_2 + e_i \quad i = 1, \ldots, 5 \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>( Z_i )</th>
<th>( A_{i1} )</th>
<th>( A_{i2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.01</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>3.52</td>
<td>0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>3</td>
<td>-5.49</td>
<td>-1.5</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>4.03</td>
<td>0.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>5</td>
<td>5.01</td>
<td>1.0</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

**LAV estimate for** \( x \) **and measurement residuals:**

\[ x^T = [3.005; -4.010] \]

\[ r^T = [0.0; 0.0125; 0.02; 0.02; 0.0] \]

**CHANGE measurement 5 from 5.01 to 15.01 (Simulated Bad Datum):**

**LAV estimate for** \( x \) **and measurement residuals:**

\[ x^T = [3.02; -4.02] \]

\[ r^T = [0.0; 0.0; 0.045; 0.01; 9.98] \]
Consider $X_1, X_2, \ldots, X_N$, a set of $N$ independent random variables where:

$$X_i \sim N(0,1)$$

Then, a new random variable $Y$ will have a $\chi^2$ distribution with $N$ degrees of freedom, i.e.:

$$\sum_{i=1}^{N} X_i^2 = Y \sim \chi^2_N$$
Now, consider the function

\[
f(x) = \sum_{i=1}^{m} R_{ii}^{-1} e_i^2 = \sum_{i=1}^{m} \left( \frac{e_i^2}{R_{ii}} \right) = \sum_{i=1}^{m} \left( e_i^N \right)^2
\]

and assuming:

\[
e_i^N \sim N(0,1)
\]

f(x) will have a \( \chi^2 \) distribution with at most \( (m-n) \) degrees of freedom.

In a power system, since at least \( n \) measurements will have to satisfy the power balance equations, at most \( (m-n) \) of the measurement errors will be linearly independent.
**Bad Data Detection**

*Chi-squares Distribution:*

\[ \Pr\{X \geq x_t\} = \int_{x_t}^{\infty} \chi^2(u) \cdot du \]

Choose \( x_t \) such that:

\[ \Pr\{X \geq x_t\} = \alpha = 0.05 \]

**Test:**

*If the measured \( X \geq x_t \), then with 0.95 probability, bad data will be suspected.*
Bad Data Detection

Detection Algorithm  $\chi^2$ -- Test

Solve the WLS estimation problem and compute the objective function:

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{\sigma_i^2}$$

Look up the value corresponding to $p$ (e.g. 95 %) probability and $(m-n)$ degrees of freedom, from the Chi-squares distribution table.

Let this value be $\chi_{(m-n), p}$ Here: $p = \Pr\{ J(x) \leq \chi_{(m-n), p} \}$

Test if

$$J(x) \geq \chi_{(m-n), p}$$

If yes, then bad data are detected.

Else, the measurements are not suspected to contain bad data.
Linear measurement model:  \[ \Delta \hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z \]

\[ \Delta \hat{z} = H \Delta \hat{x} = K \Delta z, \quad K = H (H^T R^{-1} H)^{-1} H^T R^{-1} \]

\(K\) is called the hat matrix. Now, the measurement residuals can be expressed as follows:

\[ r = \Delta z - \Delta \hat{z} \]
\[ = (I - K) \Delta z \]
\[ = (I - K)(H \Delta x + e) \]
\[ = (I - K)e \quad \text{[Note that } KH = H\text{]} \]
\[ = Se \]

where \(S\) is called the residual sensitivity matrix.
The residual covariance matrix $\Omega$ can be written as:

$$E[rr^T] = \Omega = S \cdot E[e \cdot e^T] \cdot S^T$$

$$= S \cdot R \cdot S^T = S \cdot R$$

Hence, the normalized value of the residual for measurement $i$ will be given by:

$$r_i^N = \frac{r_i}{\sqrt{\Omega_{ii}}} = \frac{r_i}{\sqrt{R_{ii}S_{ii}}}$$
Measurements can be classified as **critical** and **redundant (or non-critical)** with the following properties:

- A *critical measurement* is the one whose elimination from the measurement set will result in an *unobservable system*.

- The row/column of $S$ corresponding to a critical measurement will be zero.

- The *residuals of critical measurements* will always be zero, and therefore errors in critical measurements can not be detected.

*It can be shown that if there is a single bad data in the measurement set (provided that it is not a critical measurement) the largest normalized residual will correspond to bad datum.*
Bad Data Identification / Elimination

Two commonly used approaches:

1. Post-processing of measurement residuals – Largest normalized residuals
2. Modifying measurement weights during iterative solution of WLS estimation
Bad Data Identification

Steps of the largest normalized residual test for identification of single and non-interacting multiple bad data:

Compute the elements of the measurement residual vector:

Compute the normalized residuals

Find k such that $r_k^N$ is the largest among all $r_i^N$, $i = 1,...,m$.

If $r_k^N > c$, then the k-th measurement will be suspected as bad data.

Else, stop, no bad data will be suspected. Here, c is a chosen identification threshold, e.g. 3.0.

Eliminate the k-th measurement from the measurement set and go to step 1.
Iteratively Re-weighted Least Squares

Iterative Elimination of Bad Data

• Carry out the first iteration of the WLS estimation solution
• Calculate the measurement residuals
• Modify the measurement weights according to:

\[ \omega_i \approx \frac{k}{r_i^2} \]

• Update the weights and continue to the next iteration

This approach is computationally much cheaper than the largest normalized residual test, however:

It may not always work due to the masking of bad measurements.
Use of Synchrophasor Measurements

- Given enough phasor measurements, state estimation problem will become LINEAR, thus can be solved directly without iterations

Conventional Measurements

\[ Z = h(X) + e \]

\[ \Delta \hat{X} = (H^T R^{-1} H)^{-1} R^{-1} \Delta Z \quad \text{Iterative} \]

Phasor Measurements

\[ Z = H \cdot X + e \]

\[ \hat{X} = (H^T R^{-1} H)^{-1} R^{-1} Z \quad \text{Non-iterative} \]
Use of Synchrophasor Measurements

- Given at least one phasor measurement, there will be no need to use a reference bus in the problem formulation.
- Given unlimited number of available channels per PMU, it is sufficient to place PMUs at roughly 1/3rd of the system buses to make the entire system observable just by PMUs.

<table>
<thead>
<tr>
<th>Systems</th>
<th>No. of zero injections</th>
<th>Number of PMUs</th>
<th>Ignoring zero Injections</th>
<th>Using zero injections</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-bus</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>57-bus</td>
<td>15</td>
<td>17</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>118-bus</td>
<td>10</td>
<td>32</td>
<td>29</td>
<td></td>
</tr>
</tbody>
</table>
Performance Metrics

• State Estimation Solution

  • **Accuracy:**

    Variance of State = inverse of the gain matrix, $[G]$
    
    $= E[(x - x^*)(x - x^*)']$

  • **Convergence:**

    Condition Number = Ratio of the largest to smallest eigenvalue

    Large condition number implies an ill-conditioned problem.
Performance Metrics

• Measurement Design
  
  • **Critical Measurements:**
    
    Number of critical measurements and their types
  
  • **Local Redundancy**
    
    Number of measurements incident to a given bus
  
  • **(N-1) Robustness**
    
    Capability of the measurement configuration to render a fully observable system during single measurement and branch losses
Performance Metrics

- Measurement Quality

  - **Performance Index (WLS objective function):**

    Weighted sum of squares of residuals. Has a Chi-Squares distribution. Large numbers imply presence of bad data in the measurement set.

  - **Largest Absolute Normalized Residual:**

    If larger than 3.0, the measurement corresponding to the largest absolute value will be suspected of gross errors.

  - **Sample variance (Based on historical data):**

    Measurement weights are based on sample error variances calculated according to historical data and estimation results. They reflect the quality of individual measurements.
Summary

• State Estimation and its related functions are reviewed.
• Importance of measurement design is illustrated.
• Commonly used methods of identifying and eliminating bad data are described.
• Impact of incorporating phasor measurements on state estimation is briefly reviewed.
• Metrics for state estimation solution, measurement design and measurement quality are suggested.


