Lecture 14: Discontinuous Conduction Mode Examples

ECE 481: Power Electronics
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Buck Converter Example

Synchronous Implementation (No DCR)

\[ i_L(t) \]

\[ i_D(t) \]

\[ V_d = 12 \quad V_s = 3 \]

\[ L = \frac{15}{f} \quad T_s \geq 50 \mu s \]

\[ R = 1 \Omega \]

\[ R \to 10 \Omega \]

\[ D = \frac{2}{15} = 0.25 \quad V_s = 3 \quad i_s = \frac{3V}{10\Omega} = 0.3A \]

\[ \Delta i_L = 0.56A \]

\[ \Delta i_D \]
$K_{crit}$ and $R_{crit}$

Buck converter:

In CCM when $I_c > 0$:

$\Delta I_c = \frac{V}{E} \frac{D V_o}{R}$

In DCM when $I_c < 0$:

$\Delta I_c = \frac{V}{E} \frac{D V_o}{R}$

for CCM:

$\frac{\Delta V}{R} \geq \frac{2 L}{R T_s} = \frac{2 L}{R \tau_{crit}}$

$K = \left[ \frac{2 L}{R T_s} \right] = K_{crit}(D)$

for CCM

$R > R_{crit}(D)$

DCM Mode Boundary: Summary

$K > K_{sat}(D)$ or $R < R_{sat}(D)$ for CCM

$K < K_{sat}(D)$ or $R > R_{sat}(D)$ for DCM

Table 5.3 CCM-DCM mode boundaries for the buck, boost, and buck-boost converters

<table>
<thead>
<tr>
<th>Converter</th>
<th>$K_{sat}(D)$</th>
<th>$\max_{D \in [0,1)} (K_{sat})$</th>
<th>$R_{sat}(D)$</th>
<th>$\min_{D \in [0,1)} (R_{sat})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>$(1 - D)$</td>
<td>$\frac{1}{2L}$</td>
<td>$\frac{1}{2(1 - D) T_s}$</td>
<td>$\frac{2L}{T_s}$</td>
</tr>
<tr>
<td>Boost</td>
<td>$D (1 - D)^2$</td>
<td>$\frac{4}{2L}$</td>
<td>$\frac{2L}{D (1 - D)^2 T_s}$</td>
<td>$\frac{2L}{2 T_s}$</td>
</tr>
<tr>
<td>Buck-boost</td>
<td>$(1 - D)^2$</td>
<td>$1$</td>
<td>$\frac{2L}{(1 - D)^2 T_s}$</td>
<td>$2 \frac{L}{T_s}$</td>
</tr>
</tbody>
</table>

Chapter 5: Discontinuous conduction mode
Finding the Conversion Ratio $M(D,K)$

Analysis techniques for the discontinuous conduction mode:

- Inductor volt-second balance: $\langle v_i \rangle = \frac{1}{2} \int_{t_1}^{t_2} v_i(t) \, dt = 0$
- Capacitor charge balance: $\langle i_c \rangle = \frac{1}{2} \int_{t_1}^{t_2} i_c(t) \, dt = 0$

Small ripple approximation sometimes applies:

- $v(t) \approx V$ because $\Delta v \ll V$
- $i(t) \approx I$ is a poor approximation when $\Delta i > I$

Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

Buck Converter in DCM

Diagram showing the operation of the buck converter in DCM with subintervals and states.
Subinterval Analysis

\[ v_L = v_0 - v(t) \approx v_0 - v \quad \text{SRA still applies} \]

1. \[ i_c = i_C(t) - \frac{v(t)}{R} \approx \frac{v_0}{R} \quad \text{SRA no longer applies} \]

\[ v_L = v_0 - v(t) \approx -v \]

2. \[ i_c = i_C(t) - \frac{v(t)}{R} \approx \frac{v_0}{R} \]

\[ v_L = \Psi \]

3. \[ i_c = i_C(t) - \frac{v(t)}{R} \approx \frac{v_0}{R} \]

Waveforms in DCM

\[ v_L(t) = v_0(t) = D_1(t) v_0(t) + D_2(t) v_0(t) + D_3(t) v_0(t) \]

\[ i_c(t) = \frac{1}{R} \int v_L(t) \, dt + \frac{v(t)}{R} \quad \text{Constant}\]

\[ v_L(t) = \frac{1}{R} \int i_c(t) \, dt + \frac{v(t)}{R} \]

\[ v_L(t) = \frac{1}{R} \left[ \frac{1}{2} D_1 v_0^2 + \frac{1}{2} D_2 v_0^2 \right] \]
Solving $M(D,K)$

Two equations and two unknowns ($V$ and $D_s$):

$$\begin{align*}
V &= V_s \frac{D_1}{D_1 + D_2} \quad \text{(from inductor volt-second balance)} \\
\frac{V}{R} &= \frac{D_1 T_i}{2L} (D_1 + D_2) (V_s - V) \quad \text{(from capacitor charge balance)}
\end{align*}$$

Eliminate $D_2$, solve for $V$:

$$V = \frac{V_s}{1 + \sqrt{1 + 4K D_1}}$$

where $K = 2L / RT$,

valid for $K < K_{\text{rev}}$

Controlled duty cycle
Buck Converter $M(D,K)$

$M(D,K) = \begin{cases} \frac{2}{1 + \sqrt{1 + 4K/D^2}} & \text{for } K > \frac{1}{2D} \\ \text{CCM} & \text{for } K < \frac{1}{2D} \end{cases}$

Chapter 5: Discontinuous conduction mode

Boost Converter in DCM

Mode boundary:

- $I > \Delta I$ for CCM
- $I < \Delta I$ for DCM

Previous CCM soln:

- $i = \frac{V}{D^2 R}$
- $\Delta I = \frac{V^2}{2L} DT_c$

Chapter 5: Discontinuous conduction mode
Boost DCM Boundary

\[ \frac{V_i}{D'R} > \frac{DT_i V_i}{2L} \quad \text{for CCM} \]

\[ \frac{2L}{R T_i} > DD'^2 \quad \text{for CCM} \]

- \( K > K_{cm(D)} \) for CCM
- \( K < K_{cm(D)} \) for DCM

where \( K = \frac{2L}{R T_i} \) and \( K_{cm(D)} = DD'^1 \)

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Boost Converter Subintervals

- Subinterval 1
- Subinterval 2
- Subinterval 3

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Chapter 5: Discontinuous conduction mode
Boost Conversion Ratio

\[ v_L = v_g \]
\[ i_c = \frac{v_m}{R} \approx -\frac{V}{R} \]

\[ v_L = v_g - v_m \approx v_L - V \]
\[ i_L = i_L(t) - \frac{v(t)}{R} \approx i_L(t) - \frac{V}{R} \]

\[ v_L = \phi \]
\[ i_L(t) \quad \frac{v(t)}{R} \approx -\frac{V}{R} \]

Boost Waveforms in DCM

\[ \langle v_L \rangle = \phi = D_1 v_g + D_2 (v_g - V) + D_2 (-V) \]
\[ \phi = v_g (D_1 + D_2) + D_2 (-V) \]
\[ \frac{v}{v_g} = \frac{D_1 + D_2}{D_2} \]

\[ \langle i_L \rangle = \frac{v}{R} + \frac{1}{L} \int_{t_0}^{t} L(v(t) dt) \]
\[ = \frac{v}{R} + \langle i_0(t) \rangle \]

\[ v_L(t) \]
\[ v(t) \]
\[ i_L(t) \]
**Boost Cap-Charge Balance**

\[ \phi = \langle i_C \rangle = \langle i_{L} \rangle - \langle i_{diode} \rangle \]

\[ i_C = \phi = \frac{V}{2} + \left[ \frac{1}{2} \frac{1}{L} \frac{1}{T_s} i_{diode} \right] \frac{1}{R} \]

\[ i_{diode} = \phi = \frac{V}{2} + \frac{1}{R} \left[ \frac{1}{2} \frac{1}{L} \frac{1}{T_s} i_C \right] \]

\[ \frac{V}{R} = \frac{D \cdot D_i \cdot V \cdot T_s}{2I} \]

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**Boost DCM Conversion Ratio**

\[ V_p - V_{out} - \frac{V_d^2}{K} = 0 \]

Use quadratic formula:

\[ \frac{V}{V_r} = \frac{1}{2} \left( \sqrt{1 + 4 \frac{D_i^2}{K}} - 1 \right) \]

Note that one root leads to positive \( V \), while other leads to negative \( V \). Select positive root:

\[ \frac{V}{V_r} = M(D, K) = \left( 1 + \sqrt{1 + 4 \frac{D_i^2}{K}} \right) \frac{K}{2} \]

where valid for \( K = 2L / RT_s \)

\[ \frac{K}{K_{c.d}(D)} = \frac{1}{K_{c.d}(D)} \]

Transistor duty cycle \( D = \) interval \( T \) duty cycle \( D_i \)

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Chapter 5: Discontinuous conduction mode
**Boost Conversation Ratio**

\[
M = \begin{cases} 
\frac{1}{1-D} & \text{for } K > K_{\text{crit}} \\
\frac{1}{1 + \sqrt{1 + 4D^2 / K}} & \text{for } K < K_{\text{crit}}
\end{cases}
\]

Approximate \( M \) in DCM:

\[
M = \frac{1}{2} + \frac{D}{\sqrt{K}}
\]

**Summary of DCM Characteristics**

<table>
<thead>
<tr>
<th>Converter</th>
<th>( K_{\text{crit}}(D) )</th>
<th>DCM ( M(D,K) )</th>
<th>DCM ( D_2(D,K) )</th>
<th>CCM ( M(D) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>((1 - D)^2)</td>
<td>(\frac{1}{D} M(D,K))</td>
<td>(\frac{D}{\sqrt{K}})</td>
<td>(\frac{1}{2} - \frac{D}{\sqrt{K}})</td>
</tr>
<tr>
<td>Boost</td>
<td>(D(1 - D)^2)</td>
<td>(\frac{1 + \sqrt{1 + 4D^2 / K}}{D} M(D,K))</td>
<td>(\frac{D}{\sqrt{K}})</td>
<td>(\frac{1}{1 - D})</td>
</tr>
<tr>
<td>Buck-boost</td>
<td>((1 - D)^2)</td>
<td>(\frac{D}{\sqrt{K}})</td>
<td>(\frac{1}{1 - D})</td>
<td>(\frac{1}{2} - \frac{D}{\sqrt{K}})</td>
</tr>
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</table>

with \( K = 2L / KT_p \), DCM occurs for \( K < K_{\text{crit}} \)

\[ D_2 \rightarrow \text{from table} \]

\[ D_3 = 1 - D_1 - D_2 \]
Summary of DCM Characteristics

- DCM buck and boost characteristics are asymptotic to $M = 1$ and to the DCM buck-boost characteristic.
- DCM buck-boost characteristic is linear.
- CCM and DCM characteristics intersect at mode boundary. Actual $M$ follows characteristic having larger magnitude.
- DCM boost characteristic is nearly linear.

Chapter 5 Summary

1. The discontinuous conduction mode occurs in converters containing current- or voltage-unidirectional switches, when the inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity.
2. Conditions for operation in the discontinuous conduction mode can be found by determining when the inductor current or capacitor voltage ripples and dc components cause the switch on-state current or off-state voltage to reverse polarity.
3. The dc conversion ratio $M$ of converters operating in the discontinuous conduction mode can be found by application of the principles of inductor volt-second and capacitor charge balance.
4. Extra care is required when applying the small-ripple approximation. Some waveforms, such as the output voltage, should have small ripple which can be neglected. Other waveforms, such as one or more inductor currents, may have large ripple that cannot be ignored.
5. The characteristics of a converter changes significantly when the converter enters DCM. The output voltage becomes load-dependent, resulting in an increase in the converter output impedance.