Single Pole Response

\[ G(s) = \frac{v_2(s)}{v_1(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + \frac{1}{sC}} \]

\[ \omega_0 = \frac{1}{RC} \]

\[ f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC} \]

Plotting a Single Pole Response

1. \( \omega < \omega_0 \rightarrow \left(\frac{\omega}{\omega_0}\right)^2 \ll 1 \rightarrow 1 + \left(\frac{\omega}{\omega_0}\right)^2 \approx 1 \rightarrow \|G\|_{dB} \approx \|OdB\| \)
2. \( \omega > \omega_0 \rightarrow \left(\frac{\omega}{\omega_0}\right)^2 \gg 1 \rightarrow \left(\frac{\omega}{\omega_0}\right)^2 \approx \left(\frac{\omega}{\omega_0}\right)^2 \rightarrow \|G\|_{dB} \approx 20\log \frac{\omega}{\omega_0} \)
3. \( \omega = \omega_0 \rightarrow \|G\|_{dB} \approx 20\log \frac{1}{\frac{1}{2\pi RC}} \approx -20\log 2 \approx -3dB \)
Summary: Single Real Pole

Bode Plot: Real Zero
RHP Zero

\[ G(s) = \left(1 - \frac{s}{\omega_0}\right) \]

Zero Inverted Pole

"RHP" Zero is "Non-minimum Phase" Zero

Inverted Pole

\[ G(s) = \frac{A_0}{1 + \frac{s}{\omega_0}} \]

40° phase lag
Inverted Zero

\[ G(s) = \frac{1 + \frac{0.6}{3}}{s} = \frac{3}{s} (s^2 + 1) \]

\[ \text{Phase} = \text{angle of } \frac{3}{s} (s^2 + 1) \]

Chapter 8: Converter Transfer Functions

Multiplying Transfer Functions

\[ G_1(s) \cdot G_2(s) = A_1 e^{s \theta_1} \cdot A_2 e^{s \theta_2} \]

\[ = A_1 A_2 e^{s(\theta_1 + \theta_2)} \]

Multiply two TFs

- Phases will add
- Magnitudes will multiply
  (Add in log domain)
Example 1

\[ G(s) = \frac{G_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)} \]

with \( G_0 = 40 \Rightarrow 32 \text{ dB}, f_1 = \omega_1/2\pi = 100 \text{ Hz}, f_2 = \omega_2/2\pi = 2 \text{ kHz} \)

\[ G_0 = 40 \Rightarrow 32 \text{ dB} \]

[Diagram with frequency responses and associated equations.]
Example 2

Determine the transfer function \( A(s) \) corresponding to the following asymptotes:

\[
A(s) = A_0 \frac{\omega_n}{1 + \frac{s}{\omega_n}}
\]

What is \( A_\infty \)?

Let \( s \to 0 \), let \( \omega \to \infty \)

\[ A_\infty = A_0 \frac{\omega_n}{\omega_n} \]

\[ A_0 = A_0 \frac{\omega_n}{\omega_n} \]

\[
\begin{align*}
\| A \| & \quad \| A_0 \| \\
\frac{f_1}{10} & \quad \frac{f_2}{10} \\
0^\circ & \quad -90^\circ & \quad -45^\circ/\text{dec} & \quad +45^\circ/\text{dec} & \quad +20 \, \text{dB/dec} \\
10f_1 & \quad f_1 & \quad 10f_2 & \quad f_2
\end{align*}
\]

Chapter 8: Converter Transfer Functions
8.1.6 Resonant Poles

Example

\[ G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + \frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}} \]

Second-order denominator, of the form

\[ G(s) = \frac{1}{1 + a_1s + a_2s^2} \]

with \( a_1 = \frac{L}{R} \) and \( a_2 = LC \)

How should we construct the Bode diagram?

Standard Form for Complex Poles

\[ G(s) = \frac{1}{1 + 2\zeta\omega_0 + (\frac{s}{\omega_0})^2} \quad \text{or} \quad G(s) = \frac{1}{1 + \frac{\omega_0^2}{Q\omega_0^2} + (\frac{s}{\omega_0})^2} \]

- When the coefficients of \( s \) are real and positive, then the parameters \( \omega_0 \) and \( Q \) are also real and positive.
- The parameters \( \zeta, \omega_0, \) and \( Q \) are found by equating the coefficients of \( s \)
- The parameter \( \omega_0 \) is the angular corner frequency, and we can define \( f_0 = \frac{\omega_0}{2\pi} \)
- The parameter \( \zeta \) is called the damping factor; \( \zeta \) controls the shape of the exact curve in the vicinity of \( f = f_0 \). The roots are complex when \( \zeta < 1 \).
- In the alternative form, the parameter \( Q \) is called the quality factor; \( Q \) also controls the shape of the exact curve in the vicinity of \( f = f_0 \). The roots are complex when \( Q > 0.5 \).
The Q Factor

In a second-order system, \( \zeta \) and \( Q \) are related according to

\[
Q = \frac{\zeta}{\sqrt{2}}
\]

\( Q \) is a measure of the dissipation in the system. A more general definition of \( Q \), for sinusoidal excitation of a passive element or system is

\[
Q = 2\pi \frac{\text{(peak stored energy)}}{\text{(energy dissipated per cycle)}}
\]

For a second-order passive system, the two equations above are equivalent. We will see that \( Q \) has a simple interpretation in the Bode diagrams of second-order transfer functions.

Magnitude Asymptotes

In the form

\[
G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}
\]

let \( s = \omega \) and find magnitude:

\[
|G| = \sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}
\]

Asymptotes are

\[
|G| \rightarrow 1 \text{ for } \omega << \omega_0
\]

\[
|G| \rightarrow \left(\frac{\omega_0}{\omega}\right)^2 \text{ for } \omega >> \omega_0
\]
Exact Magnitude Curve

\[ |G(j\omega)| = \sqrt{\left(1 - \frac{(\omega / \omega_0)^2}{Q^2}\right)^2 + \frac{1}{Q^2} \left(\frac{\omega / \omega_0}{Q}\right)^2} \]

for \( Q \gg \frac{1}{2} \)

At \( \omega = \omega_0 \), the exact magnitude is

\[ |G(j\omega_0)| = Q \quad \text{or, in dB:} \quad |G(j\omega_0)|_{\text{dB}} = |Q|_{\text{dB}} \]

The exact curve has magnitude \( Q \) at \( f = f_0 \). The deviation of the exact curve from the asymptotes is \( |Q|_{\text{dB}} \)

Curves for Varying \( Q \)
Summary: Asymptotes for Complex Poles

For $Q > 0.5$

Magnitude

Phase

The Low-Q Approximation

$\|G\|_{dB}$

$f_1 = \frac{Qf_0}{F(Q)}$  

$\approx Qf_0$  

$0$ dB

$f_0$

$-20$ dB/decade

$-40$ dB/decade

$f_2 = \frac{f_0F(Q)}{Q}$

$\approx \frac{f_0}{Q}$

$10$ dB/decade