Closed-Loop Bandwidth

Consider the case where $T(s)$ can be well-approximated in the vicinity of the crossover frequency as

$$T(s) = \frac{\frac{1}{s \omega_c}}{1 + \frac{s}{\omega_c}}$$

Low-Q Case

$Q = \frac{\omega_0}{\omega_c} = \sqrt{\frac{\omega_0}{\omega_c}}$

low-Q approximation: $Q \omega_c = \omega_0$, $\frac{\omega_0}{Q} = \omega_c$
High-Q Case

\[ \omega = \sqrt{\omega_0 \omega_1} = 2\pi f_c \]

\[ Q = \frac{\omega_0}{\omega_1} = \sqrt{\frac{\omega_0}{\omega_1}} \]

Design Approach

- Assume \( G_c(s) = 1 \), and plot the resulting uncompensated loop gain \( T_u(s) \).
- Examine uncompensated loop gain to determine the needs of the compensator:
  - Is low-frequency gain amplitude large enough to result in low steady-state error?
  - Is \( \phi_m \) sufficient for stability and requirements on ringing/overshoot?
  - Is \( f_c \) high enough for a sufficiently fast response?
- Construct compensator to address shortcomings of \( T_u(s) \):
  - Use “toolbox” of compensators on following slides.
Example: Uncompensated Loop Gain

Proportional (P) Compensator

\[ G_c(s) = G_o \]
Stabilization by (P) Compensator

Another Example
Integral (I) Compensator

\[ G(s) = \frac{K}{s} \]

Stabilization by (I) Compensator
Lag (PI) Compensator

\[ G_c(s) = \frac{G_v}{1 + \frac{\omega_c}{f_c}} \]

Improves low-frequency loop gain and regulation

Example Lag Compensator Design

original (uncompensated)
loop gain is
\[ T_o(s) = \frac{P_o}{1 + \frac{\omega_o}{f_o}} \]

compensator:
\[ G_c(s) = \frac{G_v}{s} \left( 1 + \frac{\omega_c}{f_c} \right) \]

Design strategy:
choose \( G_v \) to obtain desired crossover frequency
\( \omega_o \) sufficiently low to maintain adequate phase margin
**Lead (PD) Compensator**

\[ G_c(s) = G_o \left( \frac{1 + \frac{s}{\omega_n}}{1 + \frac{s}{\omega_p}} \right) \]

Improves phase margin

**Maximum Phase Lead**

\[ f_{\text{max}} = \sqrt{\frac{f_p}{f_i}} \]

\[ \angle G_c(f_{\text{max}}) = \tan^{-1} \left( \frac{\sqrt{\frac{f_p}{f_i} - \sqrt{\frac{f_p}{f_i}}}}{2} \right) \]

\[ \frac{f_i}{f_p} = \frac{1 + \sin \theta}{1 - \sin \theta} \]
Lead Compensator Design

To optimally obtain a compensator phase lead of $\theta$ at frequency $f_0$, the pole and zero frequencies should be chosen as follows:

$$f_p = f_0 \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}}$$
$$f_z = f_0 \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}}$$

If it is desired that the magnitude of the compensator gain at $f_0$ be unity, then $G_{\omega 0}$ should be chosen as:

$$G_{\omega 0} = \frac{f_z}{f_p}$$

Example Lead Compensator Design
Combined (PID) Compensator

\[ G_c(s) = \frac{G_m}{1 + \frac{s}{\omega_h}} \left( 1 + \frac{s}{\omega_0} \right) \]

Example Design of Buck Compensator

Transistor gate driver

\[ f_s = 100 \text{ kHz} \]

Error signal

\[ V_M = 4 \text{ V} \]

Sensor gain

\[ V_{\text{ref}} = 5 \text{ V} \]