Derivation of Volt-second Balance

Inductor defining relation:
\[
\int v_L(t) = \int \frac{di_L(t)}{dt} = L \int \frac{\dot{i}_L(t)}{2} dt = L \int \dot{i}_L = L \left(\dot{i}_L(t) - \dot{i}_L(0)\right)
\]

Integrate over one complete switching period:
\[
i_L(T_s) - i_L(0) = \frac{1}{L} \int_{0}^{T_s} v_L(t) dt
\]

In periodic steady state, the net change in inductor current is zero:
\[
0 = \int_{0}^{T_s} v_L(t) dt
\]

Hence, the total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.

An equivalent form:
\[
0 = \frac{1}{L} \int_{0}^{T_s} v_L(t) dt = \left\{v(t)\right\} L
\]

The average inductor voltage is zero in steady state.

Volt-Second Balance: Direct Application

[Diagram showing two circuits: one with switch in position 1 and another with switch in position 2.]

\[
\langle v_L \rangle = \frac{1}{T_s} \int _0 ^{T_s} v_L(t) dt = \frac{1}{T_s} \left( \int_0 ^{T_s} v_L(t) dt - \int_0 ^{T_s} v_L(t) dt \right) = \frac{1}{T_s} \left( \int_0 ^{T_s} v(t') dt' \right)
\]

\[
\frac{V_s}{V_o} = M = 10
\]
Current Ripple Magnitude

\[ \Delta i_i = \frac{V_s}{L} \cdot DT_s \]

\[ \Delta i_i = \frac{V_s}{2L} \cdot DT_s \]

If I want to reduce \( \Delta i_i \) in a given application, to reduce \( \Delta i_i \): 
- \( \uparrow L \)
- \( \downarrow T_s \) (= \( \uparrow f_s \))

Capacitor Charge Balance

Dual of volt-second balance

\( \langle i_i \rangle = \langle v_s \rangle \), \( \langle i_i \rangle = 0 \)

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Derivation of Capacitor Charge Balance

Capacitor defining relation:

\[ i_c(t) = C \frac{dv_c(t)}{dt} \]

Integrate over one complete switching period:

\[ v_c(T_s) - v_c(0) = \frac{1}{C} \int_0^{T_s} i_c(t) \, dt \]

In periodic steady state, the net change in capacitor voltage is zero:

\[ 0 = \frac{1}{T_s} \int_0^{T_s} i_c(t) \, dt = \langle i_c \rangle \]

Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.

Buck Cap Charge Balance

1. \( i_c(t) = i_{c0} - \frac{v(t)}{R} \)
2. Apply small ripple approx.: \( i_c(t) = I_c \)
3. \( i_d(t) = I_d - \frac{V}{R} \)
4. Apply capacitor charge balance:
   \[ \langle i_c \rangle |_{FS} = \delta = b(I_c + \frac{V}{R}) + b(I_c - \frac{V}{R}) \]
   \[ \delta = I_c - \frac{V}{R} \]
   \[ I_c = \frac{V}{R} \]
The Boost Converter

Boost converter with ideal switch

Realization using power MOSFET and diode

more in Chapter 4

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Boost Subintervals

original converter

switch in position 1

switch in position 2

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**Boost: Subinterval 1**

\[ v_c(t) = v_i \]
\[ i_c(t) = -\frac{v_c(t)}{R} \]
\[ e(t) \rightarrow \text{SRA} \]
\[ v_c(t) = v_i \]
\[ i_c(t) = -\frac{V}{R} \]

**Boost: Subinterval 2**

\[ v_c(t) = v_i - v(t) \]
\[ i_c(t) = i_c(t) - \frac{v(t)}{R} \]
\[ e(t) \rightarrow \text{SRA} \]
\[ v_c(t) = v_i - V \]
\[ i_c(t) = \frac{V}{R} \]
Waveforms

Steady State Solution

Voltage Balance:
\[
\langle v_c \rangle = V_g - D V \quad \rightarrow \quad V = \frac{V_g}{D} \quad \rightarrow \quad M \cdot \frac{V}{V_g} = \frac{1}{1-D}
\]

Cap. Charge Balance:
\[
\langle i_c \rangle = I = D I_i - \frac{V}{R} \quad \rightarrow \quad \boxed{I = \frac{V}{RD}}
\]
Boost: Conversion Ratio

\[ M(D) = \frac{1}{D^2} = \frac{1}{1 - D} \]