More Thinking on Types of Buses

- Need to know/assume two of $|V_i|$, $\delta_i$, $P_i$ and $Q_i$
- Relax the other two within upper and lower limits
- May assume more types of buses for a variety of natures of buses

|          | $|V_i|$ | $\delta_i$ | $P_i$ | $Q_i$ |
|----------|--------|------------|-------|-------|
| $V-\delta$ | X      | X          |       |       |
| $P-Q$    |        |            | X     | X     |
| $P-V$    | X      |            |       |       |
| $Q-V$    | X      |            |       | X     |
| $P-\delta$ |        |            |       | X     |
| $Q-\delta$ |        |            |       | X     |
A more general power flow study

\[ z_{10} = j1 \]
\[ |V_1| = 1 \]
\[ \delta_1 = 0 \]
(slack bus)

\[ z_{12} = j0.4 \]
\[ z_{13} = j0.2 \]
\[ z_{20} = j0.8 \]
\[ z_{23} = j0.2 \]

\[ P_2 = 0.5 \text{pu} \]
\[ |V_2| = 1.05 \text{pu} \]

\[ |V_1| = 1 \]

\[ P_4 + jQ_4 = 1 + j0.2 \text{ pu} \]
Gauss-Seidel Method (Example 6.2)

- Solve nonlinear equation
  \( f(x) = 0 \)

1. Re-write \( x = g(x) \)
2. Select initial estimate \( x^{(0)} \)
   and error tolerance \( \varepsilon \)
3. Start iteration
   \( x^{(1)} = g(x^{(0)}) \)
   \( x^{(2)} = g(x^{(1)}) \)
   ...
   \( x^{(k+1)} = g(x^{(k)}) \)
4. Stop when \( |x^{(k+1)} - x^{(k)}| \leq \varepsilon \).

Solution: \( x = x^{(k+1)} \)

\[ f(x) = x^3 - 6x^2 + 9x - 4 = 0 \]

\[ x = -\frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9} = g(x) \]

\[ x^{(0)} = 2 \quad \varepsilon = 0.0001 \]

\[ x^{(1)} = g(x^{(0)}) = -\frac{1}{9}(2)^3 + \frac{6}{9}(2)^2 + \frac{4}{9} = 2.2222 \quad |x^{(1)} - x^{(0)}| = 0.2222 \]

\[ x^{(2)} = g(x^{(1)}) = 2.5173 \quad |x^{(2)} - x^{(1)}| = 0.2951 \]

\[ x^{(3)} = g(x^{(2)}) = 2.8966 \quad |x^{(3)} - x^{(2)}| = 0.3793 \]

\[ x^{(4)} = g(x^{(3)}) = 3.3376 \quad |x^{(4)} - x^{(3)}| = 0.4410 \]

\[ x^{(5)} = g(x^{(4)}) = 3.7398 \quad |x^{(5)} - x^{(4)}| = 0.4022 \]

\[ x^{(6)} = g(x^{(5)}) = 3.9568 \quad |x^{(6)} - x^{(5)}| = 0.2170 \]

\[ x^{(7)} = g(x^{(6)}) = 3.9988 \quad |x^{(7)} - x^{(6)}| = 0.0420 \]

\[ x^{(8)} = g(x^{(7)}) = 4.0000 \quad |x^{(8)} - x^{(7)}| = 0.0012 \]

\[ x^{(9)} = g(x^{(8)}) = 4.0000 \quad |x^{(9)} - x^{(8)}| < 0.0001 \]

3 roots: \( x_{1,2} = 1 \) and \( x_3 = 4 \)
• “Zigzag” graphical illustration
• Three questions:
  1. Can the iteration converge faster?
  2. How to find all roots?
  3. Does the iteration always converge to a root?

\[ y = g(x) = \frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9} \]
Q1: Faster iteration?

\[ x^{(k+1)} = g(x^{(k)}) = x^{(k)} + [g(x^{(k)}) - x^{(k)}] \]

Adjustment on \( x \)

- Using an acceleration factor \( \alpha \) when updating \( x^{(k)} \)

\[ x^{(k+1)} = x^{(k)} + \alpha [g(x^{(k)}) - x^{(k)}] \]
Q2: Finding all roots.

3 roots: $x_{1,2} = 1$ and $x_3 = 4$

$g(x) = -\frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9}$

Q3: Always convergent?

Initial value $x^{(0)}$ is important!
\[ f(x) = x^3 - 6x^2 + 9x - 4 = 0 \]

\[ x = g(x) = -\frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9} \]

\[ x = h(x) = -x^3 + 6x^2 - 8x + 4 \]

\[ h(x) \] is harder to converge than \( g(x) \)
A system of \( n \) equations in \( n \) variables

\[
\begin{align*}
 f_1(x_1, x_2, x_3, \ldots, x_n) &= 0 \\
 f_2(x_1, x_2, x_3, \ldots, x_n) &= 0 \\
 f_3(x_1, x_2, x_3, \ldots, x_n) &= 0 \\
 &\vdots \\
 f_n(x_1, x_2, x_3, \ldots, x_n) &= 0
\end{align*}
\]

Using an acceleration factor when updating \( x_i^{(k)} \)

\[
\begin{align*}
 x_1^{(k+1)} &= g_1(x_1^{(k)}, x_2^{(k)}, x_3^{(k)}, \ldots, x_n^{(k)}) \\
 x_2^{(k+1)} &= g_2(x_1^{(k+1)}, x_2^{(k)}, x_3^{(k)}, \ldots, x_n^{(k)}) \\
 x_3^{(k+1)} &= g_3(x_1^{(k+1)}, x_2^{(k+1)}, x_3^{(k)}, \ldots, x_n^{(k)}) \\
 &\vdots \\
 x_n^{(k+1)} &= g_n(x_1^{(k+1)}, x_2^{(k+1)}, x_3^{(k+1)}, \ldots, x_{n-1}^{(k+1)}, x_n^{(k)})
\end{align*}
\]

\[
\begin{align*}
 x_i^{(k+1)} &= g_i(x_1^{(k)}, \ldots, x_n^{(k)}) \\
 x_i^{(k+1)} &= x_i^{(k)} + \alpha(x_i^{cal} - x_i^{(k)})
\end{align*}
\]