Small Signal Discrete Time Modeling of a DCM Buck Converter

In this problem, you will investigate the control-to-output transfer function of a 12-to-3 V Buck converter shown in Fig. 1.

![Buck Converter Diagram]

Table I: Converter design specification

<table>
<thead>
<tr>
<th>$r_{on,Q}$</th>
<th>$r_{on,D}$</th>
<th>$f_s$</th>
<th>$L$</th>
<th>$V_g$</th>
<th>$I_{load}$</th>
<th>$C_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 mΩ</td>
<td>10 mΩ</td>
<td>500 kHz</td>
<td>1 µH</td>
<td>12 V</td>
<td>1 A</td>
<td>10 µF</td>
</tr>
</tbody>
</table>

When conducting, the diode and MOSFET can be modeled as on-resistances of value $r_{on,D}$ and $r_{on,Q}$, respectively. When not conducting, both can be modeled as ideal open-circuits.

Derive the control-to-output transfer function $G_{vd}(z)$ from $d(t)$ to $V_{out}$. An ADC samples the output voltage at the instant the MOSFET turns on, with zero conversion delay.

a) Find the steady-state duty cycles $D$ and $D_2$ of the converter such that the output voltage is $V_{out} = 3$ V and diode conduction is correctly modeled, where $D$ is the duty cycle of MOSFET conduction and $D_2$ is the duty cycle of diode conduction.

b) Derive the traditional averaged model of the control-to-output transfer function $G_{vd,avg}(s)$.

c) Derive a generalized state space model of the system of the form

$$x[k + 1] = f(x[k], u[k], D[k], w[k])$$

$$0 = σ(x[k], u[k], D[k], w[k])$$

Give full realizations of both equations, and state explicitly what auxiliary variable(s) are included in $w[k]$.

d) Solve the system in (c) to obtain a complete discrete time small-signal model of the DCM buck converter. Give expressions for each matrix/vector in terms of the state space models of the converter.

e) On the same axes, produce a bode plot of the averaged model $G_{vd,avg}(s)$ and your discrete time model $G_{vd}(z)$, using the built-in discrete time conversion in the `bode()` function. Comment on any differences.