Motivation

Time-Domain Analysis of Switching Transitions

(1) Assume \( C_{out} \gg C_{ds} \)
(2) Assume \( C_{ds} \) is linear (e.g., by linear equivalent)
Resonant Circuit Solution

\[ V_{C}(t=0) = V_0 \]
\[ I_C(t=0) = I_0 \]

\[ v_i(t) = \frac{V_{DC}}{\sqrt{LC}} + \left( V_o - V_{DC} \right) \cos \left( \frac{t}{\sqrt{LC}} \right) + \left( I_0 - I_{DC} \right) \frac{L}{C} \sin \left( \frac{t}{\sqrt{LC}} \right) \]

\[ i_L(t) = \frac{I_{DC}}{\sqrt{LC}} + \left( I_0 - I_{DC} \right) \cos \left( \frac{t}{\sqrt{LC}} \right) + \left( V_{DC} - V_0 \right) \frac{C}{L} \sin \left( \frac{t}{\sqrt{LC}} \right) \]

Normalisation and Notation

Notation: \[ \omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 \quad R_0 = \frac{1}{\sqrt{LC}} \]

\[ \begin{cases} v_c(t) = V_{DC} + (V_0 - V_{DC}) \cos(\omega_0 t) + R_0 (I_0 - I_{DC}) \sin(\omega_0 t) \\ i_L(t) = I_{DC} + (I_0 - I_{DC}) \cos(\omega_0 t) + \frac{1}{R_0} (V_{DC} - V_0) \sin(\omega_0 t) \end{cases} \]

Normalisation:
\[ m_c(t) = \frac{v_c(t)}{V_{base}} \]
\[ j_L(t) = \frac{i_L(t)}{I_{base}} \]
\[ \omega = \omega_0 t_i \]

Base \( V \) \rightarrow \text{Any constant voltage (you choose; some choices better than others)}

Base \( I \) \rightarrow \frac{V_{base}}{R_0}
### Circuit Analysis

**Base Case**

\[
\begin{align*}
V_{\text{base}} &= V_{\text{DC}} \\
I_{\text{base}} &= \frac{V_{\text{DC}}}{R_0}
\end{align*}
\]

**Inrush Case**

\[
\begin{align*}
v_c(t) &= V_{\text{DC}} + (V_0 - V_{\text{DC}}) \cos(\omega_0 t) + R_0 (I_0 - I_{\text{DC}}) \sin(\omega_0 t) \\
i_l(t) &= I_{\text{DC}} + (I_0 - I_{\text{DC}}) \cos(\omega_0 t) + \frac{1}{R_0} (V_{\text{DC}} - V_0) \sin(\omega_0 t)
\end{align*}
\]

**Expressions for \( m_c(\theta) \) and \( j_c(\theta) \)**

\[
\begin{align*}
m_c(\theta) &= \frac{V_c}{V_{\text{base}}} = 1 + \left( \frac{V_0}{V_{\text{DC}}} - 1 \right) \cos \theta + R_0 \frac{I_0 - I_{\text{DC}}}{V_{\text{DC}}} \sin \theta \\
j_c(\theta) &= \frac{I_c}{I_{\text{base}}} = R_0 \frac{I_{\text{DC}}}{V_{\text{DC}}} + R_0 \frac{I_0 - I_{\text{DC}}}{V_{\text{DC}}} \cos \theta + \left( \frac{1}{R_0} \frac{V_0}{V_{\text{DC}}} \right) \sin \theta
\end{align*}
\]

**Equation for a Circle in the \( m-j \) Plane**

\[
\begin{align*}
(m_c(\theta) - 1)^2 + (j_c(\theta) - R_0 \frac{I_{\text{DC}}}{V_{\text{DC}}})^2 &= \left( \frac{V_0}{V_{\text{DC}}} - 1 \right)^2 + \left( R_0 \frac{I_0 - I_{\text{DC}}}{V_{\text{DC}}} \right)^2 \\
&= (M_0 - M_{\text{DC}})^2 + (J_0 - J_{\text{DC}})^2
\end{align*}
\]

**Key Points**

- **Initial Condition**
  - \((M_0, J_0)\)
- **DC Solution**
  - \((M_{\text{DC}}, J_{\text{DC}})\)
- **Phase Plane**
  - Transforms Diff EQ's into Geometry & try
  - For L-L, L-C circuits
State Plane Analysis

DC Solution:
\[ V_c = V_{dc} \rightarrow m_c = \frac{V_{dc}}{V_{base}} \]
\[ I_c = I_{dc} \rightarrow J_c = \frac{I_{dc}}{V_{base}} \]

Initial Conditions:
\[ V_c(0) = V_0 \rightarrow m_c = \frac{V_0}{V_{base}} \]
\[ I_c(0) = I_0 \rightarrow J_c = \frac{I_0}{V_{base}} \]

\[ \text{Direction? find } \frac{dv_c}{dt} = 0 \]
\[ \text{or/and } \frac{di_c}{dt} = 0 \]
