Hybrid Dickson converter

In either subinterval, DC solution is

However, there are also pure V-C loops.
If any current flows in the V-C loops, there will still be charge-shifting loss.

Assume current is only fast DC solution:

Because $C_1$ and $C_5$ don't have zero charge over $T_s$, this operation will have some hard charging.

Split-Phase Control

$[D_x \ D_{x_b} \ D_y \ D_{y_b}] [\begin{bmatrix} \bar{a}^{x} \\ \bar{a}^{x_b} \\ \bar{a}^{y} \\ \bar{a}^{y_b} \end{bmatrix}] = \begin{bmatrix} -\frac{1}{6} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$D_x + D_{x_b} = \frac{1}{2}$

$D_y + D_{y_b} = \frac{1}{2}$
LTSpice Simulation

Switching Losses in SC Converters

Pure SC converter in SSL:
Full lossless at turn-on
Zero-current turn-off (> lossless)

Some overlap losses may occur, but with reactive loading rather than
inductively-clamped switching.
FET Turn-On (L7) \((M_2, I_L > 0)\)

**Discrete Time Modeling**

Covered in detail in EC692
Converter Analysis

Sinusoidal Analysis
Assumes high-Q tanks, small ripple & sinusoidal

State Plane
QSW & QR Converters
Limited to single-frequency, undamped resonant
Resonant SC

Multilevel
MMC

Z-source, matrix, others

Small-ripple approx & averaging
PWM (small ripple)

Averaging
(waveform, switch, state space)

Switched Capacitor Charge

Switched Circuits

original converter

switch in position 1

switch in position 2

LTI equivalents
Historical Perspective

Robert D Middlebrook
PhD, Stanford, 1955
CalTech Professor, 1955-1998

Slobodan Cúk
PhD CalTech, 1976
CalTech Prof, 1977-1999

Modelling, analysis, and design of switching converters

Model a switched system as an averaged, time-invariant system with

\[ \dot{x}(t) = Ax(t) + Bu(t) \]

where

\[ A = DA_1 + D'A_2 \]
\[ B = DB_1 + D'B_2 \]


Linear Circuit Modeling Using State Space

In switch position 1

\[
\begin{aligned}
  v_g(t) - v_c(t) &= L \frac{di_L(t)}{dt} \\
  i_L(t) - \frac{v_c(t)}{R} &= C \frac{dv_c(t)}{dt}
\end{aligned}
\]
Linear Circuit Modeling Using State Space

In switch position 1

\[
\begin{cases}
v_g(t) - v_c(t) = L \frac{di_L(t)}{dt} \\
i_L(t) - \frac{v_c(t)}{R} = C \frac{dv_c(t)}{dt}
\end{cases}
\]

Which can be written, in state space, form as

\[
\frac{d}{dt} \begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1 & -1/R \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} v_g(t)
\]

Or, generally,

\[\dot{x}(t) = A_1 x(t) + B_1 u(t)\]

In the second switch position, we will have a new (linear) circuit with

\[\dot{x}(t) = A_2 x(t) + B_2 u(t)\]

Switching Signal

In a PWM converter with two switch positions, the two linear circuits combine according to a switching function \(s(t)\)

\[
\dot{x}(t) = [A_1 s(t) + A_2 s'(t)]x(t) + [B_1 s(t) + B_2 s'(t)]u(t)
\]

where

\[s(t) = \begin{cases} 
1, & \text{if } nT_s < t < (n + D)T_s \\
0, & \text{if } (n + D)T_s < t < (n + 1)T_s
\end{cases}\]

\[s'(t) = 1 - s(t)\]
SMPS State Space

In traditional state space modeling of linear systems

\[ \dot{x}(t) = Ax(t) + Bu(t) \]

with \( u(t) \) containing a control input. When \( A \) and \( B \) are constant, this is a linear system. However, we have

\[ \dot{x}(t) = [A_1 s(t) + A_2 s'(t)]x(t) + [B_1 s(t) + B_2 s'(t)]u(t) \]

or, equivalently

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t) \]

which is nonlinear: how do we deal with it?

Converting to Linear System

Assume that our system model

\[ \dot{x}(t) = [A_1 s(t) + A_2 s'(t)]x(t) + [B_1 s(t) + B_2 s'(t)]u(t) \]

can be approximated by some linear system

\[ \dot{x}(t) = Ax(t) + Bu(t) \]

which removes the nonlinearity of the system

– Nonlinearities came from switching
– Expect that switching dynamics will be lost

Note: This system is now linear in \( x(t) \) and \( u(t) \), but not in our control signal, \( s(t) \)
Approximate Steady State Waveforms

\[
\langle x(t) \rangle = \frac{1}{T_s} \int_{0}^{T_s} x(t) \, dt
\]
Approximate Steady State Waveforms

The Averaging Approximation

If waveforms can be approximated as linear

\[
\dot{x}(t) = \begin{cases} 
A_1 \langle x(t) \rangle + B_1 \langle u(t) \rangle, & \text{if } nT_s < t < (n + D)T_s \\
A_2 \langle x(t) \rangle + B_2 \langle u(t) \rangle, & \text{if } (n + D)T_s < t < (n + 1)T_s 
\end{cases}
\]

so the average slope is

\[
\langle \dot{x}(t) \rangle = \frac{1}{T_s} (A_1 \langle x(t) \rangle + B_1 \langle u(t) \rangle)DT_s + (A_2 \langle x(t) \rangle + B_2 \langle u(t) \rangle)(1 - D)T_s
\]

or, rearranging

\[
\langle \dot{x}(t) \rangle = (DA_1 + D'A_2)\langle x(t) \rangle + (DB_1 + D'B_2)\langle u(t) \rangle
\]