B. Pattern Formation

Differentiation & Pattern Formation

- A central problem in development: How do cells differentiate to fulfill different purposes?
- How do complex systems generate spatial & temporal structure?
- CAs are natural models of intercellular communication

Plecostomus
Activation & Inhibition in Pattern Formation

- Color patterns typically have a characteristic length scale
- Independent of cell size and animal size
- Achieved by:
  - short-range activation $\Rightarrow$ local uniformity
  - long-range inhibition $\Rightarrow$ separation
Interaction Parameters

- \( R_1 \) and \( R_2 \) are the interaction ranges
- \( J_1 \) and \( J_2 \) are the interaction strengths

CA Activation/Inhibition Model

- Let states \( s_i \in \{-1, +1\} \)
- and \( h \) be a bias parameter
- and \( r_{ij} \) be the distance between cells \( i \) and \( j \)
- Then the state update rule is:

\[
\begin{align*}
    s_i(t+1) &= \text{sign} \left[ h + J_1 \sum_{j: r_{ij} \leq R_1} s_j(t) + J_2 \sum_{j: R_1 < r_{ij} < R_2} s_j(t) \right]
\end{align*}
\]

Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation

RunAICA.nlogo
Example

$$(R_1=1, R_2=6, J_1=1, J_2=-0.1, h=0)$$

Effect of Bias

$$(h = -6, -3, -1; 1, 3, 6)$$

Effect of Interaction Ranges

$$(R_1=1, R_2=6, h=0)$$

$$(R_1=1.5, R_2=8, h=0)$$

$$(R_1=1, R_2=6, h=-3)$$

$$(R_1=1.5, R_2=8, h=-3)$$
Differential Interaction Ranges

- How can a system using strictly local interactions discriminate between states at long and short range?
- E.g. cells in developing organism
- Can use two different *morphogens* diffusing at two different rates
  - activator diffuses slowly (short range)
  - inhibitor diffuses rapidly (long range)

Digression on Diffusion

- Simple 2-D diffusion equation:
  \[ \dot{A}(x, y) = D \nabla^2 A(x, y) \]
- Recall the 2-D Laplacian:
  \[ \nabla^2 A(x, y) = \frac{\partial^2 A(x, y)}{\partial x^2} + \frac{\partial^2 A(x, y)}{\partial y^2} \]
- The Laplacian (like 2nd derivative) is:
  - positive in a local minimum
  - negative in a local maximum

Reaction-Diffusion System

\[
\begin{align*}
\frac{\partial A}{\partial t} &= D \nabla^2 A + f_A(A, I) \\
\frac{\partial I}{\partial t} &= D \nabla^2 I + f_I(A, I)
\end{align*}
\]

\[ \dot{\mathbf{c}} = D \nabla^2 \mathbf{c} + \mathbf{f}(\mathbf{c}), \text{ where } \mathbf{c} = \begin{pmatrix} A \\ I \end{pmatrix} \]
General Reaction-Diffusion System

\[ \frac{\partial c_i}{\partial t} = \sum_{\alpha} k_\alpha V_{\alpha} \left( \prod_{k=1}^{n} c_k^{m_{\alpha}} \right) - \nabla \cdot \mathbf{j} \]

where \( \mathbf{j} = \bar{\mu}_i c_i - \text{div} \mathbf{D}_i c_i \) (flux)

where \( k_\alpha \) = rate constant for reaction \( \alpha \)
and \( V_{\alpha} \) = stoichiometric coefficient
and \( m_{\alpha} \) = a non-negative integer
and \( \bar{\mu}_i \) = drift vector
and \( \mathbf{D}_i \) = diffusivity matrix

\[ \text{div} \mathbf{D}_i = -\sum_{\alpha} \sum_{k} \mathbf{D}_i \frac{\partial c_k}{\partial x_k} \]

Framework for Complexity

- change = source terms + transport terms
- source terms = local coupling
  = interactions local to a small region
- transport terms = spatial coupling
  = interactions with contiguous regions
  = advection + diffusion
    - advection: non-dissipative, time-reversible
    - diffusion: dissipative, irreversible

NetLogo Simulation of Reaction-Diffusion System

1. Diffuse activator in X and Y directions
2. Diffuse inhibitor in X and Y directions
3. Each patch performs:
   stimulation = bias + activator – inhibitor + noise
   if stimulation > 0 then
      set activator and inhibitor to 100
   else
      set activator and inhibitor to 0
Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation

Run Pattern.nlogo

Continuous-time Activator-Inhibitor System

- Activator $A$ and inhibitor $I$ may diffuse at different rates in $x$ and $y$ directions.
- Cell becomes more active if activator + bias exceeds inhibitor.
- Otherwise, less active.
- $A$ and $I$ are limited to $[0, 100]$ (depletion/saturation).

\[
\frac{\partial A}{\partial t} = d_A \frac{\partial^2 A}{\partial x^2} + d_A \frac{\partial^2 A}{\partial y^2} + k_A (A + B - I)
\]

\[
\frac{\partial I}{\partial t} = d_I \frac{\partial^2 I}{\partial x^2} + d_I \frac{\partial^2 I}{\partial y^2} + k_I (A + B - I)
\]

Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation with Continuous State Change

Run Activator-Inhibitor.nlogo
Turing Patterns

• Alan Turing studied the mathematics of reaction-diffusion systems


• The resulting patterns are known as *Turing patterns*.

Observations

• With local activation and lateral inhibition

• And with a random initial state

• You can expect to get Turing patterns

• These are stationary states (dynamic equilibria)

• Macroscopically, Class I behavior
  – Microscopically, may be class III

A Key Element of Self-Organization

• Activation vs. Inhibition

• Cooperation vs. Competition

• Amplification vs. Stabilization

• Growth vs. Limit

• Positive Feedback vs. Negative Feedback
  – Positive feedback creates
  – Negative feedback shapes
Reaction-Diffusion Computing

- Has been used for image processing
  - diffusion $\Rightarrow$ noise filtering
  - reaction $\Rightarrow$ contrast enhancement
- Depending on parameters, RD computing can:
  - restore broken contours
  - detect edges
  - improve contrast

Image Processing in BZ Medium

- (A) boundary detection, (B) contour enhancement,
  (C) shape enhancement, (D) feature enhancement

Voronoi Diagrams

- Given a set of generating points:
- Construct a polygon around each generating point of set, so all points in a polygon are closer to its generating point than to any other generating points.
Some Uses of Voronoi Diagrams

- Collision-free path planning
- Determination of service areas for power substations
- Nearest-neighbor pattern classification
- Determination of largest empty figure

Computation of Voronoi Diagram by Reaction-Diffusion Processor

Mixed Cell Voronoi Diagram
Path Planning via BZ medium: No Obstacles

Path Planning via BZ medium: Circular Obstacles

Mobile Robot with Onboard Chemical Reactor
Bibliography for Reaction-Diffusion Computing
