B. Pattern Formation

Differentiation & Pattern Formation

- A central problem in development: How do cells differentiate to fulfill different purposes?
- How do complex systems generate spatial & temporal structure?
- CAs are natural models of intercellular communication

Activation & Inhibition in Pattern Formation

- Color patterns typically have a characteristic length scale
- Independent of cell size and animal size
- Achieved by:
  - short-range activation $\Rightarrow$ local uniformity
  - long-range inhibition $\Rightarrow$ separation
Interaction Parameters

- $R_1$ and $R_2$ are the interaction ranges
- $J_1$ and $J_2$ are the interaction strengths

CA Activation/Inhibition Model

- Let states $s_i \in \{-1, +1\}$
- and $h$ be a bias parameter
- and $r_{ij}$ be the distance between cells $i$ and $j$
- Then the state update rule is:

$$ s_i(t + 1) = \text{sign} \left[ h + J_1 \sum_{s_j(t) \cap r_{ij} < R_1} + J_2 \sum_{R_1 \leq r_{ij} < R_2} \right] $$

Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation

RunAICA.nlogo

Example

$(R_1=1, R_2=6, J_1=1, J_2=-0.1, h=0)$

Effect of Bias

$(h = -6, -3, -1; 1, 3, 6)$

Effect of Interaction Ranges

$(R_2=6, R_1=1, h=0)$

$(R_2=8, R_1=1, h=0)$

$(R_2=6, R_1=1.5, h=0)$

$(R_2=6, R_1=1.5, h=-3)$
Part 2B: Pattern Formation

Differential Interaction Ranges

• How can a system using strictly local interactions discriminate between states at long and short range?
• E.g. cells in developing organism
• Can use two different morphogens diffusing at two different rates
  – activator diffuses slowly (short range)
  – inhibitor diffuses rapidly (long range)

Digression on Diffusion

• Simple 2-D diffusion equation:
  \[ \dot{A}(x,y) = D \nabla^2 A(x,y) \]
• Recall the 2-D Laplacian:
  \[ \nabla^2 A(x,y) = \frac{\partial^2 A(x,y)}{\partial x^2} + \frac{\partial^2 A(x,y)}{\partial y^2} \]
• The Laplacian (like 2nd derivative) is:
  – positive in a local minimum
  – negative in a local maximum

Reaction-Diffusion System

\[ \frac{\partial A}{\partial t} = D_x \nabla^2 A + f_a(A,I) \]
\[ \frac{\partial I}{\partial t} = D_I \nabla^2 I + f_i(A,I) \]

\[ \dot{c} = D \nabla^2 c + f(c), \text{ where } c = \begin{bmatrix} A \\ I \end{bmatrix} \]

General Reaction-Diffusion System

\[ \frac{\partial c_i}{\partial t} = \sum_{k=1}^{m} k_i V_{ik} \left( \prod_{k=1}^{m} c_{i}^{m_k} \right) - \nabla \cdot J_i \]

where \( J_i = \mu_i c_i - \text{div } D_i c_i \) (flux)

Framework for Complexity

• change = source terms + transport terms
• source terms = local coupling
  = interactions local to a small region
• transport terms = spatial coupling
  = interactions with contiguous regions
  = advection + diffusion
  – advection: non-dissipative, time-reversible
  – diffusion: dissipative, irreversible

NetLogo Simulation of Reaction-Diffusion System

1. Diffuse activator in X and Y directions
2. Diffuse inhibitor in X and Y directions
3. Each patch performs:
   stimulation = bias + activator – inhibitor + noise
   if stimulation > 0 then
     set activator and inhibitor to 100
   else
     set activator and inhibitor to 0
Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation

Run Pattern.nlogo

Continuous-time Activator-Inhibitor System

- Activator $A$ and inhibitor $I$ may diffuse at different rates in $x$ and $y$ directions
- Cell becomes more active if activator + bias exceeds inhibitor
- Otherwise, less active
- $A$ and $I$ are limited to $[0, 100]$ (depletion/saturation)

\[
\frac{\partial A}{\partial t} = d_A \frac{\partial^2 A}{\partial x^2} + d_A \frac{\partial^2 A}{\partial y^2} + k_a (A + B - I)
\]

\[
\frac{\partial I}{\partial t} = d_I \frac{\partial^2 I}{\partial x^2} + d_I \frac{\partial^2 I}{\partial y^2} + k_l (A + B - I)
\]

Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation with Continuous State Change

Run Activator-Inhibitor.nlogo

Turing Patterns

- Alan Turing studied the mathematics of reaction-diffusion systems
- The resulting patterns are known as Turing patterns

Observations

- With local activation and lateral inhibition
- And with a random initial state
- You can expect to get Turing patterns
- These are stationary states (dynamic equilibria)
- Macroscopically, Class I behavior
  - Microscopically, may be class III

A Key Element of Self-Organization

- Activation vs. Inhibition
- Cooperation vs. Competition
- Amplification vs. Stabilization
- Growth vs. Limit
- Positive Feedback vs. Negative Feedback
  - Positive feedback creates
  - Negative feedback shapes
Part 2B: Pattern Formation

Reaction-Diffusion Computing
- Has been used for image processing
  - diffusion \(\Rightarrow\) noise filtering
  - reaction \(\Rightarrow\) contrast enhancement
- Depending on parameters, RD computing can:
  - restore broken contours
  - detect edges
  - improve contrast

Image Processing in BZ Medium
- (A) boundary detection
- (B) contour enhancement
- (C) shape enhancement
- (D) feature enhancement

Voronoi Diagrams
- Given a set of generating points:
- Construct a polygon around each generating point, so all points in a polygon are closer to its generating point than to any other generating points.

Some Uses of Voronoi Diagrams
- Collision-free path planning
- Determination of service areas for power substations
- Nearest-neighbor pattern classification
- Determination of largest empty figure

Computation of Voronoi Diagram by Reaction-Diffusion Processor

Mixed Cell Voronoi Diagram
Part 2B: Pattern Formation

Path Planning via BZ medium: No Obstacles

Path Planning via BZ medium: Circular Obstacles

Mobile Robot with Onboard Chemical Reactor

Actual Path: Pd Processor

Actual Path: BZ Processor
Bibliography for Reaction-Diffusion Computing