Equivalent Circuits with Multiple Damper Windings (e.g. Round-rotor Machines)

- **d axis:**
  \[ L_{fd} \triangleq L_F - M_R \quad R_{fd} \triangleq R_F \]
  \[ \psi_{fd} \triangleq \psi_F \quad e_{fd} \triangleq e_F \]
  \[ L_{1d} \triangleq L_D - M_R \quad R_{1d} \triangleq R_D \]
  \[ \psi_{1d} \triangleq \psi_D \]

\( M_R-L_{ad} \approx 0 \) is named \( L_{fkd1} \) in some literature to model rotor mutual flux leakage, i.e. the flux linking the rotor’s field and damper windings but not stator windings.

- **q axis:**
  \[ L_{1q} \triangleq L_Q - L_{aq} \quad R_{1q} \triangleq R_Q \]
  \[ \psi_{1q} \triangleq \psi_Q \]
  \[ L_{2q} \triangleq L_G - L_{aq} \quad R_{2q} \triangleq R_G \]
  \[ \psi_{2q} \triangleq \psi_G \]

**Subscript Notations:**
- \((\_fd)\) ~ field winding quantities
- \((\_kd)\) ~ k-th d-axis damper winding quantities
- \((\_kq)\) ~ k-th q-axis damper winding quantities

(a) d-axis equivalent circuit

(b) q-axis equivalent circuit
Example: a model with 3 rotor windings in each of d- and q-axis equivalent circuits

  - The proposed equivalent circuits are expected to contain sufficient details to model all machines
  - Parameters are estimated by frequency response tests

![Diagram of synchronous machine equivalent circuit](image1)

![Graph showing simulated and measured responses](image2)

*Figure S-2. Simulation of Nanticoke Generator Power and Field Current During a Transient Caused By Line Switching.*
Example 3.1 (Kundur’s book)

A 555 MVA, 24kV, 0.9p.f., 60Hz, 3 phase, 2 pole synchronous generator has the following inductances an resistances associated with the stator and field windings:

\begin{align*}
  l_{aa} &= 3.2758 + 0.0458 \cos(2\theta) \quad \text{mH} \\
  l_{ab} &= -1.6379 - 0.0458 \cos(2\theta + \pi/3) \quad \text{mH} \\
  l_{aF} &= 40.0 \cos\theta \quad \text{mH} \\
  L_F &= 576.92 \quad \text{mH} \\
  R_a &= 0.0031 \quad \Omega \\
  R_F &= 0.0715 \quad \Omega
\end{align*}

a. Determine $L_d$ and $L_q$ in H

b. If the stator leakage inductance $L_l$ is 0.4129 mH, determine $L_{ad}$ and $L_{aq}$ in H

c. Using the machine rated values as the base values for the stator quantities, determine the per unit values of the following in the $L_{ad}$ base reciprocal per unit system (assuming $L_{ad} = M_F = M_R$ in per unit): $L_l, L_{ad}, L_{aq}, L_d, L_q, M_F, L_F, L_{fd}, R_a$ and $R_F$

Solution:

a.  
\begin{align*}
  l_{aa} &= L_s + L_m \cos 2\theta = 3.2758 + 0.0458 \cos 2\theta \quad \text{mH} \\
  l_{ab} &= -M_s - L_m \cos(2\theta + \pi/3) = -1.6379 - 0.0458 \cos(2\theta + \pi/3) \quad \text{mH} \\
  l_{aF} &= M_F \cos \theta = 40.0 \cos \theta \quad \text{mH} \\
  L_d &= L_s + M_s + 3L_m/2 = 3.2758 + 1.6379 + \frac{3}{2} \times 0.0458 = 4.9825 \quad \text{mH} \\
  L_q &= L_s + M_s - 3L_m/2 = 3.2758 - 1.6379 - \frac{3}{2} \times 0.0458 = 4.8451 \quad \text{mH}
\end{align*}

b.  
\begin{align*}
  L_{ad} &= L_d - L_l = 4.9825 - 0.4129 = 4.5696 \quad \text{mH} \\
  L_{aq} &= L_q - L_l = 4.4851 - 0.4129 = 4.432 \quad \text{mH}
\end{align*}
c. \( 3\text{-phase }VA_{\text{base}} = 555 \text{ MVA} \)

\[
E_{\text{RMS base}} = 24/\sqrt{3} = 13.856 \text{ kV}
\]

\[
e_{s\text{ base (peak)}} = \sqrt{2} \times E_{\text{RMS base}} = \sqrt{2} \times 13.856 = 19.596 \text{ kV}
\]

\[
I_{\text{RMS base}} = \frac{3\text{-phase }VA_{\text{base}}}{(3 \times E_{\text{RMS base}})} = \frac{555 \times 10^6}{(3 \times 13.856 \times 10^3)} = 13.351 \times 10^3 \text{ A}
\]

\[
i_{s\text{ base (peak)}} = \sqrt{2} \times I_{\text{RMS base}} = \sqrt{2} \times 13.351 \times 10^3 = 18.881 \times 10^3 \text{ A}
\]

\[
Z_{s\text{ base}} = \frac{e_{s\text{ base}}}{i_{s\text{ base}}} = \frac{19.596 \times 10^3}{(18.881 \times 10^3)} = 1.03784 \text{ } \Omega
\]

\[
\omega_{\text{base}} = 2\pi \times 60 = 377 \text{ elec. rad/s}
\]

\[
L_{s\text{ base}} = \frac{Z_{s\text{ base}}}{\omega_{\text{base}}} = \frac{1.03784}{377 \times 10^3} = 2.753 \text{ mH}
\]

\[
i_{F\text{ base}} = \frac{L_{ad}/M_{F} \times i_{s\text{ base}}}{4.5696/40 \times 18.8815 \times 10^3} = 2158.0 \text{ A}
\]

\[
e_{F\text{ base}} = \frac{3\text{-phase }VA_{\text{base}}/i_{F\text{ base}}}{555 \times 10^6/2158} = 257.183 \text{ kV}
\]

\[
Z_{F\text{ base}} = \frac{e_{F\text{ base}}}{i_{F\text{ base}}} = \frac{257.183 \times 10^3}{2158} = 119.18 \text{ } \Omega
\]

\[
L_{F\text{ base}} = \frac{Z_{F\text{ base}}}{\omega_{\text{base}}} = \frac{119.18 \times 10^3}{377} = 316.12 \text{ mH}
\]

\[
M_{F\text{ base}} = \frac{L_{F\text{ base}}}{M_{F\text{ base}}} = \frac{576.92}{316.12} = 1.825 \text{ pu}
\]

Then per unit values are:

\[
L_{l} = \frac{L_{l}/L_{s\text{ base}}}{0.4129/2.753} = 0.15 \text{ pu}
\]

\[
R_{F} = \frac{R_{F}/Z_{F\text{ base}}}{0.0031/1.03784} = 0.0006 \text{ pu}
\]

\[
L_{ad} = 4.5696/2.753 = 1.66 \text{ pu}
\]

\[
M_{F} = \frac{M_{F}/M_{F\text{ base}}}{400/241} = 1.66 \text{ pu}
\]

\[
L_{aq} = 4.432/2.753 = 1.61 \text{ pu}
\]

\[
L_{d} = \frac{L_{d} + L_{ad}}{0.15 + 1.66} = 1.81 \text{ pu}
\]

\[
L_{q} = \frac{L_{q} + L_{aq}}{0.15 + 1.61} = 1.76 \text{ pu}
\]

\[
R_{a} = \frac{R_{a}}{1.03784} = 0.003 \text{ pu}
\]
Steady-state Analysis

- All derivatives ($pX$) are zero:

\[ p\omega_r = 0 \rightarrow \omega_r = 1 \text{ and } L = X \text{ in p.u.} \]

\[ p\psi_{fd} = 0 \rightarrow e_{fd} = R_{fd}i_{fd} \]

\[ p\psi_{1d} = 0 \rightarrow i_{1d} = 0 \quad \psi_d = -L_d i_d + L_{ad} i_{fd} \]

\[ p\psi_{1q} = 0 \rightarrow i_{1q} = 0 \quad \psi_q = -L_q i_q \]

\[ p\psi_{d} = 0 \rightarrow e_d = \psi_q - R_a i_d = L_q i_q - R_a i_d \]

\[ p\psi_{q} = 0 \rightarrow e_q = \psi_d - R_a i_q = -L_d i_d + L_{ad} i_{fd} - R_a i_q \]

Voltage and flux equations:

\[ e_{fd} = R_{fd}i_{fd} \quad \psi_{fd} = (L_{ad} + L_{fd})i_{fd} - L_{ad}i_d \]

\[ e_d = X_q i_q - R_a i_d \quad \psi_{1d} = L_{ad}(i_{fd} - i_d) \]

\[ e_q = -X_d i_d + X_{ad} i_{fd} - R_a i_q \quad \psi_{1q} = \psi_{2q} = -L_{aq} i_q \]

- Single equivalent circuit for both d and q axes:

\[ \tilde{I}_t = i_d + j i_q \]

\[ \tilde{E}_t = e_d + j e_q = X_q i_q - R_a i_d - j X_d i_d + j X_{ad} i_{fd} - j R_a i_q \]

\[ \tilde{E}_t = \tilde{E}_q - (R_a + j X_q) \tilde{I}_t \quad \text{where } \tilde{E}_q = j[X_{ad} i_{fd} - (X_d - X_q) i_d] \]

If saliency is neglected:

\[ E_q = X_{ad} i_{fd} \]

\[ X_d = X_q = X_s \text{ (synchronous reactance)} \]
Computing per-unit steady-state values

\[ \omega_r = 1 \text{p.u.} \]

- **Active and Reactive Powers**

\[
S = \frac{\tilde{E}_t \tilde{I}_t^*}{E_t I_t} = (e_d + j e_q)(i_d - j i_q) = (e_d i_d + e_q i_q) + j (e_q i_d - e_d i_q)
\]

where \( e_d = -\omega_r \psi_q - R_a i_d \)

\[ e_q = \omega_r \psi_d - R_a i_q \]

\[
P_t = e_d i_d + e_q i_q = \omega_r (\psi_d i_q - \psi_q i_d) - R_a (i_d^2 + i_q^2) = P_e - R_a (i_d^2 + i_q^2)
\]

\[
Q_t = e_q i_d - e_d i_q
\]

- **Air-gap torque (or electric torque)**

\[
T_e = \frac{P_e}{\omega_r} = \psi_d i_q - \psi_q i_d = P_t + R_a (i_d^2 + i_q^2)
\]

\[ \tilde{E}_t = \tilde{E}_q - (R_a + j X_q) \tilde{I}_t \]

- **Rotor angle \( \delta \)**

\[ \delta = \delta_i + \angle \tilde{E}_t \]

\[
\delta_i = \tan^{-1} \left( \frac{X_q I_t \cos \phi - R_a I_t \sin \phi}{E_t + R_a I_t \cos \phi + X_q I_t \sin \phi} \right)
\]

\[
I_t = \frac{\sqrt{P_t^2 + Q_t^2}}{E_t} \quad \phi = \cos^{-1} \left( \frac{P_t}{E_t I_t} \right)
\]
**Representation of Magnetic Saturation**

- **Assumptions for stability studies**
  - The leakage fluxes are not significantly affected by saturation of the iron portion, so $L_l$ is constant and only $L_{ad}$ and $L_{aq}$ saturate in equivalent circuits.

  \[
  L_{adu} \rightarrow L_{ad} \quad L_{aqu} \rightarrow L_{aq} \quad \text{where } L_{adu} \text{ and } L_{aqu} \text{ denote their unsaturated values}
  \]

  - The leakage fluxes do not contribute to the iron saturation. Thus, saturation is determined only by the air-gap flux linkage.

  \[
  \tilde{\psi}_{at} = \psi_{ad} + j\psi_{aq} = \psi_d + L_l i_d + j\psi_q + jL_l i_q
  \]

  \[
  \psi_{at} = \sqrt{\psi_{ad}^2 + \psi_{aq}^2}
  \]

- Saturation relationship $\psi_{at}$ vs. $i_{fd}$ (or MMF) under loaded conditions is the same as under no-load conditions, so only the open-circuit characteristic (OCC) is considered.

- No magnetic coupling between $d$ and $q$ axes, saturations on $L_{ad}$ and $L_{aq}$ can be modeled individually.
Estimating Saturation Factors $K_{sd}$ and $K_{sd}$

\[ L_{ad} = K_{sd} L_{adu} \quad L_{aq} = K_{sq} L_{aqu} \]

- Salient pole machines
  - The path for $q$-axis flux is largely in air, so $L_{aq}$ does not vary significantly with saturation of the iron portion of the path
  - Assume $K_{sq} = 1.0$ for all loading conditions.

- Round rotor machines
  - There is a magnetic saturation in both axes, but the saturation data in $q$ axis is usually not available
  - Assume $K_{sq} = K_{sd}$

- Thus, we focus on estimating $K_{sd}$

\[
K_{sd} = \frac{L_{ad}}{L_{adu}} = \frac{\psi_{at}}{\psi_{at0}} = \frac{\psi_{at}}{(\psi_{at} + \psi_{I})} = \frac{I_0}{I}
\]

(See Kundur’s Example 3.3 on Estimating $K_{sd}$ for different loading conditions)
Modeling of the Saturation Characteristic

- $\psi_I$ is modeled by 3 approximate functions
  - **Segment I** ($\psi_{at}<\psi_{T1}$):
    \[ \psi_I = \psi_{at0} - \psi_{at0} = 0 \]
  - **Segment II** ($\psi_{T1}<\psi_{at}<\psi_{T2}$):
    \[ \psi_I = A_{sat} e^{B_{sat}}(\psi_{at}-\psi_{T1}) \]
  - **Segment III** ($\psi_{at}>\psi_{T2}$):
    \[ \psi_I = \psi_{G2} + L_{ratio}(\psi_{at}-\psi_{T2}) - \psi_{at} \]

  - Note: segments I and II are not connected since when $\psi_{at} = \psi_{T1}$, $\psi_I = A_{sat} \neq 0$ (usually small)
  - Segments II and III are assumed to be connected at $\psi_{at} = \psi_{T2}$ to solve $\psi_{G2}$

\[ A_{sat} e^{B_{sat}}(\psi_{T2}-\psi_{T1}) = \psi_{G2} + L_{ratio}(\psi_{T2}-\psi_{T2}) - \psi_{T2} \]

\[ \psi_{G2} = \psi_{T2} + A_{sat} e^{B_{sat}}(\psi_{T2}-\psi_{T1}) \]

Five independent parameters: $A_{sat}$, $B_{sat}$, $\psi_{T1}$, $\psi_{T2}$ and $L_{ratio}$
Synchronous Machine Model DG1S1

Model Descriptions

This model uses parameters in basic form and type 1 saturation model.

Data Format

IBUS, 'DG1S1', I, MVA, X_{sd}, X_{sq}, X_{d}, X_{q}, R_s, X_{sd1}, X_{qd1}, R_{sd1}, X_{sd2}, X_{qd2}, R_{sd2}, X_{sd3}, R_{sd3}, H, K_d, a, A_{sat}, B_{sat}, \psi_L, \psi_M, RS /

Parameter Descriptions

IBUS - Bus number, name, or generator equipment name of the machine.
I - ID of the machine (may or may not be enclosed in single quotes).
MVA - MVA base of the machine. If not specified (i.e., no value or zero is entered), the MVA base of the matched generator in powerflow data will be used.
X_{sd} - Unsaturated direct axis mutual reactance in per unit on machine MVA base.
X_{sq} - Unsaturated quadrature axis mutual reactance in per unit on machine MVA base.
X_{d} - Leakage reactance in per unit on machine MVA base.
X_{q} - Leakage reactance in per unit on machine MVA base.
R_s - Armature resistance in per unit on machine MVA base.
R_d - Field winding leakage reactance in per unit on machine MVA base.
R_{q} - Field winding resistance in per unit on machines MVA base.
X_{sd1} - First quadrature axis damper winding leakage reactance in per unit on machine MVA base.
R_{sd1} - First quadrature axis damper winding resistance in per unit on machine MVA base.
X_{sd2} - First direct axis damper winding leakage reactance in per unit on machine MVA base.
R_{sd2} - First direct axis damper winding resistance in per unit on machine MVA base.
X_{sd3} - Second quadrature axis damper winding leakage reactance in per unit on machine MVA base.
R_{sd3} - Second quadrature axis damper winding resistance in per unit on machine MVA base.
X_{sd4} - Second direct axis damper winding leakage reactance in per unit on machine MVA base.
R_{sd4} - Second direct axis damper winding resistance in per unit on machine MVA base.
X_{sd5} - Third quadrature axis damper winding leakage reactance in per unit on machine MVA base.
R_{sd5} - Third quadrature axis damper winding resistance in per unit on machine MVA base.
H - Inertia time constant of the machine in MW-second/MVA.
K_p - Damping coefficient in (p.u. torque)/(p.u. speed deviation).
\alpha - This parameter is used only for synchronous motors, as the exponential in the load characteristic of the motor. T_m = K_\alpha\psi^2 (K is determined by TSAT based on the initial condition). It is ignored for generator model.
A_{sat} - Coefficient in saturation characteristic.
B_{sat} - Coefficient in saturation characteristic.
\psi_L - Flux linkage on the saturation curve where the Region II characteristic starts.
\psi_M - Flux linkage on the saturation curve where the Region III characteristic starts.
RS - Ratio of the slopes of air-gap line and the Region III characteristic.

Here \psi_L = \psi_{T1}, \psi_M = \psi_{T2} and RS = L_{ratio}
1.20 GENSACE

Salient Pole Generator Model (Exponential Saturation on Both Axes)

This model is located at system bus \#_____. IBUS.

Machine identifier \#_____. ID.

This model uses CONs starting with \#_____. J, EJ, FJ, and STATES starting with \#_____. K.

The machine MVA is \#_____. for each of \#_____. units = \#_____. MBASE.

ZSOURCE for this machine is \#_____. \( j \) \#_____. on the above MBASE.

### CONs Table

<table>
<thead>
<tr>
<th>CONs</th>
<th>#</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td></td>
<td>( T_{\text{do}} &gt; 0 ) (sec)</td>
<td></td>
</tr>
<tr>
<td>J+1</td>
<td></td>
<td>( T_{\text{do}} &gt; 0 ) (sec)</td>
<td></td>
</tr>
<tr>
<td>J+2</td>
<td></td>
<td>( T_{\text{do}}^* &gt; 0 ) (sec)</td>
<td></td>
</tr>
<tr>
<td>J+3</td>
<td></td>
<td>( H, \text{ inertia} )</td>
<td></td>
</tr>
<tr>
<td>J+4</td>
<td></td>
<td>( D, \text{ speed damping} )</td>
<td></td>
</tr>
<tr>
<td>J+5</td>
<td></td>
<td>( X_d )</td>
<td></td>
</tr>
<tr>
<td>J+6</td>
<td></td>
<td>( X_q )</td>
<td></td>
</tr>
<tr>
<td>J+7</td>
<td></td>
<td>( X_d' )</td>
<td></td>
</tr>
<tr>
<td>J+8</td>
<td></td>
<td>( X_q' = X_e' )</td>
<td></td>
</tr>
<tr>
<td>J+9</td>
<td></td>
<td>( Xi )</td>
<td></td>
</tr>
<tr>
<td>J+10</td>
<td></td>
<td>( S(1.0) )</td>
<td></td>
</tr>
<tr>
<td>J+11</td>
<td></td>
<td>( S(1.2) )</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( X_d, X_q, X_d', X_q', X_e, X_i, H, \) and \( D \) are in pu, machine MVA base. \( X_q' \) must be equal to \( X_d' \).

### STATES Table

<table>
<thead>
<tr>
<th>STATES</th>
<th>#</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td></td>
<td>( E_q' )</td>
</tr>
<tr>
<td>K+1</td>
<td></td>
<td>( \psi_q )</td>
</tr>
<tr>
<td>K+2</td>
<td></td>
<td>( \psi_{kd} )</td>
</tr>
<tr>
<td>K+3</td>
<td></td>
<td>( \Delta \text{ speed (pu)} )</td>
</tr>
<tr>
<td>K+4</td>
<td></td>
<td>( \text{Angle (radians)} )</td>
</tr>
</tbody>
</table>

From Anderson's book

**Fig. 5.25** Estimating saturation as an exponential function.

\[
S(1.0) = S_{G1} = \frac{i_{F1} - i_{F0}}{i_{F0}}
\]

\[
S(1.2) = S_{G2} = \frac{i_{F3} - i_{F2}}{i_{F2}} = \frac{i_{F3} - 1.2i_{F0}}{1.2i_{F0}}
\]

\[
S_G V_t = A_G e^{B_G (V_t - 0.8)} \sim \psi_I = A_{sat} e^{B_{sat} (\psi_{at} - \psi_{T1})}
\]

\[
A_G = S_{G1}^2 / 1.2 S_{G2} \quad B_G = 5 \ln (1.2 S_{G2} / S_{G1})
\]
Example 3.2 in Kundur’s Book

The following are the parameters in per unit on machine rating of a 555 MVA, 24 kV, 0.9 p.f., 60 Hz, 3600 RPM turbine-generator:\(^1\):

\[
\begin{align*}
L_{ad} &= 1.66 & L_{aq} &= 1.61 & L_f &= 0.15 & R_a &= 0.003 \\
L_{fd} &= 0.165 & R_{fd} &= 0.0006 & L_{1d} &= 0.1713 & R_{1d} &= 0.0284 \\
L_{1q} &= 0.7252 & R_{1q} &= 0.00619 & L_{2q} &= 0.125 & R_{2q} &= 0.02368 \\
M_R & \text{ is assumed to be equal to } L_{ad}.
\end{align*}
\]

(a) When the generator is delivering rated MVA at 0.9 p.f. (lag) and rated terminal voltage, compute the following:

(i) Internal angle \( \delta_i \) in electrical degrees
(ii) Per unit values of \( e_d, e_q, i_d, i_q, i_{1d}, i_{1q}, i_{2d}, i_{2q}, e_{fd}, e_{fd}, \psi_{1d}, \psi_{1q}, \psi_{2d}, \psi_{2q} \)
(iii) Air-gap torque \( T_e \) in per unit and in newton-meters

Assume that the effect of magnetic saturation at the given operating condition is to reduce \( L_{ad} \) and \( L_{aq} \) to 83.5% of the values given above.

(b) Compute the internal angle \( \delta_i \) and field current \( i_{fd} \) for the above operating condition, using the approximate equivalent circuit of Figure 3.22. Neglect \( R_a \).

Solution

(a) With the given operating condition, the per unit values of terminal quantities are

\[
P = 0.9, \quad Q = 0.436, \quad E_f = 1.0, \quad I_f = 1.0, \quad \phi = 25.84^\circ
\]
The saturated values of the inductances are

\[ L_{ad} = 0.835 \times 1.66 = 1.386 \]
\[ L_{aq} = 0.835 \times 1.61 = 1.344 \]
\[ L_d = L_{ad} + L_l = 1.386 + 0.15 = 1.536 \]
\[ L_q = L_{aq} + L_l = 1.344 + 0.15 = 1.494 \]

Following the procedure outlined in Section 3.6.5,

(i) \[ \delta_i = \tan^{-1} \left( \frac{1.494 \times 1.0 \times 0.9 - 0.003 \times 1.0 \times 0.436}{1.0 + 0.003 \times 1.0 \times 0.9 + 1.494 \times 1.0 \times 0.436} \right) \]
\[ = \tan^{-1}(0.812) = 39.1 \text{ electrical degrees} \]

(ii) \[ e_d = E_1 \sin \delta_i = 1.0 \sin 39.1 = 0.631 \text{ pu} \]
\[ e_q = E_1 \cos \delta_i = 1.0 \cos 39.1 = 0.776 \text{ pu} \]
\[ i_d = I_1 \sin (\delta_i + \phi) = 1.0 \sin(39.1 + 25.84) = 0.906 \text{ pu} \]
\[ i_q = I_1 \cos (\delta_i + \phi) = 1.0 \cos(39.1 + 25.84) = 0.423 \text{ pu} \]
\[ i_{fd} = \frac{e_q + R_a i_q + X_{ad} i_d}{X_{ad}} \]
\[ = \frac{0.776 + 0.003 \times 0.423 + 1.536 \times 0.906}{1.386} \]
\[ = 1.565 \text{ pu} \]

\[ e_{fd} = R_{fd} i_{fd} = 0.0006 \times 1.565 \]
\[ = 0.000939 \text{ pu} \]
\[ \psi_{fd} = (L_{ad} + L_{fd}) i_{fd} - L_{ad} i_d \]
\[ = (1.386 + 0.165) \times 1.565 - 1.386 \times 0.907 \]
\[ = 1.17 \text{ pu} \]
\[ \psi_{1d} = L_{ad} (i_{fd} - i_d) \]
\[ = 1.386 \times (1.565 - 0.906) \]
\[ = 0.913 \text{ pu} \]
\[ \psi_{1q} = \psi_{2q} = -L_{aq} i_q = -1.344 \times 0.423 \]
\[ = -0.569 \text{ pu} \]

Under steady state,

\[ i_{1d} = i_{1q} = i_{2q} = 0 \]
(iii) Air-gap torque

\[ T_e = P_t + I_t^2 R_e \]
\[ = 0.9 + 1.0^2 \times 0.003 \]
\[ = 0.903 \text{ pu} \]

\[ T_{base} = \frac{\text{MVA}_{base} \times 10^6}{\omega_{m_{base}}} \]
\[ = \frac{555 \times 10^6}{2\pi \times 60} = 1.472 \times 10^6 \text{ N} \cdot \text{m} \]

Therefore,

\[ T_e = 0.903 \times 1.472 \times 10^6 \]
\[ = 1.329 \times 10^6 \text{ N} \cdot \text{m} \]

(b) Using the saturated value of \( X_{ad} \),

\[ E_q = X_{ad} i_{fd} = 1.386 i_{fd} \]

\[ X_s = X_{ad} + X_l = 1.386 + 0.15 = 1.536 \]

From the equivalent circuit of Figure 3.22, with \( \tilde{E}_t \) as reference phasor,

\[ \tilde{E}_q = \tilde{E}_t + jX_s \tilde{I}_t \]
\[ = 1.0 + j1.536(0.9 - j0.436) \]
\[ = 1.670 + j1.382 \]
\[ = 2.17 \angle 39.6^\circ \text{ pu} \]

\[ \delta_i = 39.6^\circ \approx 39.1^\circ \]

Therefore,

\[ i_{fd} = \frac{E_q}{X_{ad}} = \frac{2.17}{1.386} = 1.566 \text{ pu} \approx 1.565 \text{ pu} \]
Sub-transient and Transient Analysis

- Following a disturbance, currents are induced in rotor circuits. Some of these induced rotor currents decay more rapidly than others.
  - **Sub-transient parameters**: influencing rapidly decaying (cycles) components
  - **Transient parameters**: influencing the slowly decaying (seconds) components
  - **Synchronous parameters**: influencing sustained (steady state) components

![Diagram showing sub-transient, transient, and steady-state periods](image)

**Figure 3.27** Fundamental frequency component of armature current
Transient Phenomena

- Study transient behavior of a simple RL circuit

\[ v(t) = V_m \sin(\omega t + \alpha) \cdot u(t) \quad \text{Unit step function} \]

\[ Ri(t) + L \frac{di(t)}{dt} = v(t) \]

- Apply Laplace Transform

\[ RI(s) + L[sI(s) - i(0)] = V(s) \quad \rightarrow \quad I(s) = \frac{V(s)}{R} \cdot \frac{1}{1 + s\tau} = \frac{\Psi(s)}{R/s} \cdot \frac{1}{1 + s\tau} \quad \text{where} \quad \tau = \frac{L}{R} \]

- Apply Inverse Laplace Transform to \( I(s) \)

\[ i(t) = I_m \sin(\omega t + \alpha - \gamma) - I_m e^{-t/\tau} \sin(\alpha - \gamma) \]

Steady-state (sinusoidal component) \hspace{1cm} dc offset (transient component)

- Saadat’s Example 8.1

\[ R=0.125\Omega, \quad L=10\text{mH}, \quad V_m=151V \]

\[ \tau=L/R=0.08\text{s} \quad \text{(time for decaying to 37%)} \]