2.2 Modeling of Loads

1. Chapter 7 of Kundur’s book
Background

• The representation of loads has not received as much attention as other components in power systems, but it may have significant impacts on stability analysis results.

• Accurate load modeling is difficult because:
  – at high-voltage levels, loads must be aggregated for stability studies,
  – a power system has a large number of diverse load components,
  – ownerships and locations of load devices are not directly accessible to the electricity utility (e.g. transmission and distribution are owned by different companies)
  – there is no precise information on load composition, which changes with time of day and week, seasons and weather, and
  – load characteristics are uncertain, particularly for large voltage or frequency variations
**Bus Load**

- Defined as a portion of the actual system, which is not explicitly represented in the system model but treated as a single power-consuming device connected to a bus in the system model.

- In this context, “bus load” includes connected load devices together with some or all of the following:
  - Substation step-down transformers
  - Sub-transmission feeders
  - Primary distribution feeders
  - Distributions transformers
  - Secondary distribution feeders
  - Shunt capacitors
  - Voltage regulators
  - Customer wiring, transformers and capacitors

- If we want to represent the “bus load” accurately, those elements listed must be accounted for.

- To a large extent, this depends on what is and is not represented in the system model. For bulk power system studies, much of the sub-transmission as well as the distribution system are omitted.
Approaches for Load Modeling

• “Bottom-up” (theoretical aggregation):
  – Analytically, by lumping similar loads based on the load type and then using pre-determined values for each parameter of the load
  – Referred to as “component-based approach”

• “Top-down” (identification):
  – Selecting a load model structure and then performing parameter estimation using an appropriate identification technique based on field measurements
  – Referred to as “measurement-based approach”
Component-based approach

- **Load Characteristics**: A set of parameters or functions, e.g. power factor, $P(V,f)$ and $Q(V,f)$, describing behaviors of a specified load. This could be applied to either a specific load device or an aggregation of multiple load devices (e.g. a load component, a load class and the total bus load).

- **Load Component**: The aggregate equivalent of all load devices of similar types, e.g. heater, air conditioner, lighting, etc.

- **Load Composition**: Fractional composition of the load by Load Components
  - Could be fraction applied to a Bus Load or to a specific Load Class
  - As an example, composition for an internet data center could be 40~50% computer servers, 40~50% HVAC loads and 10~15% lighting loads

- **Load Class**: A category of load, e.g. residential, commercial or industrial. For load modeling purposes, it is useful to group loads into several classes, which each have similar load composition and characteristics

- **Load class mix**: Fractional composition of the Bus Load by Load Class.
Component-based approach

Load Bus

Load Class Mix, e.g.
- 80% residential + 20% commercial

Load Class, e.g.
- Residential
- Commercial
- Industrial

Load Characteristics e.g.
- Power factor
- $P(V,f)$, $Q(V,f)$
- Motor

Load Composition e.g.
- 20% lighting + 40% heating + 40% AC

Load Component, e.g.
- Heater load
- Air Conditioner load
- Lighting load

Load Device, e.g.
- Heater
- Air Conditioner
- Light

Figure 7.21 Component-based modelling approach
Load Model

• **Static Load Model:**
  – $P$ and $Q$ at time $t$ are expressed as algebraic equations on values of bus voltage magnitude $V$ and frequency $f$ only at time point $t$.
  \[ P( V(t), f(t) ) , \quad Q( V(t), f(t) ) \]
  – Represents resistive/ lighting loads or simplified dynamic loads

• **Dynamic Load Model:**
  – $P$ and $Q$ at time $t$ are expressed as difference/differential equations on $V$ and $f$ at $t$ and past time instants
  \[ P( V(\tau \leq t), f(\tau \leq t) ) \quad \text{and} \quad Q( V(\tau \leq t), f(\tau \leq t) ) \]
  – Represents induction motor loads and others
Static Load Models

• Polynomial load model (ZIP Model):
  \[ P = P_0(p_1\bar{V}^2 + p_2\bar{V} + p_3) \]
  \[ Q = Q_0(q_1\bar{V}^2 + q_2\bar{V} + q_3) \]

  \( p_1 + p_2 + p_3 = 1 \) and \( q_1 + q_2 + q_3 = 1 \) if \( P_0, Q_0 \) and \( V_0 \) are rated values

- It is composed of constant impedance (Z), constant current (I) and constant power (P or Q) components.

• Exponential load model
  \[ P = P_0(\bar{V})^a \]
  \[ Q = Q_0(\bar{V})^b \]

- Estimation of \( a \) and \( b \):
  Around \( V_0 \),
  \[ \frac{\partial P}{\partial \bar{V}} \approx a \]
  \[ \frac{\partial Q}{\partial \bar{V}} \approx b \]

<table>
<thead>
<tr>
<th>Value of ( a ) (or ( b ))</th>
<th>Corresponding ZIP model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Constant power</td>
</tr>
<tr>
<td>1</td>
<td>Constant current</td>
</tr>
<tr>
<td>2</td>
<td>Constant impedance</td>
</tr>
</tbody>
</table>

Usually, \( a=0.5 \sim 1.8 \) and \( b=1.5 \sim 6 \)
Frequency Dependency

• With a frequency deviation $\Delta f = f - f_0$
  
  $P = P_0(\bar{V})^a(1 + K_{pf}\Delta f)$
  
  $Q = Q_0(\bar{V})^b(1 + K_{qf}\Delta f)$

- Unlike the speed of a generator, the frequency of a bus voltage is not a state variable in the system model for stability analysis.

- $\Delta f$ can be computed by taking the numerical derivative of the voltage phase angle.

• Comprehensive static model: summation of ZIP and exponential models
  
  $P = P_0(P_{ZIP} + P_{EX1} + P_{EX2})$

Typically, $K_{pf} = \partial P/\partial f = 0$ to 3.0,

$K_{qf} = \partial Q/\partial f = -2.0$ to 0
## Static Load Models

### Static Characteristics of Load Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Power factor</th>
<th>$a$</th>
<th>$b$</th>
<th>$K_{pf}$</th>
<th>$K_{qf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air conditioner</td>
<td>0.90</td>
<td>0.088</td>
<td>2.5</td>
<td>0.98</td>
<td>-1.3</td>
</tr>
<tr>
<td>3-phase central</td>
<td>0.96</td>
<td>0.202</td>
<td>2.3</td>
<td>0.90</td>
<td>-2.7</td>
</tr>
<tr>
<td>1-phase central</td>
<td>0.82</td>
<td>0.468</td>
<td>2.5</td>
<td>0.56</td>
<td>-2.8</td>
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<tr>
<td>Window type</td>
<td>1.0</td>
<td>2.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Water heaters,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range top, oven,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deep fryer</td>
<td>0.99</td>
<td>1.8</td>
<td>3.6</td>
<td>0</td>
<td>-1.4</td>
</tr>
<tr>
<td>Dishwasher</td>
<td>0.65</td>
<td>0.08</td>
<td>1.6</td>
<td>3.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Clothes washer</td>
<td>0.99</td>
<td>2.0</td>
<td>3.2</td>
<td>0</td>
<td>-2.5</td>
</tr>
<tr>
<td>Clothes dryer</td>
<td>0.8</td>
<td>0.77</td>
<td>2.5</td>
<td>0.53</td>
<td>-1.5</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>0.8</td>
<td>2.0</td>
<td>5.1</td>
<td>0</td>
<td>-4.5</td>
</tr>
<tr>
<td>Television</td>
<td>1.0</td>
<td>1.55</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Incandescent lights</td>
<td>0.9</td>
<td>0.96</td>
<td>7.4</td>
<td>1.0</td>
<td>-2.8</td>
</tr>
<tr>
<td>Fluorescent lights</td>
<td>0.88</td>
<td>0.07</td>
<td>0.5</td>
<td>2.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Industrial motors</td>
<td>0.87</td>
<td>0.08</td>
<td>1.6</td>
<td>2.9</td>
<td>1.7</td>
</tr>
<tr>
<td>Fan motors</td>
<td>0.85</td>
<td>1.4</td>
<td>1.4</td>
<td>5.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Agricultural pumps</td>
<td>0.70</td>
<td>2.3</td>
<td>1.6</td>
<td>-1.0</td>
<td>-1.0</td>
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<tr>
<td>Arc furnace</td>
<td>0.64</td>
<td>3.4</td>
<td>1.5</td>
<td>0</td>
<td>-11.8</td>
</tr>
<tr>
<td>Transformer (unloaded)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

### Static Characteristics of Load Classes

<table>
<thead>
<tr>
<th>Load class</th>
<th>Power factor</th>
<th>$a$</th>
<th>$b$</th>
<th>$K_{pf}$</th>
<th>$K_{qf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summer</td>
<td>0.9</td>
<td>1.2</td>
<td>2.9</td>
<td>0.8</td>
<td>-2.2</td>
</tr>
<tr>
<td>Winter</td>
<td>0.99</td>
<td>1.5</td>
<td>3.2</td>
<td>1.0</td>
<td>-1.5</td>
</tr>
<tr>
<td>Commercial</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summer</td>
<td>0.85</td>
<td>0.99</td>
<td>3.5</td>
<td>1.2</td>
<td>-1.6</td>
</tr>
<tr>
<td>Winter</td>
<td>0.9</td>
<td>1.3</td>
<td>3.1</td>
<td>1.5</td>
<td>-1.1</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.85</td>
<td>0.18</td>
<td>6.0</td>
<td>2.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Power Plant Auxiliaries</td>
<td>0.8</td>
<td>0.1</td>
<td>1.8</td>
<td>2.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

\[
P = P_0(\bar{V})^a(1 + K_{pf} \Delta f) \\
Q = Q_0(\bar{V})^b(1 + K_{qf} \Delta f)
\]

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<tr>
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<td>0</td>
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<tr>
<td>1</td>
<td>Constant current</td>
</tr>
<tr>
<td>2</td>
<td>Constant impedance</td>
</tr>
</tbody>
</table>
Common Practice in Stability Studies

- In the absence of information on load characteristics:
  - When $V$ is around $V_0$
    - $a=1$: $P$ as constant current
    - $b=2$: $Q$ as constant impedance
  - When $V$ is below a threshold (e.g. $0.6\sim0.7V_0$), convert the load to constant impedance to avoid computational problems
Motor loads

- Motor loads consume **60-70% of the total energy** supplied by a power system.
- Motors could **drop 70% of system load** during a fault or during voltage excursions following the fault. This will improve (impact) stability of receiving-end (sending-end) generators.
- Air-conditioner compressor induction motors count up to **50% of system load in summer**
  - It stalls if the voltage is low (e.g. 50-65% of the nominal voltage for 3 cycles) due to a fault; once stalled, it behaves like a constant impedance load (with a low power factor) drawing 2-3 times of the rated current.
  - The thermal overload protection disconnects the motor if it remains stalled for considerable amount of time (typically, 15s if stalled at 50% voltage).
  - It recovers if voltage goes back to 70% of the nominal voltage. That **delays the voltage recovery**.
  - In areas with a high percentage of air conditioner loads, delayed voltage recovery (e.g. FIDVR issues), short-term voltage stability and fast voltage collapse are main stability concerns.
Impacts of Induction Motors on System Oscillation Damping and Voltage Stability

Sources:
- Measurement-Based Load Modeling, EPRI Report, Product ID:1014402, 2006
Modeling of Induction Motors

\[ s = \frac{n_s - n_r}{n_s} \]
\[ T \propto \frac{P_{sh}}{n_r} \]

**Figure 7.8** Alternative form of induction machine equivalent circuit
Discharge Lighting Loads

- Mercury vapor, sodium vapor, etc.
- About 20% of the load in commercial areas
- Unlike incandescent lights, discharge lights extinguish at voltages of 70-80% of rated voltage
  - This causes load to drop following fault clearing.
  - That will either improve or impact stability depending on the location
- When the voltage recovers, they restart after a 1-2 sec delay

*Figure 4. When voltages swing below 80% following fault clearing, discharge lighting will extinguish, reduce system load, and stabilize voltage between 70 and 80%. This will improve.*
Thermostat-Controlled Loads

- Space heaters, water heaters, molding and packaging machines, soldering machines, etc.
- **Short term**: constant resistance.
- **Long term**: constant power with a suitable time constant.

*Figure 5. When voltage is low, thermostats on “resistance type loads” remain closed longer, effectively restoring the load to a constant power characteristic in the steady-state.*
Measurement-based Approach for Load Modeling

- Load characteristics are measured at representative substations and feeders at selected times of the day and season.
  - Steady-state $\partial P/\partial V$, $\partial P/\partial f$, $\partial Q/\partial V$ and $\partial Q/\partial f$
  - Dynamic load-voltage characteristics

- These are used to extrapolate the parameters of loads throughout the system
Field Test

Sources:

- Measurement-Based Load Modeling, EPRI Report, Product ID:1014402, 2006

Figure 6-52
Comparison of Measured and Estimated (Using Converged Parameters) Real and Reactive Power – ZIP + Motor Model Structure
Parameter Estimation

Table 6-16
Converged Parameters – ZIP + Motor Model Structure

Parameter Derivation Results

Load Model Configuration - ZIP static model + induction model

\[
\begin{align*}
\Delta P &= \Delta P_0 \left[ a_P \left( \frac{V}{V_0} \right)^2 + b_P \left( \frac{V}{V_0} \right) + c_P \right] + k_P \Delta f \\
\Delta Q &= \Delta Q_0 \left[ a_Q \left( \frac{V}{V_0} \right)^2 + b_Q \left( \frac{V}{V_0} \right) + c_Q \right] + k_Q \Delta f \\
k_p &= k_q = 0 \quad c_p = 1 - a_p - b_p \quad c_q = 1 - a_q - b_q
\end{align*}
\]

Parameters for static load - P

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Derived Value</th>
<th>Deviation%</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.5</td>
<td>0.3216</td>
<td>-33.6735</td>
</tr>
<tr>
<td>ap</td>
<td>0.4</td>
<td>-6.8823</td>
<td>-1495.5969</td>
</tr>
<tr>
<td>bp</td>
<td>0.3</td>
<td>-2.1110</td>
<td>-803.592</td>
</tr>
</tbody>
</table>

Parameters for static load - Q

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Derived Value</th>
<th>Deviation%</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.6</td>
<td>0.9408</td>
<td>88.1551</td>
</tr>
<tr>
<td>aq</td>
<td>0.4</td>
<td>-12.2013</td>
<td>-3150.33</td>
</tr>
<tr>
<td>bq</td>
<td>0.3</td>
<td>-9.8494</td>
<td>-3383.1359</td>
</tr>
</tbody>
</table>

Parameters for dynamic (motor) load

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Derived Value</th>
<th>Deviation%</th>
</tr>
</thead>
<tbody>
<tr>
<td>xi</td>
<td>0.16</td>
<td>0.4995</td>
<td>212.197</td>
</tr>
<tr>
<td>xiA</td>
<td>0.12</td>
<td>0.4990</td>
<td>315.7978</td>
</tr>
<tr>
<td>xiM</td>
<td>3.2</td>
<td>2.9983</td>
<td>-9.5074</td>
</tr>
<tr>
<td>R1</td>
<td>0.019</td>
<td>0.4996</td>
<td>2675.6405</td>
</tr>
<tr>
<td>RA</td>
<td>0.031</td>
<td>0.4999</td>
<td>1512.5999</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>0.8652</td>
<td>-13.4815</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0.3934</td>
<td>Inf</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0.0008</td>
<td>Inf</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0.0841</td>
<td>Inf</td>
</tr>
<tr>
<td>H</td>
<td>0.7</td>
<td>0.1690</td>
<td>-75.8586</td>
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<tr>
<td>T</td>
<td>0.8622</td>
<td>0.4799</td>
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</table>