Homework #3: Generator and Load Modeling

**Question 1:** Prove these two formulas respectively on $L_d'$ and $L_d''$ on Slide #43 (Table 4.1 in Kundur’s book)

\[
L_d' \frac{T_4}{T_1} = L_d' + \frac{L_{ad}}{L_{fd}}/L_{1d}
\]

\[
L_d'' \frac{T_0}{T_3} = L_1 + \frac{L_{ad}}{L_{fd}}/L_{1d}
\]

**Question 2:** Prove the differential equation on $E_d'$ in the 2-axis generator model:

\[
T_d \frac{d}{dt} E_d' = -E_d' + (X_q' - X_q')i_d
\]

where $E_d' = -\omega L_{ad} \psi_{1q} / L_Q$

**Question 3:** Consider the following equivalent circuits for a 384 MVA, 24kV, 0.85 power factor, 60Hz, 3 phase, 2 pole synchronous generator.

![d-axis equivalent circuit](image)

![q-axis equivalent circuit](image)

It has the following parameters:

- $L_d = 0.183$ pu
- $R_d = 0.0014$ pu
- $L_{ad} = 1.5949$ pu
- $L_{aq} = 1.5749$ pu
- $L_{fd} = 0.7017$ pu
- $R_{fd} = 0.0012$ pu
- $L_{1d} = 0.0845$ pu
- $R_{1d} = 0.0222$ pu
- $L_{1q} = 0.401$ pu
- $R_{1q} = 0.00523$ pu
- $L_{2q} = 0.0641$ pu
- $R_{2q} = 0.01431$ pu

Stored energy at rated speed = 1006.5 MW·s

Damping coefficient $K_D = 0$

1. Ignoring saturation, calculate transient and subtransient reactance parameters in per unit values and open-circuit time constants using the formulas in Slide #42

\[
X_d', X_d'', X_q', X_q'', T_d', T_d'', T_q', T_q''
\]
2. Assuming the following open-circuit saturation curve for both $d$- and $q$-axis saturation characteristics, draw the saturation curve (refer to Kundur’s Figure 3.30 or Slide #33)

\[
A_{sat} = 0.03125 \quad B_{sat} = 6.931 \\
\psi_{T1} = 0.8 \text{ pu} \quad \psi_{T2} = 1.0 \text{ pu} \quad L_{ratio} = 1.5
\]

3. With the armature terminal voltage at rated value, consider two operating conditions with the following steady-state generator outputs

Output 1: $P_t = 307$ MW $Q_t = 115$ MVAr
Output 2: $P_t = 345$ MW $Q_t = -154$ MVAr

For each of the two conditions,

i) Compute factor $K_{sd}$ (assuming $K_{sq}=K_{sd}$), internal rotor angle $\delta_i$, and per unit values of $\psi_{ds}$, $E_q$, $e_d$, $e_q$, $i_d$, $i_q$, $i_{1d}$, $i_{1q}$, $i_{2d}$, $e_{1d}$, $\psi_{1d}$, $\psi_{1q}$, $\psi_{2d}$,

ii) Calculate the phasors of terminal voltage $E_t$, terminal current $I_t$, sub-transient voltage $E''$ and transient voltage $E'$. Draw the phasor diagram about those phasors and voltages on $R_a$, $X_d$, $X_q$, $X''_d$, $X''_q$, $X'_q$, $X'_d$, $X''_q$. Are the heads of phasors $E_q$, $E'$ and $E''$ on the same straight line and why?

iii) Calculate steady-state air-gap torque $T_e$ in per unit and N·m. How much is $T_m$ in per unit?

iv) Consider the classic model for the generator and assume that it is connected to a load through reactance $X_l = 0.1$ pu as shown by the figure. Calculate the per unit voltage magnitude $V_t$, real power $P$ and reactive power $Q$ of the load.

If the load of the bus under the current condition can be described by the following frequency dependent exponential load model, where $V_\phi$, $P_0$ and $Q_0$ take the values of $V_t$, $P$ and $Q$ calculated above and $f_0=60$Hz

\[
P= P_0 (V_t/V_\phi)^0.9 \times [1+1.2\times (f/f_0)/f_0] \\
Q= Q_0 (V_t/V_\phi)^2 \times [1-1.5\times (f/f_0)/f_0]
\]

If at two time points $t_1$ and $t_2$, actual measurements of $f$ and $V_t$ are

\[
\begin{align*}
\text{at } t_1 & \quad f=59.75 \text{ Hz} \quad V_t=0.99 \text{ pu} \\
\text{at } t_2 & \quad f=60.05 \text{ Hz} \quad V_t=0.96 \text{ pu}
\end{align*}
\]

Assume $T_m$ to be constant. Calculate $d\Delta\omega/\Delta t$ in rad/s$^2$ at $t_1$ and $t_2$. 