Homework #6

1. (10 points) Problem 11.5 on Saadat’s book (see the appendix for Problem 11.4)
Two synchronous generators represented by a constant voltage behind transient reactance are connected by a pure reactance $X = 0.3$ per unit, as shown in Figure 11.34. The generator inertia constants are $H_1 = 4.0 \text{MJ/MVA}$ and $H_2 = 6 \text{MJ/MVA}$, and the transient reactances are $X_1' = 0.16$ and $X_2' = 0.20$ per unit. The system is operating in the steady state with $E_1' = 1.2$, $P_{m1} = 1.5$, and $E_2' = 1.1$, $P_{m2} = 1.0$ per unit. Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Referring to Problem 11.4, reduce the two-machine system to an equivalent one-machine against an infinite bus. Find the inertia constant of the equivalent machine, the mechanical input power, and the amplitude of its power angle curve, and obtain the equivalent swing equation in terms of $\delta$.

2. (30 points) The circuit model below represents the steady-state operating condition of a single-machine-infinite-bus system. $E_t=E_b=1.0\text{pu}$. The generator is represented by the classical model with $H=10 \text{ (MWs/MVA)}$ and $K_D=0$. Consider a contingency with this sequence of events:
- at $t=0\text{s}$, a three-phase short circuit fault happens at one end of a transmission line as indicated;
- at $t_1$, the breakers on both ends of that line are opened to clear the fault, and $\delta(t_1)$ reaches $\delta_1$;
- at $t_2$, when $\delta(t_2)$ reaches $\delta_2$, those two breakers succeed in reclosing the line since the fault disappears during $t_1$ to $t_2$. 

FIGURE 11.34
System of Problem 11.5.
If $\delta_1=60^\circ$ and $\delta_2=80^\circ$, apply the Equal-Area Criterion to judge the system’s transient stability with respect to the contingency described above. Sketch the $P-\delta$ curve of the generator under that contingency, and indicate the path of the system state and the accelerating and decelerating areas. If the system is transiently stable, what is the maximum rotor angle $\delta_m$?

3-7 (60 points) Problems 11.14-11.17 on Saadat’s book (see the appendix for Problem 11.6)

11.14. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit. Use $eacpower(P_m, E, V, X)$ to find
(a) The maximum power input that can be added without loss of synchronism.
(b) Repeat (a) with zero initial power input. Assume the generator internal voltage remains constant at the value computed in (a).

11.15. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit.
(a) A temporary three-phase fault occurs at the sending end of one of the transmission lines. When the fault is cleared, both lines are intact. Using equal area criterion, determine the critical clearing angle and the critical fault clearing time. Use $eacfault(P_m, E, V, X_1, X_2, X_3)$ to check the result and to display the power-angle plot.
(b) A three-phase fault occurs at the middle of one of the lines, the fault is cleared, and the faulted line is isolated. Determine the critical clearing angle. Use $eacfault(P_m, E, V, X_1, X_2, X_3)$ to check the results and to display the power-angle plot.

11.16. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit. A three-phase fault at the middle of one line is cleared by isolating the faulted circuit simultaneously at both ends.
(a) The fault is cleared in 0.2 second. Obtain the numerical solution of the swing equation for 1.5 seconds. Select one of the functions $swingmeu$, $swingrk2$, or $swingrk4$.
(b) Repeat the simulation and obtain the swing plots when fault is cleared in 0.4 second, and for the critical clearing time.

11.17. Consider the power system network of Example 11.7 with the described operating condition. A three-phase fault occurs on line 1–5 near bus 5 and is cleared by the simultaneous opening of breakers at both ends of the line. Using the $trstab$ program, perform a transient stability analysis. Determine the system stability for
(a) When the fault is cleared in 0.2 second
(b) When the fault is cleared in 0.4 second
(c) Repeat the simulation to determine the critical clearing time.
Appendix - Saadat’s Problems 11.4 and 11.6

11.4. The swing equations of two interconnected synchronous machines are written as

\[ \frac{H_1}{f_0} \frac{d^2 \delta_1}{dt^2} = P_{m1} - P_{e1} \]

\[ \frac{H_2}{f_0} \frac{d^2 \delta_2}{dt^2} = P_{m2} - P_{e2} \]

Denote the relative power angle between the two machines by \( \delta = \delta_1 - \delta_2 \). Obtain a swing equation equivalent to that of a single machine in terms of \( \delta \), and show that

\[ \frac{H}{f_0} \frac{d^2 \delta}{dt^2} = P_m - P_e \]

where

\[ H = \frac{H_1 H_2}{H_1 + H_2} \]

\[ P_m = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} \]

\[ P_e = \frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2} \]

11.6. A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown in Figure 11.35. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is 1.1. The infinite bus voltage \( V = 1.0 \angle 0^\circ \) per unit. Determine the generator excitation voltage and obtain the swing equation as given by (11.36).

![Diagram of the system](image)

**FIGURE 11.35**
System of Problem 11.6.