ECE315 - Signals and Systems, Spring 2019

Homework 8, Due 04/16/19

Note that some problems are from Oppenheim’s book and the answers were given at the end of the book. So if you just provide the answer without detailed steps, you will get zero point for that problem. This homework focuses on how to utilize FT properties and pairs.

1. On how to use the analysis or synthesis equations to calculate the Fourier transform of signal \( x(t) \) (or \( x[n] \)) and the inverse Fourier transform of \( X(j\omega) \) (or \( X(e^{j\omega}) \)).

   (a) \( x(t) = e^{-2|t-1|} \)

   (b) \( x(t) = \frac{d}{dt} \{ u(-2-t) + u(t-2) \} \)

   (c) \( x(t) = 1 + \cos(6\pi t + \frac{\pi}{8}) \) (Note that this is a periodic signal)

   (d) \( X(j\omega) \) is complex where

   \[
   |X(j\omega)| = 2\{u(\omega + 3) - u(\omega - 3)\}
   \]

   \[
   \angle X(j\omega) = -\frac{3}{2}\omega + \pi
   \]

   (e) \( x[n] = \delta[n+2] - \delta[n-2] \)

   (f) \( X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} [2\pi \delta(\omega - 2\pi k) + \pi \delta(\omega - \frac{\pi}{2} - 2\pi k) + \pi \delta(\omega + \frac{\pi}{2} - 2\pi k)] \)

   (g) \( |X(e^{j\omega})| = \begin{cases} 
   1, & 0 \leq |\omega| \leq \frac{\pi}{4} \\
   0, & \frac{\pi}{4} \leq |\omega| \leq \pi 
   \end{cases} \) and \( \angle X(e^{j\omega}) = -\frac{3\omega}{\pi} \)

2. On how to use the Fourier transform properties and transform pairs.

   (a) Assume the Fourier transform of \( x(t) \) is \( X(j\omega) \). Use it to represent the Fourier transform to

   \[
   x_1(t) = x(1 - 3t) + x(-6 + t).
   \]

   (b) Assume the Fourier transform of \( x(t) \) is \( X(j\omega) \). Use it to represent the Fourier transform to

   \[
   x_2(t) = \frac{d^2}{dt^2} x(3t - 1).
   \]
(c) Determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither, without evaluating the inverse of the signal.

i. \(X(j\omega) = \sum_{k=-\infty}^{\infty} \left( \frac{1}{2} \right)^{|k|} \delta(\omega - k\pi/4)\)

ii. \(X(j\omega) = \frac{\sin(\omega)}{\omega} e^{j(2\omega + \pi/2)}\)

iii. \(X(j\omega) = u(\omega) - u(\omega - 2)\)

(d) For the following signal

\[
x(t) = \begin{cases} 
0, & t < -1/2 \\
 t + 1/2, & -1/2 \leq t \leq 1/2 \\
1, & t > 1/2 
\end{cases}
\]

i. Determine \(X(j\omega)\). Hint: use the differentiation and integration properties and the Fourier transform pair for the rectangular pulse.

ii. Calculate the Fourier transform of the even part of \(x(t)\). Is it the same as the real part of \(X(j\omega)\)?

iii. Calculate the Fourier transform of the odd part of \(x(t)\).

(e) Find the Fourier transform of \(x(t) = \frac{t}{2} \left( \frac{\sin(t)}{\pi t} \right)^2\). Use Parseval’s relation to determine the numerical value of \(A = \int_{-\infty}^{\infty} t^2 \left( \frac{\sin(t)}{\pi t} \right)^4 dt\).

3. The following four facts are given about a real signal \(x[n]\) with Fourier transform \(X(e^{j\omega})\):

(a) \(x[n] = 0\) for \(n > 0\)

(b) \(x[0] > 0\)

(c) \(Im\{X(e^{j\omega})\} = \sin(\omega) - \sin(2\omega)\)

(d) \(\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3\)

Determine \(x[n]\).