1. (40 pts) In class, we have gone through an exercise that uses complex exponentials to represent the periodic square wave with $T = 4$, and the width of the square being 1. That is,

$$x(t) = \begin{cases} 
1 & |t| < \frac{1}{2} \\
0 & \frac{1}{2} < |t| < 2
\end{cases}$$

The Fourier series coefficient is calculated as

$$a_k = \frac{\sin(\pi k/2)}{k\pi}$$

with $a_0 = 1/2$. Since $x(t)$ is real, we can use the trigonometric form of Fourier series representation, i.e.,

$$x(t) = a_0 + 2 \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)$$

Since $a_k$ is real, $A_k = |a_k|$ and $\theta_k = 0$. That is,

$$x(t) = a_0 + 2 \sum_{k=1}^{\infty} |a_k| \cos(k\omega_0 t)$$

Write a MATLAB function $fs(N)$ that reconstructs an approximated version of $x(t)$ with finite sums.

$$\tilde{x}(t) = a_0 + 2 \sum_{k=1}^{N} |a_k| \cos(k\omega_0 t)$$

Submit the source code of your function. Also submit a plot with $2 \times 2$ subplots with $N = 10, 50, 100, \text{ and } 1000$. Explain Gibb’s phenomenon.
2. (30 pts) Comparison of performing convolution in the spatial vs. frequency domain.

- **Convolve** [http://web.eecs.utk.edu/~qi/ece315/project/cameraman.pgm](http://web.eecs.utk.edu/~qi/ece315/project/cameraman.pgm) with convolution kernel \( h \) (i.e., a 2-D impulse response)

\[
h = \frac{1}{9} \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

using both spatial domain method (using the MATLAB function `conv2()`) and frequency domain method (see details below). Compare the results both visually and by reporting the excucion time. Use the MATLAB function `tic` and `tac`.

- Generate convolution kernels of different sizes (e.g., \(7 \times 7, 25 \times 25, 49 \times 49\), etc.) and compare the performance. You should see that the convolution results from the spatial and frequency domains look exactly the same. Plot a figure of \(2 \times 1\) subplots with the original, results from spatial convolution as well as frequency-domain convolution. Also plot a bar chart or a table comparing the different run time from the two methods when different kernel sizes are used.

- Generate the autocorrelation image of `cameraman.pgm` in both spatial and frequency domains and compare the execution time. Autocorrelation is like a convolution with itself where the kernel is the image itself. Comment on what you see in the result. Can you reason why it behaves that way? What would be a good application for autocorrelation? Add the run time difference of autocorrelation in the above table or bar chart. Comment on the trend you observe.

3. (30 pts) Pitch analysis. Pitch can be derived using autocorrelation, Cepstrum analysis, etc. Pitch characterizes the peak energy levels of the signal. Here is the function to derive pitch contour [http://web.eecs.utk.edu/~qi/ece315/project/pitch.m](http://web.eecs.utk.edu/~qi/ece315/project/pitch.m). Go through the code and understand how the contour is generated. Explain in the report how to calculate autocorrelation and how the code generates pitch contour. Plot the pitch contour of all your 4 recorded speeches with the course title. Do they look similar? Pick a speech file from a classmate and plot the pitch contour. Do they look similar or very different? Here is the [http://web.eecs.utk.edu/~qi/ece315/project/pitchacorr.m](http://web.eecs.utk.edu/~qi/ece315/project/pitchacorr.m) function called by `pitch()`.