This exam is closed book and I will hand out an equation sheet. Unless stated otherwise, all values are given in p.u. and represent balanced three-phase 60Hz systems. BE SURE TO SHOW YOUR WORK CLEARLY AND FULLY.

1. **PV Curves (10 points):** For the two bus system below, sketch the PV curve for a load with a unity power factor and one with a leading power factor of 0.5. Label your curves and indicate values that are easily calculated.
2. **Load frequency control (20 points):** Consider a two area power system. Both areas have two generating units. The governors are adjusted for a 5% droop in area A and 7.5% in area B. The unit capacities and area load are given below:

<table>
<thead>
<tr>
<th>Area A</th>
<th>Area B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1 = 200$ MW</td>
<td>$G_1 = 100$ MW</td>
</tr>
<tr>
<td>$G_2 = 400$ MW</td>
<td>$G_2 = 500$ MW</td>
</tr>
<tr>
<td>Load = 300 MW</td>
<td>Load = 200 MW</td>
</tr>
</tbody>
</table>

Initially, there is a 100 MW load transfer from Area A to Area B and the system is operating at 60 Hz. If the load increases by 150 MW in area A, find:

a) the initial change in system frequency,

b) the area control error (ACE) in both areas assuming the bias is 40 MW/0.1 Hz in both A and B. Indicate whether the ACE signal indicates an increase or a decrease is needed in that area.

Choose a common base, say 500 MW, and calculate frequency response characteristic.

\[ R_i = \frac{500}{200} = 0.25 \text{ p.u.} \]
\[ R_e = \frac{500}{400} = 0.125 \text{ p.u.} \]
\[ B_A = \frac{1}{R_i} + \frac{1}{R_e} = 2.4 \text{ p.u.} \]

\[ R_i = \frac{500}{100} = 5 \text{ p.u.} \]
\[ R_e = 0.075 \text{ p.u.} \]
\[ B_B = \frac{1}{R_i} + \frac{1}{R_e} = 16 \text{ p.u.} \]

\[ \Delta P_A = \frac{150 \text{ MW}}{500 \text{ MW}} = 0.3 \text{ p.u.} = -(B_A + B_B) \Delta f \Rightarrow \Delta f = -0.0075 \text{ p.u.} = -0.45 \text{ Hz} \]

(iii) \[ \text{ACE} = \Delta P_{tie} + B_A \Delta f \quad B = \frac{400}{500} = 0.8 \text{ p.u.} = 48 \text{ p.u.} \]

(iv) \[ \Delta P_A = B_A \Delta f = 0.9 \times 0.3 \text{ p.u.} = 0.27 \text{ p.u.} \]
\[ \Delta P_B = B_B \Delta f = 0.12 \text{ p.u.} \]
\[ \Delta P_{tie} = \Delta P_A - \Delta P_B = 0.12 \text{ p.u.} \]

\[ \text{ACE}_A = -0.12 + 48(-0.0075) = -0.48 \text{ p.u.} \]
\[ \text{ACE}_B = -0.12 + 48(-0.0075) = -0.24 \text{ p.u.} \]

Both give incorrect signal.
3. Power flow solutions – Newton-Raphson algorithm (25 points): For the system below, use the Newton-Raphson algorithm to find the first update for the voltage magnitude and angles at all buses. Assume a flat start.

\[ y_{bus} = \begin{bmatrix} -30 & 15 & 15 \\ 15 & 25 & 10 \\ 15 & 10 & -25 \end{bmatrix} \]

Bus 1
(slack bus) \( \bar{V}_1 = 1.00 \angle 0^0 \)
\( \bar{y}_{23} = -j15 \)

Bus 2
\( P_2 = 2.25 \quad V_2 = 1.00 \)
\( \bar{y}_{23} = -j10 \)

Bus 3
\( P_3 = -3.0 \quad Q_3 = -1.5 \)
(PQ bus)

(i) Find load flow given flat start \( Q_{at \text{ load}} \)

\( f_2 = 0, f_3 = 0, y_3 = 0 \)

(ii) Then mismatches are

\( \Delta P_2 = P_2 - f_2 = 225 \)
\( \Delta P_3 = P_3 - f_3 = -3 \)
\( \Delta Q_3 = Q_3 - y_3 = -1.5 \)

(iii) Find Jacobian at starting point

\[
\bar{J} = \begin{bmatrix}
225/25 & 225/25 & 225/25 \\
225/25 & 225/25 & 225/25 \\
225/25 & 225/25 & 225/25 \\
\end{bmatrix}
\]

(iv) Then \( \bar{J}^{-1} = \begin{bmatrix} 0.048 & 0.019 & 0.019 & 0.048 & 0.048 \\
0.019 & 0.048 & 0.019 & 0.048 & 0.048 \\
0 & 0 & 0 & 0 & 0 \end{bmatrix} \)

(v) \[ \begin{bmatrix} 8_2 \\ 8_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \bar{J}^{-1} \begin{bmatrix} 2.25 \\ -3 \\ -1.5 \end{bmatrix} = \begin{bmatrix} 0.05 \\ -0.10 \\ 0.94 \end{bmatrix} \]

\( \bar{V} = \begin{bmatrix} 1.06 \\ 1.2866 \\ 0.94 \end{bmatrix} \left( \begin{bmatrix} 5.73 \end{bmatrix} \right) \)
4. STATE ESTIMATION (25 POINTS): Consider the 6 bus system with the one-line diagram as sketched below. The buses are numbered clockwise starting from the upper left. Use the DC load flow approximation with all bus voltages equal to 1.0 p.u. to answer the following.

- Select and find the location for a minimal number of measurements that provides for observability. For your selected measurements, generate the measurement matrix $H$.
- Select and place the minimal number of additional measurements needed to detect all errors in your measurement system.
- For this set of measurements that can detect all errors, what is the rank of $HP$? Show clearly your reasoning.
- Extra credit: Assume the generator at bus 3 has tripped off so that no power is injected at bus 3. How many measurements would be needed for observability? Explain your answer.

(i) We need 5 measurements - choose flows 1-2, 2-3, 3-4, 4-5, 5-6 and let bus 1 be reference. Flows are then found from $\sum_j (S_i - S_j)$ so

\[
H = \begin{bmatrix}
-10 & 0 & 0 & 0 & 0 \\
10 & -10 & 0 & 0 & 0 \\
0 & 5 & -5 & 0 & 0 \\
0 & 0 & 10 & -10 & 0 \\
0 & 0 & 0 & 10 & -10
\end{bmatrix}
\]

(ii) Easiest way to ensure error detection is to add the flow from 6-1

(iii) Rank of $H = 5$ (observability gives maximum rank)

(iv) Then you can think of injection at 3 as a pseudo-measurement (value known although not measured). Thus, only measurements would be needed for observability.
5. Short Answer (20 points)

(a) In the power system security framework, what is meant by a ‘n-1’ secure? (Be specific.)

Following any single contingency (outage), all operating requirements (voltage, loading, etc.) are still met.

(b) What are the known variables (i.e., specified) at each bus type in the power flow equations?

- Slack: \( V_c \)
- Generator: \( P_c, V_c \)
- Load: \( Q_c \)

(c) What are the unknown variables (i.e., not specified) at each bus type in the power flow equations?

- Slack: \( P_c, Q_c \)
- Generator: \( V_c \)
- Load: \( V_c \)

(d) Consider a system with two areas, say area A and area B. Area A and Area B have the bias set at exactly the natural governor response for its area. If Area B experiences a 100 MW increase in load, then what will be the Area Control Error (ACE) for Area A and for Area B:

- Area B: \(-100\) MW
- Area A: \(0\) MW