1. Swing equations (20 points): A generator is connected to three parallel lossless transmission lines as sketched below. A fault occurs at the location indicated in the diagram. When the fault is cleared this line is taken out of service. Answer the following:
(a) Find the swing equation for the pre-fault, during fault and post-fault conditions.
(b) Sketch the power angle curves labeling them as pre-fault, during fault or post-fault as appropriate.
2. **Stability considerations with swing equation (15 points):** Assume the same system as in problem one again using the classical model. At some time after the fault has cleared, the rotor angle is found to be $\delta = 2.1$ rad. Find the electrical power output and the rotor acceleration. Can you determine if the system is stable with the information given? Why or why not?

- **Post-fault**
  \[ P_e = 2.18 \sin \delta \quad \delta = 2.1 \text{ rad} \quad P_e = 1.88 \]
  \[ \dot{\delta}(t) = 200(0.6 - P_e) = -80.6 \text{ rad/s}^2 \]

- Stability – for this case one could determine stability
  i) Note in the worst case the fault had just cleared when $\delta = 2.1$ rad (120.3°) or just consider $\delta = 0.6$
  ii) Also even if the fault never clears the machine will start to decelerate so just check

\[ A_a = \int (0.6 - 1.412 \sin \delta) \, d\delta \]

\[ A_d = \int (1.412 \sin \delta - 0.6) \, d\delta \]

![Diagram showing the swing equation and stability analysis](image-url)
3. **Synchronous machine modeling – algebraic equations (25 points):** Continuing with the system in problem one but now you are to consider the d-q axis model (model 2 - neglecting damper winding transients and armature resistance). Instead of specifying \( E_q \), the terminal voltage is given as \( E_v = 1.05 \). In addition, you are given the parameter values of \( X_d^r = 0.3 \) and \( X_q = 0.5 \). Answer the following:

a) Find the steady-state voltage behind the reactance, \( E_q \), and steady-state rotor angle, \( \delta \).

b) Sketch a phasor diagram that shows the circuit quantities in this two-axis model. Specifically, your phasor diagram should show the following vector quantities: \( E_d, E_q, \delta, E_v, \) \( \delta_d, \delta_q, X_d, X_\delta, X_q \), and \( E_B \) (infinite bus voltage). You can use two diagrams to show this if it makes the diagram less crowded. Be sure to clearly label your diagram(s).

\[ P_e = \frac{E_t E_v}{X_d^r} \sin \delta = 3.94 \sin 8^\circ \]
\[ x_c = 0.811 / 0.811 / 0.8 = 0.27 \]
\[ V_e = 0.6 \Rightarrow S_0 = 8.76^\circ \]
\[ I_e = \frac{E_c - E_v}{x_c} = 0.62 \angle -132.7^\circ \]

\[ (ii) \text{ Find terminal current: let } E_v \text{ be reference} \]

\[ i_d = I_v \cos (90 - \psi) \quad \psi = \delta + 132.7^\circ \]

\[ = 0.36 \]

\[ (iii) \quad E_q' = E_v + (X_q' - X_q) i_d = 1.12 \]

![Diagram showing d-q axis model](image-url)
4. Critical clearing time (20 points): A generator is connected to a transmission line as sketched below. A fault occurs at the location indicated in the diagram (next to the bus). The fault is temporary so that when it clears the line remains in service. Find the critical clearing angle and critical clearing time to ensure stability.

\[ E_q = 1.2 \angle \delta \]
\[ P_m = 0.4 \]
\[ H = 3.0 \text{ s} \]

**i) Pre-fault**
\[ P_e = \frac{E_q E_b \sin \delta}{X_{eq}} \quad x_{eq} = 0.6 \]

**ii) Post-fault**
\[ P_e = 0.5 \sin \delta \quad \text{with} \quad P_m = 0.4 \]
\[ \delta_0 = 53.13^\circ \]
\[ \delta_{max} = 126.87^\circ \]

**During fault**
\[ P_e = 0 \]

\[ s_{cc} = \int (P_m - P_e) \, ds = \int_0^{s_{max}} (0.4 \, ds - 0.4 \, (6\alpha - 60)) \]

\[ A_a = \int s_{cc} (P_m - P_e) \, ds = \int_0^{s_{max}} (0.5 \sin \delta + 0.4) \, ds = -0.5 \cos \delta \bigg|_{s_{cc}}^{s_{max}} - 0.4 \bigg|_{s_{cc}}^{s_{max}} \]

\[ A_d = -\int s_{cc} (P_m - P_e) \, ds = \int_0^{s_{max}} (0.5 \sin \delta - 0.4) \, ds = -0.5 \cos \delta \bigg|_{s_{cc}}^{s_{max}} - 0.4 \bigg|_{s_{cc}}^{s_{max}} \]

\[ A_a = A_d \implies 0 = 0.5 \cos s_{cc} - 0.5 \cos \delta_{max} - 0.4 (s_{max} - 60) \]

\[ \cos s_{cc} = 0.430 \quad s_{cc} = 64.56^\circ \]

**iii) Then find**
\[ S(t) = \frac{11}{H} P_m = 0.11 \quad S(t) = 477e^2 + 4^\circ \]
\[ t_{cc} = 0.126 \text{ s} \]
5. Short answer (20 points):

a) What is (are) the input(s) to the power system stabilizer? What is the function of the stabilizer in generator control? (Be specific.)
- Frequency deviation is input
- PSS provides additional damping torque to reduce oscillations

(b) What method of analysis is appropriate for small disturbance stability?
- Linear (eigenvalue analysis)

(c) What method of analysis is appropriate for transient stability?
- Non-linear - equal area criterion

d) Consider a generator modeled by the swing equation tied to an infinite bus. Explain in words why the steady-state rotor angle cannot exceed 90 degrees.

Past 90° degrees, the rotor will not want to return to the same position following any perturbation. That is, a slight increase in angle will result in acceleration, and a slight decrease in angle will result in deceleration.