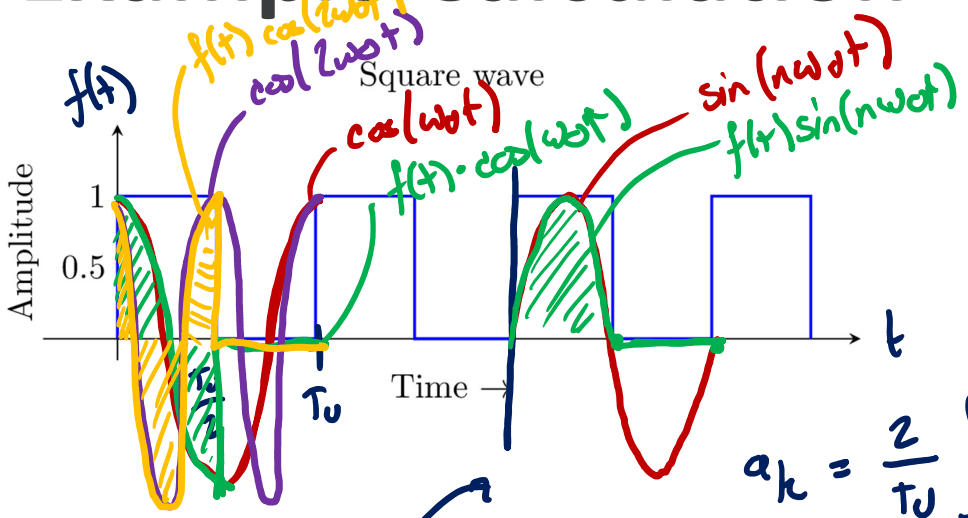
Example Calculation



find Fourier series coefficients

$$a_0 = \frac{1}{T_0}\int_0^{T_0} f(t)\,dt$$

$$= \frac{1}{T_0}\left(1 \cdot \frac{T_0}{2}\right) = \frac{1}{2}$$

$$a_k = \frac{2}{T_0}\int_0^{T_0} f(t)\cos(k\omega_0 t)\,dt$$

$= \phi$ for $k = 1$, $= \phi$ for $k = 2 \ldots$

$$\boxed{a_k = \phi \quad \text{for all } k} \quad (\forall k)$$

$T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$

$$b_k = \frac{2}{T_0}\int_0^{T_0} f(t)\sin(k\omega_0 t)\,dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2} 1 \cdot \sin(k\omega_0 t)\,dt = \frac{2}{T_0}\left[-\frac{1}{k\omega_0}\cos(k\omega_0 t)\right]\Big|_0^{T_0/2}$$

$$= \frac{\omega_0}{\pi}\left[-\frac{1}{k\omega_0}\cos(k\pi) + \frac{1}{k\omega_0}(1)\right]$$

$$= b_k = \begin{cases} \frac{2}{k\pi}, & k \text{ odd} \\ \phi, & k \text{ even} \end{cases}$$

Symmetry in Fourier Series

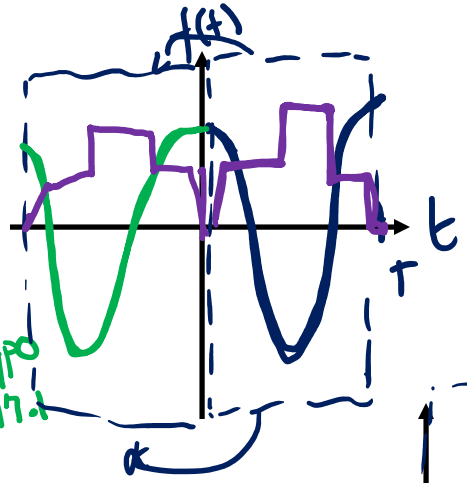
Even functions

$f(t) = f(-t)$

$b_k = 0$

Note: Book has typo in Table 17.1

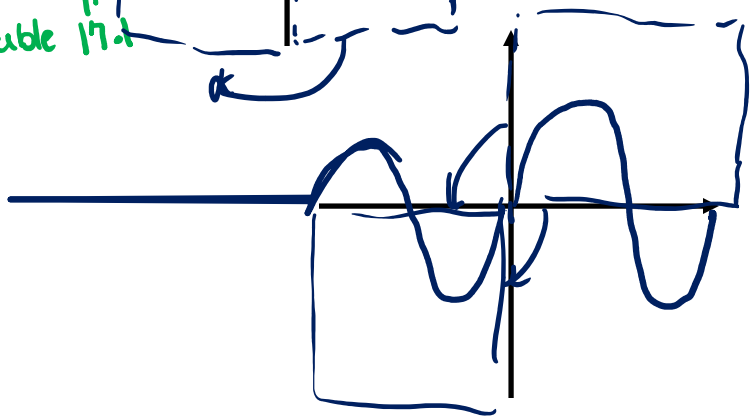
(with DC component ignored)



Odd functions

$f(t) = -f(-t)$

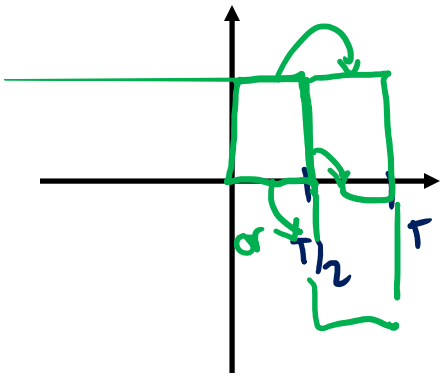
$a_k = 0$



Half-wave symmetric functions

$f(t \pm T_0/2) = f(t)$ or $-f(t)$

$a_k,\ b_k = 0$ for even k



Application: Digital Communication