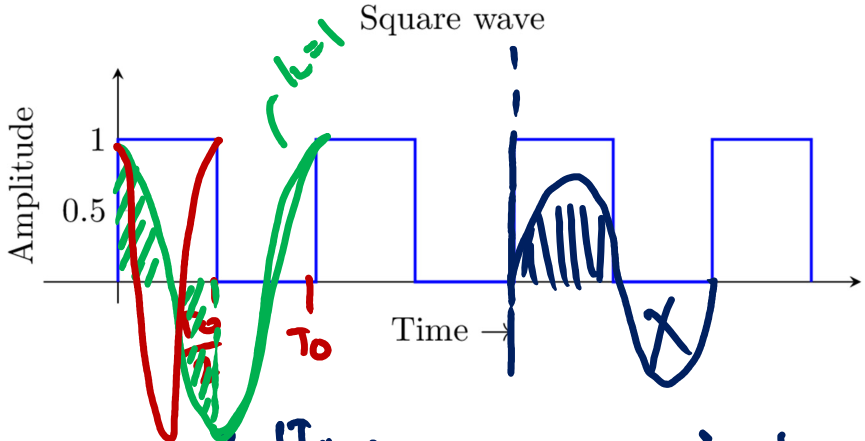
Example Calculation



$\omega_0 = \frac{2\pi}{T_0}$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} f(t)\,dt = 1/2$$

$$= \frac{1}{T_0}\int_0^{T_0/2} 1 \cdot dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} \emptyset\, dt$$

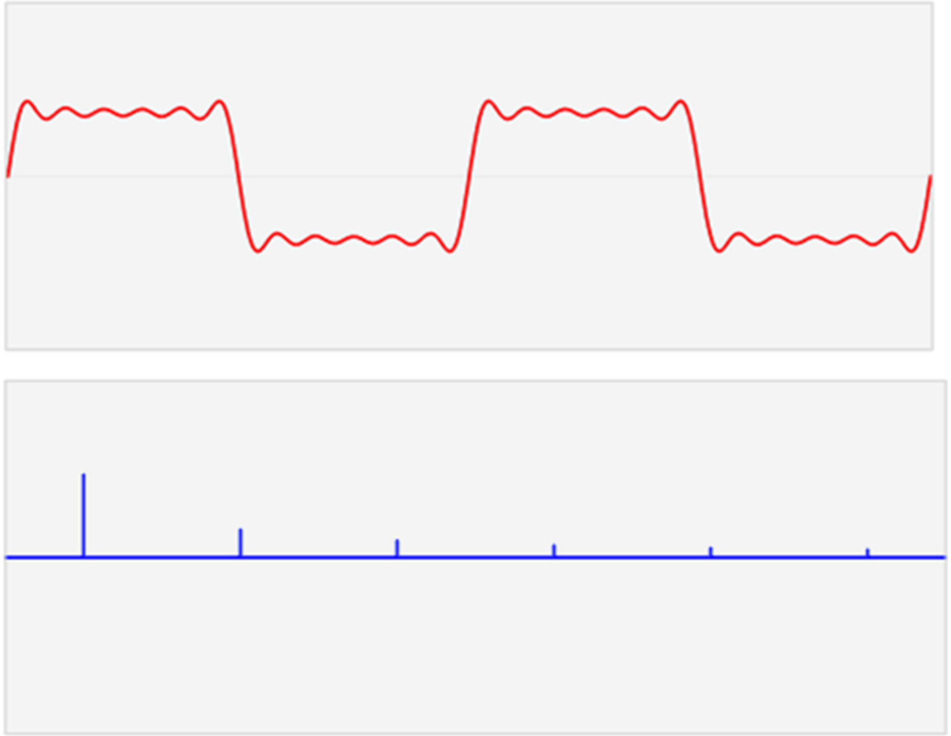
$$a_k = \frac{2}{T_0}\int_0^{T_0} f(t)\cos(k\omega_0 t)\,dt = \emptyset \text{ for all } k$$

$$b_k = \frac{2}{T_0}\int_0^{T_0} f(t)\sin(k\omega_0 t)\,dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2} (1)\sin(k\omega_0 t)\,dt$$

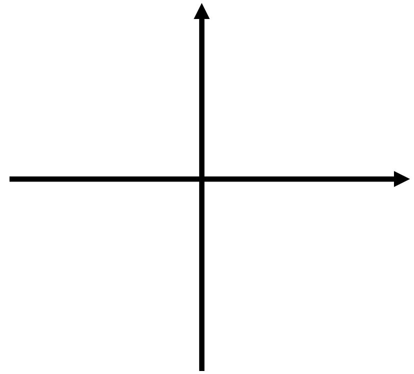
Fourier Series & Frequency Domain

Input Spectrum



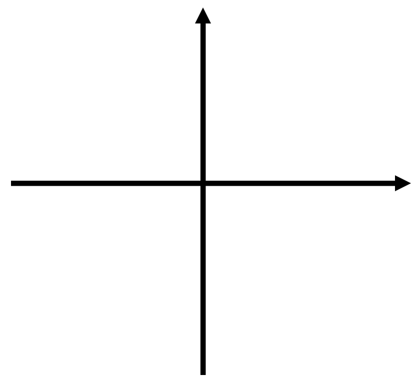
Symmetry in Fourier Series

Even functions



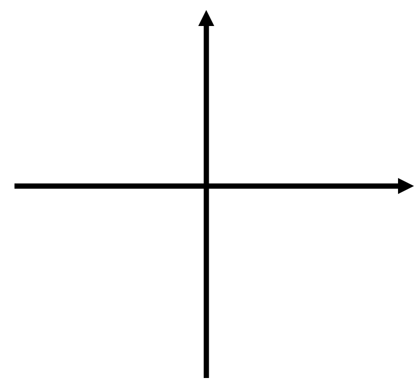
$$b_k = 0$$

Odd functions



$$a_k = 0$$

Half-wave symmetric functions



$a_k,\ b_k = 0$ for even k

Fourier Series Representation

Assume we have some function $f(t)$ which is periodic with period $T_0 = \frac{2\pi}{\omega_0}$

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$f(t)$ can be expressed this way if

1. $f(t)$ is single-valued
2. $\int_{t_0}^{t_0+T_0} |f(t)|\, dt$ exists
3. $f(t)$ had finite discontinuities and max/min per period

$$a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \cos(k\omega_0 t)\, dt$$

$$b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \sin(k\omega_0 t)\, dt$$

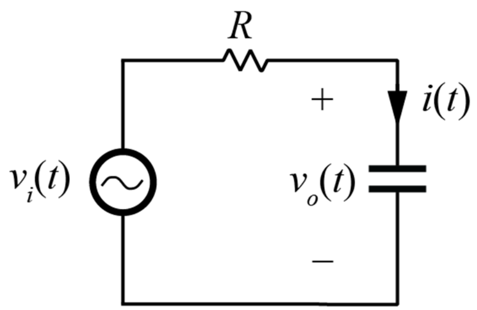
Alternate forms

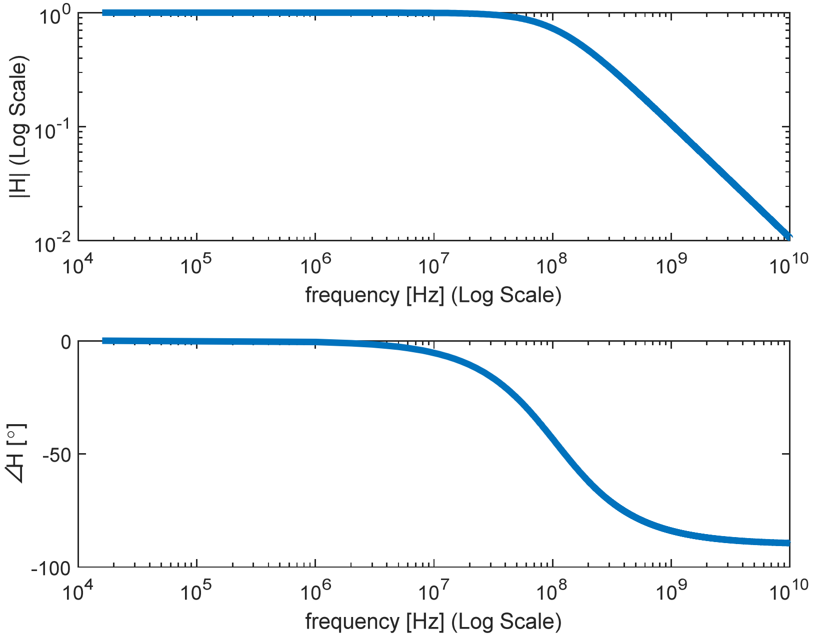
$$f(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \varphi_k) \quad \begin{cases} A_k = \sqrt{{a_k}^2 + {b_k}^2} \\ \varphi_k = \tan^{-1}\left(\frac{b_k}{a_k}\right) \end{cases}$$

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad \begin{cases} c_k = \frac{1}{2}(a_k - jb_k) \\ c_{-k} = \frac{1}{2}(a_k + jb_k) \\ c_0 = a_0 \end{cases}$$

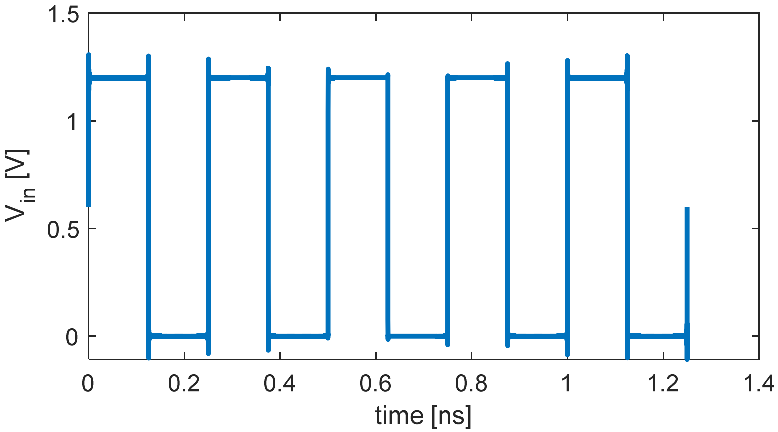
$$c_k = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} f(t) e^{-jk\omega_0 t} dt$$

Application: Digital Communication

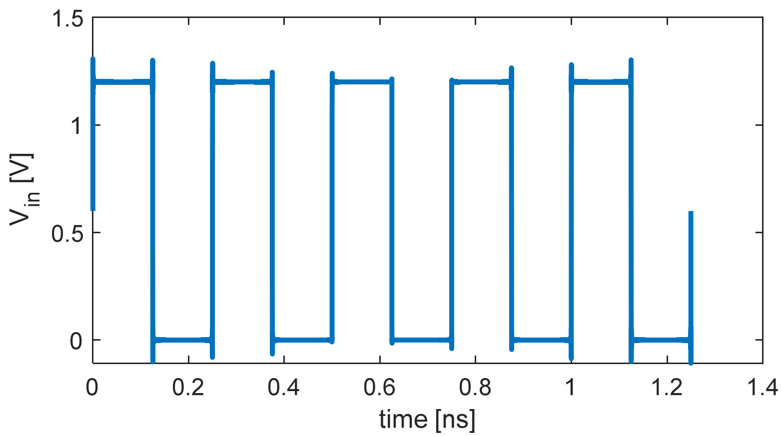


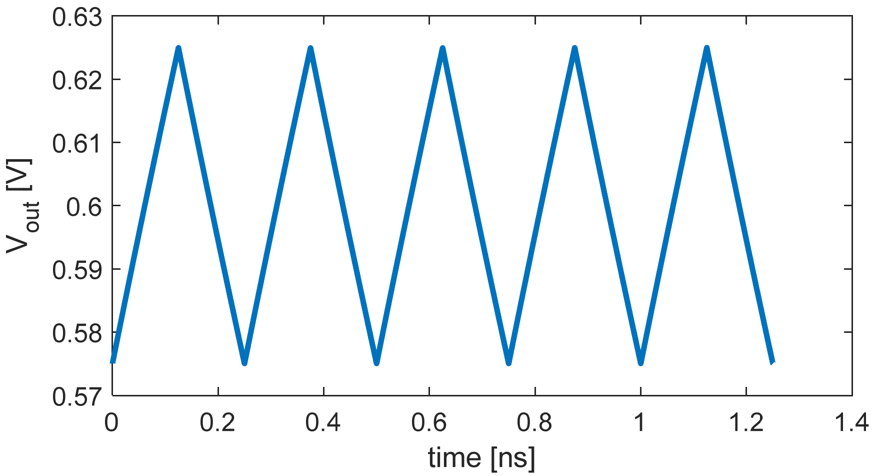


Applying Superposition

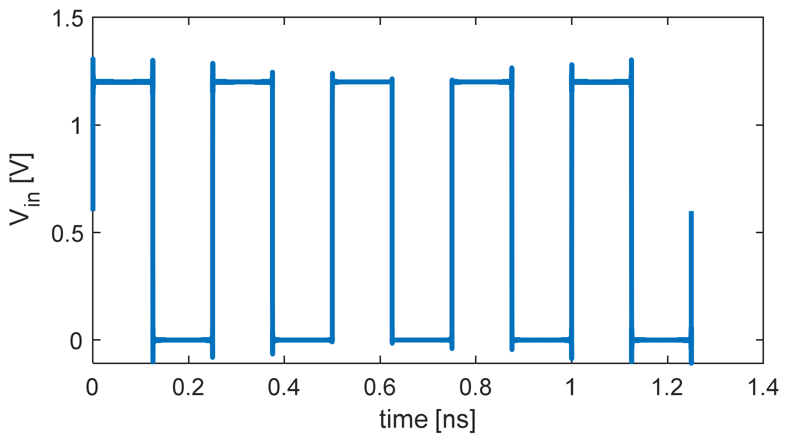


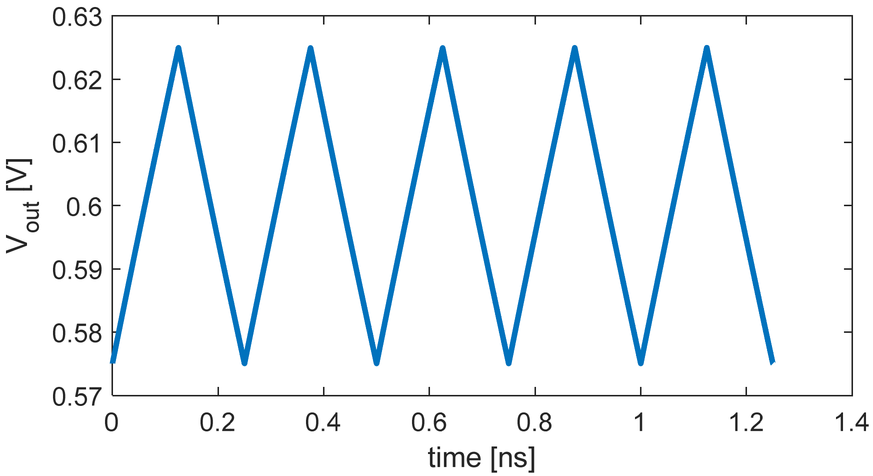
Calculated Output Voltage

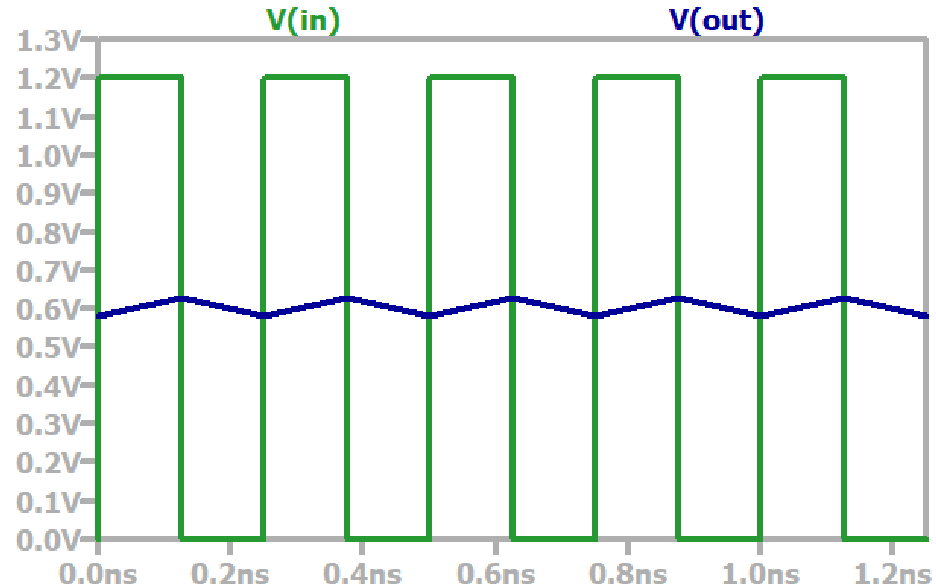


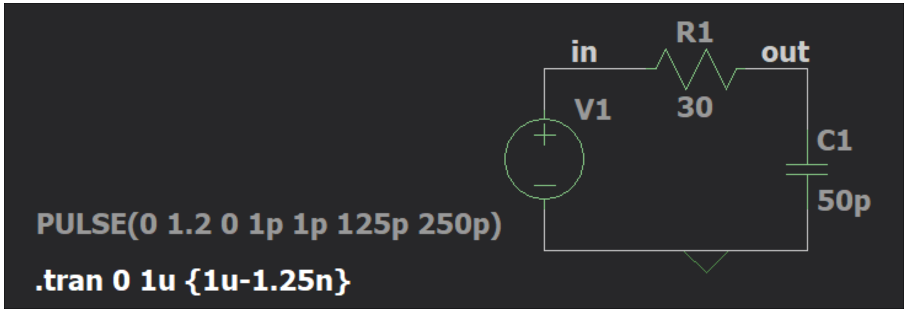


Simulation Verification









Frequency Domain Interpretation