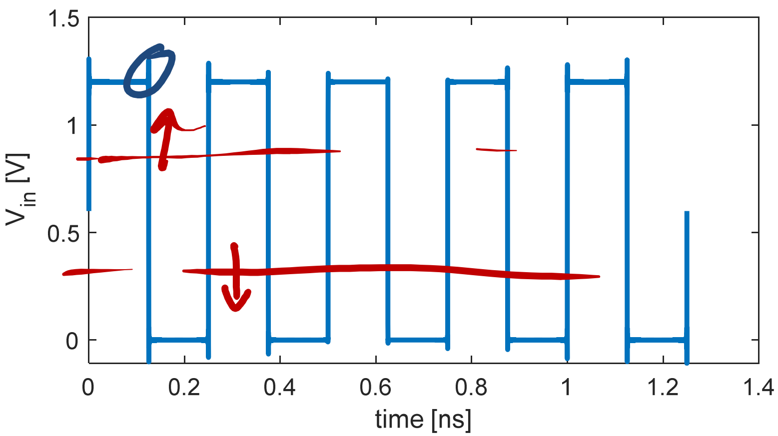
Applying Superposition



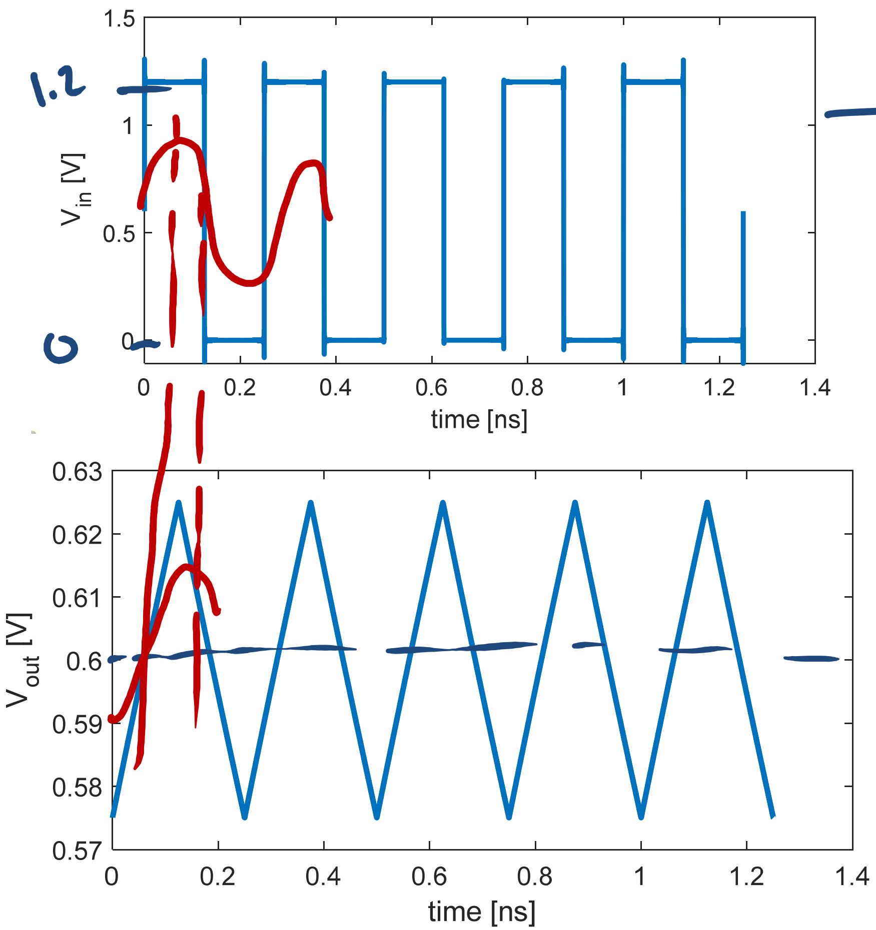
$$f(t) \approx \underline{a_0} + \sum_{k=1}^{\infty} \underline{b_k} \sin(k\omega_0 t)$$

← first $1 \le k \le 1000$

$$H(jk\omega_0) = \frac{1}{jk\omega_0 RC + 1}$$

$$v_o(t) \approx a_0 |H(j0)| + \sum_{k=1}^{1000} |H(jk\omega_0)|\, b_k \sin[k\omega_0 t + \angle H(jk\omega_0)]$$

Calculated Output Voltage



b_k

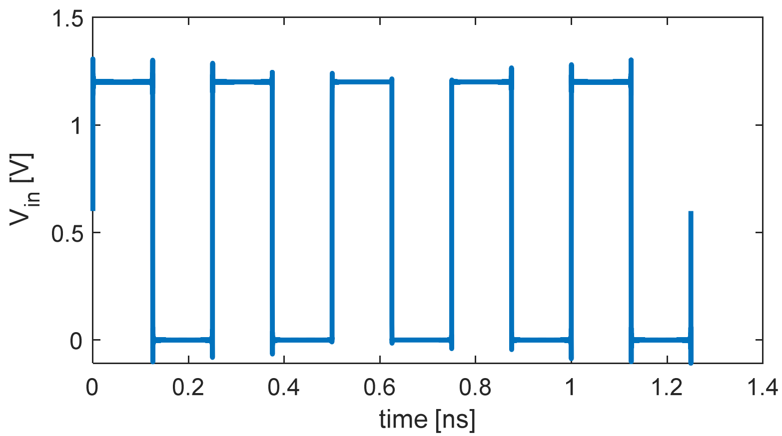
multiply by $|H(jk\omega_0)|$

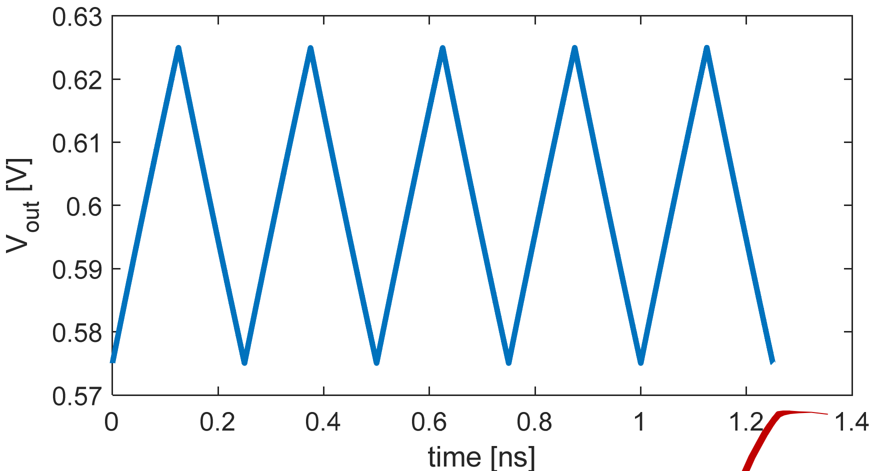
phase shift by $\angle H(jk\omega_0)$

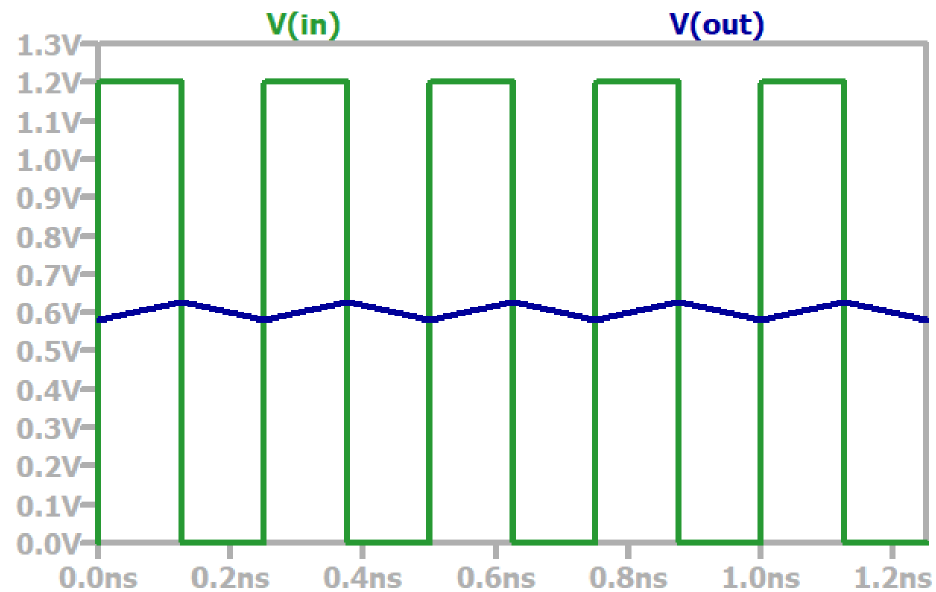
add back up

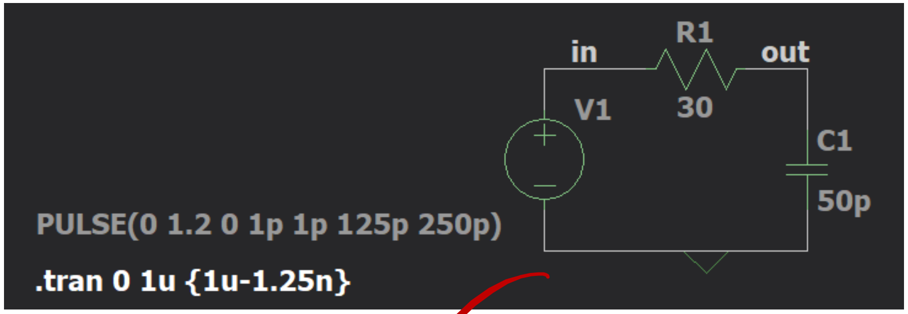
$k = 1 \ldots 1000$

Simulation Verification

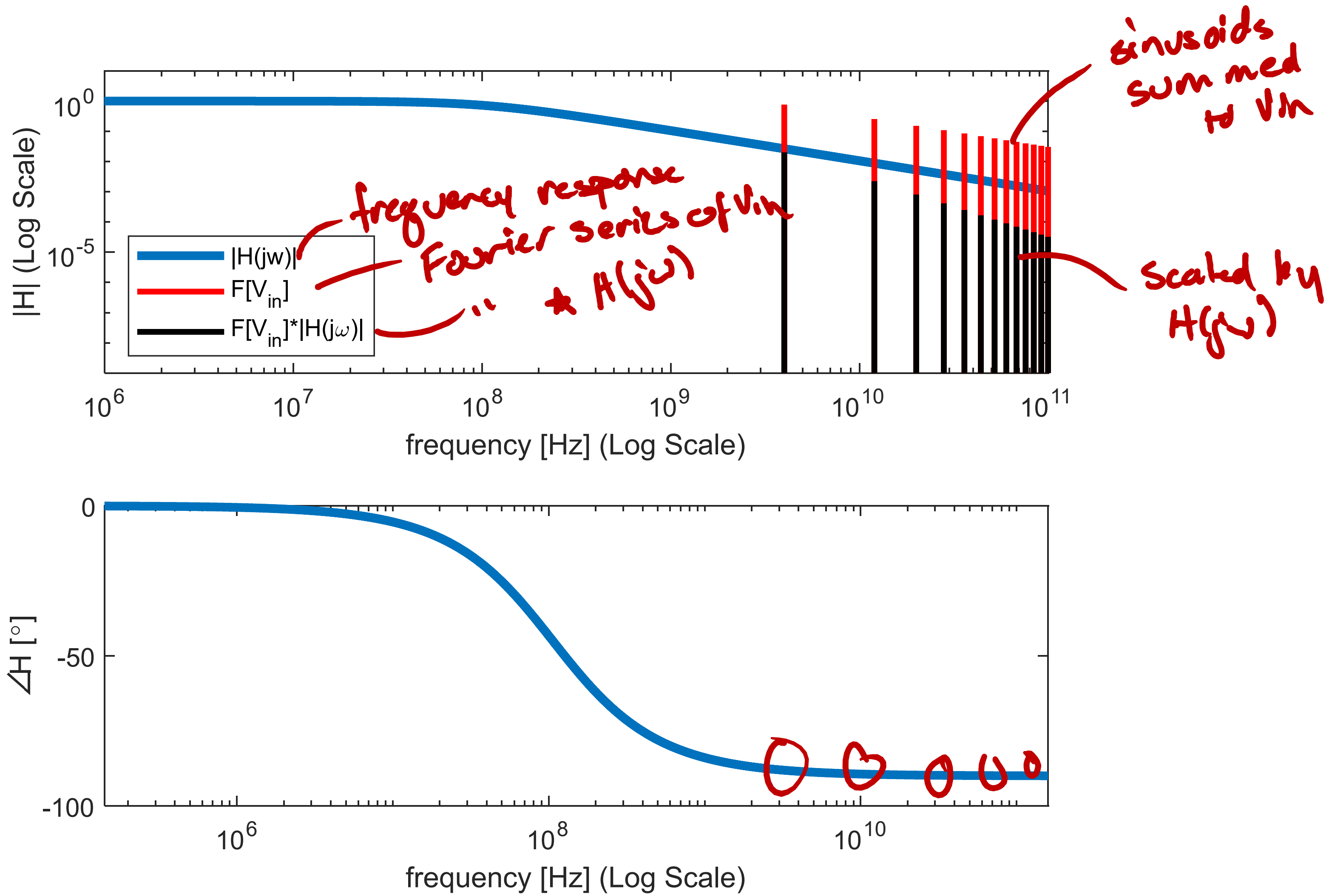


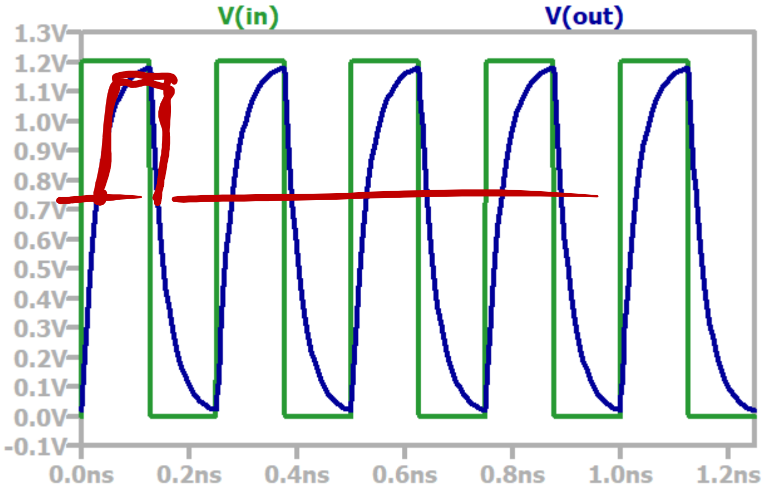




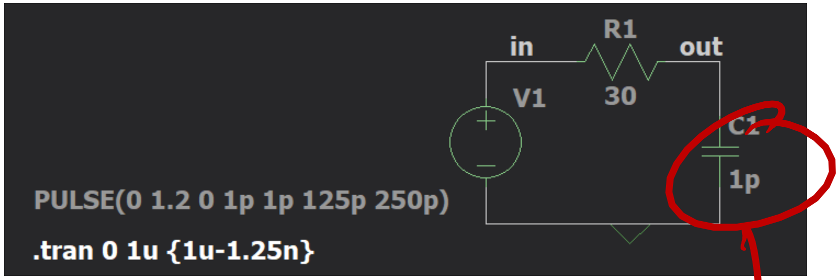


Frequency Domain Interpretation





V(in)
V(out)
1.3V
1.2V
1.1V
1.0V
0.9V
0.8V
0.7V
0.6V
0.5V
0.4V
0.3V
0.2V
0.1V
0.0V
-0.1V
0.0ns
0.2ns
0.4ns
0.6ns
0.8ns
1.0ns
1.2ns

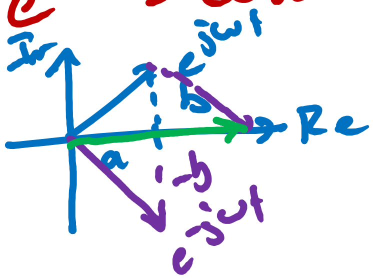


in
R1
out
V1
30
C1
1p
PULSE(0 1.2 0 1p 1p 125p 250p)
.tran 0 1u {1u-1.25n}

50pF → 1pF

Complex Form of Fourier Series

Euler: $e^{j\omega t} = \cos\omega t + j\sin\omega t$



a+bj a-bj

$$\left\{ \begin{aligned} \cos(\omega t) &= \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \\ \sin(\omega t) &= \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right) \end{aligned} \right.$$

Fourier Series

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \frac{1}{2}\left(e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right) + \frac{b_k}{2j}\left(e^{jk\omega_0 t} - e^{-jk\omega_0 t}\right)$$

$$f(t) = a_0 + \sum_{k=1}^{\infty} e^{jk\omega_0 t}\underbrace{\left(\frac{a_k - b_k j}{2}\right)}_{c_k \text{ for } k>0} + e^{-jk\omega_0 t}\underbrace{\left(\frac{a_k + b_k j}{2}\right)}_{c_k \text{ for } k<0}$$

$$f(t) = a_0 + \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

Fourier Series Representation

Assume we have some function $f(t)$ which is periodic with period $T_0 = \frac{2\pi}{\omega_0}$

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$f(t)$ can be expressed this way if

1. $f(t)$ is single-valued
2. $\int_{t_0}^{t_0+T_0} |f(t)|\, dt$ exists
3. $f(t)$ had finite discontinuities and max/min per period

$$\boxed{a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \cos(k\omega_0 t)\, dt}$$

$$\boxed{b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \sin(k\omega_0 t)\, dt}$$

Alternate forms

$$f(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \varphi_k) \quad \begin{cases} A_k = \sqrt{{a_k}^2 + {b_k}^2} \\ \varphi_k = \tan^{-1}\left(\frac{b_k}{a_k}\right) \end{cases}$$

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad \begin{cases} c_k = \frac{1}{2}(a_k - jb_k) \\ c_{-k} = \frac{1}{2}(a_k + jb_k) \\ c_0 = a_0 \end{cases}$$

$$\boxed{c_k = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} f(t) e^{-jk\omega_0 t}\, dt}$$

Input Spectrum